

Mathematical Model of a Two-Factor Transportation Problem With Weighting Coefficients

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Abstract—The article presents a two-factor transportation problem with weighting coefficients, which is formulated as a problem of finding the most profitable plan for the transportation of homogeneous cargo from points of departure to points of consumption in the conditions of two factors and the presence of weighting coefficients. The task is to develop a mathematical model of this problem. It is proposed to use the method of reducing the initial problem to the form of a classical transportation problem for the use of any of the existing solution algorithms in the future. The content of the developed step-by-step algorithm for reducing a two-factor transportation problem with weighting coefficients to the form of a classical transportation problem is presented, and the corresponding general scheme is given. The conclusions are drawn and the advantages of developing a software product for solving a two-factor transportation problem with weighting coefficients using the developed method are argued.

Index Terms—transportation problem, mathematical model, tariff matrix, factor, weighting coefficient, transportation plan

I. INTRODUCTION

In today's world, there are increasingly complex transportation situations where several factors need to be taken into account simultaneously, such as transportation costs, delivery times, environmental performance, etc. Such problems can arise in various fields such as logistics, supply chain, transportation planning, etc. Solving a two-factor transportation

problem with weighting coefficients is of practical importance for businesses and organizations, as it allows them to find the optimal allocation of resources that meets their needs and takes into account the importance of different factors. Effective algorithms for solving such problems can help reduce costs, improve logistics processes, and increase the competitiveness of enterprises.

In the transport logistics system, one of the important issues to be studied is the problem of decision-making [1]. Since this system is characterized by the problem of finding the optimal route, taking into account the need to reduce time, financial, and other transportation costs [2]. This problem became even more important during the military invasion of Ukraine [3].

II. THEORETICAL BACKGROUND

Genetic algorithms were used by Burduk and Musiał [4] to tackle the optimization problem. They discussed genetic algorithms' characteristics and capacity to address computational issues. The authors employed the MATLAB program to address the issue at hand.

Prifti et al. [5] studied a linear programming problem in an Albanian corporation, which aimed to minimize transportation expenses. The problem was solved using three methods: the North West Corner Method, the Least Cost Method, and

Vogel's Approximation Method. The authors found that Vogel's Approximation Method produced the best results by considering the capacity provided by the two manufacturing facilities and the demand from the nine geographical locations.

Pop et al. [6] studied a transportation problem in a supply chain that spans from producers to consumers via distribution hubs. They proposed a hybrid technique that combines a steady-state genetic algorithm with a localized exploration process to solve the optimization challenge.

Chyzhmotria et al. [7] proposed an algorithm to transform input data for a dual-factor multivariate transportation issue with weighting coefficients into a format suitable for solving the conventional transportation problem using established techniques.

Pandian and Natarajan [8] proposed a straightforward and accessible approach to address a dual-phase transportation quandary using the zero point method. The approach yields multiple solutions to the two-stage transportation problem and assists decision-makers in logistics-related challenges by facilitating their decision-making process.

Garajová and Rada [9] proposed a model for the interval transportation problem that aims to discover an optimal delivery plan with minimal expenses for conveying a specific product from supply centers to customers. The model employs an interval programming technique to depict the uncertainty arising from imprecise supply and demand figures and inaccurate transportation costs.

Cosma et al. [10] proposed a hybrid genetic algorithm that integrates a linear programming optimization task to address a particular instance of a dual-phase fixed-charge transportation issue.

Deng et al. [11] proposed a hybrid transportation problem model that incorporates both centralized and decentralized transportation approaches. The model aims to minimize overall expenses by optimizing routes, considering cost reductions, road section capacity, congestion penalties, pre-delivery expenses, and strict time windows. To tackle this model, they proposed a hybrid genetic search algorithm.

Islam et al. [12] studied vehicle scheduling within a supply chain network involving a third-party logistics company. They employed a metaheuristic algorithm named Chemical Reaction Optimization (CRO) to address this challenge.

Transportation problem is a complex decision-making challenge with uncertainty. The goal is to find the best way to transport cargo, considering multiple objectives such as cost, time, labor, and damage, while following route capacity limits. Gupta et al. [13] proposed a multi-objective optimization model for a comprehensive stochastic transportation issue that aims to identify the most optimal transportation strategy for achieving the maximum cargo volume while adhering to specific capacity limitations for each route.

The analysis of scientific studies has led to the conclusion that this type of problem is not described in the scientific literature, and therefore, there are currently no analogues of the algorithm for solving problems of this type.

The study of a two-factor transportation problem with weighting coefficients is important for the research and practice community aimed at finding new approaches, methods, and algorithms in the field of transportation process optimization. The *purpose* of this study is to develop a mathematical model, method, and algorithm for solving this problem to increase the efficiency of management decision-making, optimize resource allocation, and improve the overall efficiency of the enterprise.

III. RESULTS

There are two main types of transportation problems: one based on the cost criterion, which aims to find a transportation plan with the lowest transportation cost, and another based on the time criterion, which prioritizes the time of cargo transportation.

In addition to the classic transportation problems, there are several other variations, such as a transportation problem with prohibitions and a two-stage transportation problem [14].

The two-factor transportation problem is an extension of the classical transportation problem, where it is necessary to solve the optimization problem of allocating resources (e.g., goods or services) from suppliers to recipients at minimum cost. However, the two-factor transportation problem takes into account additional factors or constraints that have weighting coefficients.

Weighting coefficients are introduced to take into account the different weights or priorities of different factors in the resource allocation process. For example, coefficients can be set to reflect transportation costs, storage costs, delivery times, cargo safety, integrity, environmental pollution, or other factors that affect allocation efficiency. It should be noted that some of the factors may require maximizing the results.

The existing methods for solving the classical transport problem cannot be applied to this type of problem, so the authors propose a separate method and algorithm for solving it.

Suppose that there are a_i (for point A_i) units of a certain homogeneous cargo at m points of origin A_1, A_2, \dots, A_m . This cargo must be delivered to n consumers B_1, B_2, \dots, B_n in the number of b_j units (for consumer B_j). The tariffs c_{ij} for transportation of a unit of cargo from the i -th point of departure to the j -th point of consumption are known according to the first factor. Also known are the tariffs t_{ij} of transportation of a unit of cargo from the i -th point of departure to the j -th point of consumption according to the second factor.

The tariffs c_{ij} make up the tariff matrix C for the first factor:

$$C = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \dots & \dots & \dots & \dots \\ c_{m1} & c_{m2} & \dots & c_{mn} \end{pmatrix} \quad (1)$$

The tariffs t_{ij} make up the tariff matrix T for the second factor:

$$T = \begin{pmatrix} t_{11} & t_{12} & \dots & t_{1n} \\ t_{21} & t_{22} & \dots & t_{2n} \\ \dots & \dots & \dots & \dots \\ t_{m1} & t_{m2} & \dots & t_{mn} \end{pmatrix} \quad (2)$$

The weighting coefficients for each of the two factors for each supplier and each consumer make adjustments to the mathematical model.

Suppose that at the supply point A_i the weighting coefficients are k_1^i and k_2^i for each of the factors, respectively, and $k_1^i + k_2^i = 1$. Similarly, at consumption point B_j , the weighting coefficients will be k_1^j and k_2^j for each of the factors, respectively, with $k_1^j + k_2^j = 1$.

For each supplier-consumer pair, we will take the average weighting coefficient for each of the factors for further use:

$$k_1^{ij} = \frac{k_1^i + k_1^j}{2}, \quad k_2^{ij} = \frac{k_2^i + k_2^j}{2} \quad (3)$$

The following transformations will allow us to bring the problem to the form of a classical transportation problem, which will allow us to use any existing solution algorithm in the future.

To equalize the mutual influence of the factors on each other, the next step is to scale the numerical ranges for the tariff matrices C and T . To do this, we first need to find the maximum value for each of the tariff matrices:

$$c_{max} = \max(c_{ij}), \quad t_{max} = \max(t_{ij}) \quad (4)$$

Next, increase all c_{ij} values of the tariff matrix C by t_{max} times, and all t_{ij} values of the tariff matrix T by c_{max} times:

$$c'_{ij} = c_{ij} \cdot t_{max}, \quad t'_{ij} = t_{ij} \cdot c_{max} \quad (5)$$

We will summarize the tariffs of the two factors, taking into account the weighting coefficients, using the formula:

$$u_{ij} = c'_{ij} \cdot k_1^{ij} + t'_{ij} \cdot k_2^{ij} \quad (6)$$

The tariffs u_{ij} will make up the tariff matrix U of the reduced problem:

$$U = \begin{pmatrix} u_{11} & u_{12} & \dots & u_{1n} \\ u_{21} & u_{22} & \dots & u_{2n} \\ \dots & \dots & \dots & \dots \\ u_{m1} & u_{m2} & \dots & u_{mn} \end{pmatrix} \quad (7)$$

Let x_{ij} be the number of units of a product that is transported from point A_i to point B_j . Then the mathematical model of the problem is as follows:

$$F(X) = \sum_{i=1}^m \sum_{j=1}^n u_{ij} x_{ij} \rightarrow \min \quad (8)$$

under the conditions of

$$\sum_{i=1}^m x_{ij} = b_j \quad (j = \overline{1, n}) \quad (9)$$

$$\sum_{j=1}^n x_{ij} = a_i \quad (i = \overline{1, m}) \quad (10)$$

$$x_{ij} \geq 0 \quad (i = \overline{1, m}; j = \overline{1, n}) \quad (11)$$

Equality (9) means that all consumption points have received the goods in full. Equality (10) means that all points of departure are empty. Condition (11) excludes transportation in the opposite direction.

A transportation problem plan is a matrix $X = (x_{ij})$ ($i = \overline{1, m}; j = \overline{1, n}$) for any nonnegative solution of the system of linear equations (9) and (10).

An optimal plan for a two-factor transportation problem with weighting coefficients is a plan $X^* = (x_{ji})$ ($i = \overline{1, m}; j = \overline{1, n}$), at which function (8) takes on a minimum value.

For factors that, by their nature, require maximizing the results, at the beginning of the algorithm, it will be necessary to switch to the minimization task by rotating the values of the corresponding tariff matrix: $c''_{ij} = 1/c_{ij}$, $t''_{ij} = 1/t_{ij}$.

The optimal cargo transportation plan for a two-factor transportation problem with weighting coefficients $X^* = (x_{ji})$ ($i = \overline{1, m}; j = \overline{1, n}$) allows obtaining the values of two objective functions for each of the factors:

$$F_1(X) = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}, \quad F_2(X) = \sum_{i=1}^m \sum_{j=1}^n t_{ij} x_{ij} \quad (12)$$

The value of the objective functions will be the total cost of cargo transportation for each of the factors in the corresponding units of measurement. The general scheme of the developed method of reducing the two-factor transportation problem with weighting coefficients to the form of a classical transportation problem is shown in Fig. 1.

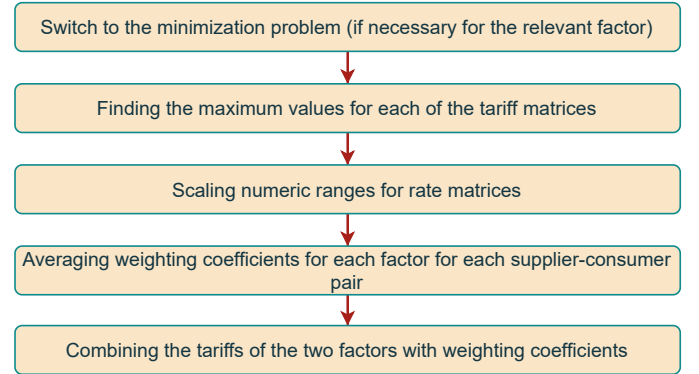


Fig. 1. General scheme of the method of reducing a two-factor transportation problem with weighting coefficients to the form of a classical transportation problem.

In terms of parallelisation of calculations, we obtain the algorithm shown in Fig. 2.

As mentioned above, this method cannot be analysed in comparison with other methods, since the latter cannot be applied to this type of problem. Therefore, at this stage, we can only check the proposed algorithm and its results for adequacy.

The proposed model's and developed algorithm adequacy can be analyzed using the following example. Suppose there

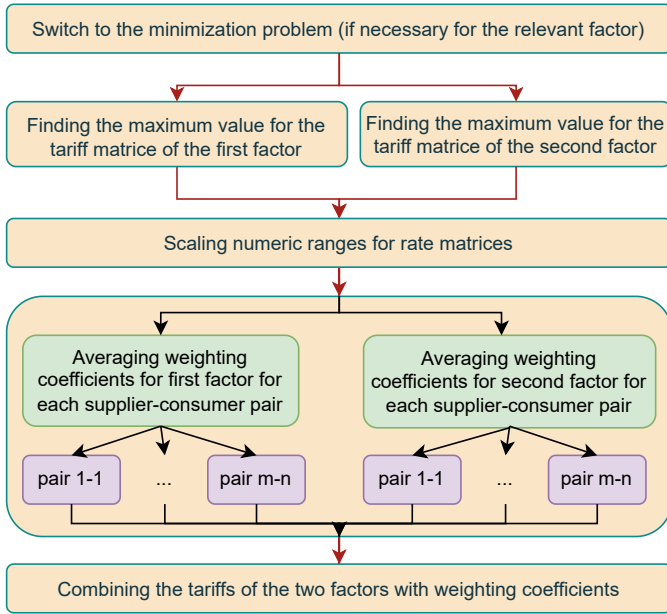


Fig. 2. General scheme of the method of reducing a two-factor transportation problem with weighting coefficients to the form of a classical transportation problem in terms of parallelisation.

are 4 points of departure and 4 points of consumption. The stocks of goods at the points of departure are 5050, 2050, 1250, and 1300 units, respectively. Consumers need 2400, 3500, 1250, and 2500 units, respectively.

The tariff matrices for the first and second factors are as follows (see table I).

TABLE I
THE TARIFF MATRICES FOR THE FIRST AND SECOND FACTORS

	Factor 1				Factor 2			
	B1	B2	B3	B4	B1	B2	B3	B4
A1	140	20	30	50	10	5	12	7
A2	55	15	65	25	18	20	12	3
A3	60	20	35	110	1	7	6	15
A4	75	80	90	55	5	3	5	12

The weighting coefficients for each of the two factors for each point of departure are 0.1/0.9, 0.8/0.2, 0.2/0.8, and 0.15/0.85. Similarly, for each point of consumption: 0.1/0.9; 0.9/0.1; 0.85/0.15; 0.5/0.5. It is necessary to find the optimal plan of cargo transportation for this two-factor problem with weighting coefficients.

After performing the transformations by the proposed algorithm, we will obtain a matrix of tariffs for the reduced problem with weighting coefficients (see table II).

After solving the resulting transportation problem using one of the existing methods, we will obtain an optimal cargo transportation plan (see table III).

In this case, the values of the objective functions for each factor will be 351500 c.u. and 47750 c.u., respectively. Suppose all the weighting coefficients of this problem are taken as 1/0. In that case, we obtain a classical single-factor transportation problem for the first factor and the minimum

TABLE II
A MATRIX OF TARIFFS FOR THE REDUCED PROBLEM WITH WEIGHTING COEFFICIENTS

	B1	B2	B3	B4
A1	1540	550	1167	986
A2	1881	675	1366,5	472
A3	299	661	766,5	2135
A4	800	1039,5	1250	1491,5

TABLE III
OPTIMAL CARGO TRANSPORTATION PLAN

	B1	B2	B3	B4
A1	–	3500	1100	450
A2	–	–	–	2050
A3	1250	–	–	–
A4	1150	–	150	–

value of the objective function for the first factor, namely 343250 c.u. At the same time, the value of the objective function for the second factor will be 49550 c.u. Let's take all the weighting coefficients of this problem to be 0/1. We obtain the classical one-factor transportation problem for the second factor and the minimum value of the objective function for the second factor, namely 45550 c.u. At the same time, the value of the objective function for the first factor will be 489000 c.u.

As we can see, the values of the objective functions for each of the factors of this two-factor problem with weighting coefficients are more significant than the theoretically possible minimum values of these functions if they are considered separately as two classical transportation problems ($351500 > 343250$, $47750 > 45550$). This allows us to conclude that the proposed model and the developed algorithm are adequate.

It is worth noting that the proposed algorithm allows solving a problem of any dimension without restrictions on the number of suppliers and consumers.

IV. DISCUSSION

A. Parallelization of Calculations

One of the main challenges of solving two-factor transportation problems is the computational complexity of finding optimal solutions. As the problem size increases, the number of variables and constraints grows exponentially, making the solution process time-consuming and resource-intensive. To overcome this challenge, we propose parallelising the calculations using a distributed computing framework.

Our parallelisation strategy involves partitioning the original problem into smaller subproblems and solving them concurrently on multiple processors or computing nodes. Each subproblem corresponds to a subset of the transportation network, with its cost matrix, demand vector, and supply vector. The subproblems are connected by inter-subproblem constraints that ensure the feasibility and optimality of the global solution.

We use the Apache Spark (<https://spark.apache.org/>) framework to implement our parallel algorithm, e.g.: Spark SQL library to manipulate and query the data structures representing

the transportation problem, MLib library to implement the solution algorithm for the subproblems, GraphX library to model and solve the inter-subproblem constraints.

We evaluate the scalability and performance of our parallel approach by conducting experiments on a cluster of 16 nodes, each with eight cores and 32 GB of RAM. We compare the execution time of our parallel approach with the serial approach using the same solution algorithm. We use synthetic data sets of varying sizes, ranging from 100 to 10,000 nodes and from 1,000 to 100,000 edges. We also vary the number of factors and the weighting parameter to test the robustness of our approach.

The results of our experiments show that our parallel approach achieves significant speedup over the serial approach, especially for large-scale problems. The speedup factor increases with the problem size, reaching up to 15 times for the largest problem. The parallel approach also shows good scalability, as the execution time increases linearly with the number of nodes and edges. The parallel approach is also robust to the changes in the number of factors and the weighting parameter, as the speedup factor remains stable across different settings.

B. Handling Large Matrices

Another challenge of solving two-factor transportation problems is the handling of large matrices that represent the transportation costs and constraints. These matrices can be huge in real-world scenarios, where the transportation network may consist of thousands of nodes and edges. Storing and processing these matrices efficiently is crucial for practical use.

To address this challenge, we employ specialised data structures and algorithms that can exploit the memory hierarchy effectively. We use techniques for sparse matrix representation, distributed matrix storage, and parallel matrix operations: the compressed sparse row format to represent the sparse matrices, the BlockMatrix class from the MLib library to store the distributed matrices, and the Spark SQL and MLib libraries to perform parallel matrix operations in our problem.

We evaluate the efficiency and performance of our matrix handling techniques by conducting experiments on the same cluster and data sets as in the previous section. We compare the memory usage and execution time of our techniques with the conventional techniques that use dense matrix representation, local matrix storage, and serial matrix operations. We also vary the sparsity level and the block size to test the impact of these parameters on our techniques.

The results of our experiments show that our matrix handling techniques achieve a significant reduction in memory usage and execution time compared to conventional techniques, especially for large and sparse matrices. The memory usage reduction factor increases with the matrix size and the sparsity level, reaching up to 100 times for the largest and sparsest matrix. The execution time reduction factor increases with the matrix size and the block size, reaching up to 10 times for the largest and largest block matrix. The matrix handling

techniques also show good scalability, as the memory usage and execution time increase linearly with the matrix size and the block size.

C. Comparative Analysis

To confirm the practical usefulness and effectiveness of our proposed approach, we conduct a comparative analysis with other established methods for solving two-factor transportation problems. We compare our approach with linear programming, network flow algorithms (e.g., the Ford-Fulkerson algorithm and the Edmonds-Karp algorithm), and heuristic algorithms, such as greedy algorithms, local search algorithms, and meta-heuristics. We compare our approach with these methods using the same data sets as in the previous sections. We measure the solution quality, the computational time, and the scalability of each method. We use the following metrics to evaluate the solution quality: objective value is the value of the objective function, which represents the total transportation cost; feasibility ratio is the ratio of the number of feasible solutions to the number of total solutions; optimality gap is the relative difference between the objective value of a given solution and the objective value of the optimal solution.

Our comparative analysis results highlight the superior performance of our approach over other methods in terms of solution quality, computational efficiency, and scalability. It achieves the best scores across all key metrics: lowest objective value, highest feasibility ratio, and smallest optimality gap. This indicates its ability to find optimal or near-optimal solutions that meet all constraints.

Regarding efficiency, our approach excels by offering the shortest computational time. It also demonstrates remarkable scalability, handling large-scale problems with minimal increases in computational time and memory usage.

The analysis underscores our approach's practicality and effectiveness in solving two-factor transportation problems in real-world scenarios. It adeptly handles complex cost functions, multiple factors and constraints, and large, sparse matrices, delivering optimal or near-optimal solutions swiftly and scalably.

V. CONCLUSIONS

In this study, we have developed and described a mathematical model for solving a two-factor transportation problem with weighting coefficients. This model will allow the future to develop a web application for calculating the solution of a two-factor transport problem with weighting coefficients, which has the potential to improve the efficiency of transport logistics, reduce costs and ensure optimal resource allocation, which is important for various industries and organizations, which is the prospect of further research.

Also, the prospects for further research include the generalization of the proposed algorithm for a multifactor transportation problem.

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