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Machine learning for prediction of emergent economy dynamics

Arnold E. Kiv[1], Vladimir N. Soloviev[2], Serhiy O. Semerikov[2,3,4,5], Hanna B. Danylchuk[6], Liubov O. Kibalnyk[6], Andriy V. Matviychuk[7] and Andrii M. Striuk[3]

[1]*Ben-Gurion University of the Negev, P.O.B. 653, Beer Sheva, 8410501, Israel*

[2]*Kryvyi Rih State Pedagogical University, 54 Gagarin Ave., Kryvyi Rih, 50086, Ukraine*

[3]*Kryvyi Rih National University, 11 Vitalii Matusevych Str., Kryvyi Rih, 50027, Ukraine*

[4]*Institute of Information Technologies and Learning Tools of the NAES of Ukraine, 9 M. Berlynskoho Str., Kyiv, 04060, Ukraine*

[5]*University of Educational Management, 52A Sichovykh Striltsiv Str., Kyiv, 04053, Ukraine*

[6]*The Bohdan Khmelnytsky National University of Cherkasy, 81 Shevchenko Blvd., Cherkasy, 18031, Ukraine*

[7]*Kyiv National Economic University named after Vadym Hetman, 54/1 Peremogy Ave., Kyiv, 03680, Ukraine*

Abstract

This is an introductory text to a collection of selected papers and revised from the M3E2 2021: 9th International Conference on Monitoring, Modeling & Management of Emergent Economy, which held in Odessa National University of Economics, Odessa, Ukraine, on the May 26-28, 2021. It consists of introduction, conference review and some observations about the event and its future.

Keywords

dynamics of emergent markets in crisis and post-crisis period, econophysics, global challenges for economic theory and practice in Europe, information systems and technologies in economics, innovation models of economic development, modeling of hospitality sphere development, models of global transformations, monitoring, modeling and forecasting in the banking sector, monitoring, modeling, forecasting and preemption of crisis in socio-economic systems, risk management models in emergent economy

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kiv.arnold20@gmail.com (A. E. Kiv); vnsoloviev2016@gmail.com (V. N. Soloviev); semerikov@gmail.com (S. O. Semerikov); abdanilchuk@gmail.com (H. B. Danylchuk); liubovkibalnyk@gmail.com (L. O. Kibalnyk); editor@nfmte.com (A. V. Matviychuk); andrey.n.stryuk@gmail.com (A. M. Striuk)

https://ieeexplore.ieee.org/author/38339185000 (A. E. Kiv); https://kdpu.edu.ua/personal/vmsoloviov.html (V. N. Soloviev); https://kdpu.edu.ua/semerikov (S. O. Semerikov); https://scholar.google.com.ua/citations?user=bfPE5scAAAAJ (H. B. Danylchuk); https://scholar.google.com.ua/citations?user=tPw46YYAAAAJ (L. O. Kibalnyk); https://kneu.edu.ua/en/depts9/k_ekon_matematychn_modeljuvannja/vykladachi_kmm/Matvijchuk.A.V/ (A. V. Matviychuk); http://mpz.knu.edu.ua/pro-kafedru/vikladachi/224-andrii-striuk (A. M. Striuk)

0000-0002-0991-2343 (A. E. Kiv); 0000-0002-4945-202X (V. N. Soloviev); 0000-0003-0789-0272 (S. O. Semerikov); 0000-0002-9909-2165 (H. B. Danylchuk); 0000-0001-7659-5627 (L. O. Kibalnyk); 0000-0002-8911-5677 (A. V. Matviychuk); 0000-0001-9240-1976 (A. M. Striuk)

1. Introduction

The development of human society, its various spheres of activity and functioning in recent years is characterized by the emergence of new challenges and threats. Humanity is facing many global problems of development of technology and the increasing scientific and technological progress. In addition to the usual environmental and man-made disasters, financial crises, the new ones appeared. The COVID-19 pandemic, for example. As a result, technologies and online communication instruments began to develop at an increasing pace. The need to work and study remotely has forced people to master new methods and tools of digital technology.

Due to the limited movement and transportation, as well as due to the reduction of production and business closure, enterprises, regions, national economies and transnational capital found themselves in a difficult situation. They experienced a decline in production and trade, bankruptcy, reduced profitability, slower growth etc. This is just a small list of the effects of the pandemic on the world economy.

The causes, mechanisms and consequences of such processes are of particular concern to the scientific community, which is the first to try to understand the depth and importance of these changes. This year's theme of the International Conference on Monitoring, Modeling & Management of Emergent Economy has especially relevant issues that experts of various fields are trying to raise and solve in their works.

The authors' attempt to find out the causes of the crisis and the possibility of using modern ICT to solve existing problems or prevent economic, political and environmental threats, need to be especially considered. The countries with emerging economies are particularly vulnerable to new challenges, so participation in constructive scientific discourse is very important. Modern challenges contribute to the search for new approaches to solving these problems.

In their research scientists focus on economic, financial security and sustainability of enterprises and regions; digitalization of all spheres of human life; modern methods of management and marketing activities; development and analysis of the financial market and cryptocurrency market; modeling and forecasting of international economic activity of various business entities; especially relevant methods of machine learning and fuzzy logic; solving the problems of various sectors of the economy of Ukraine and other countries, especially countries with emerging economies.

The subject of the works included in the proceedings it necessary to search for a new scientific paradigm in a constantly changing environment. After all, new challenges and threats are a certain stimulus for the development of scientific thought. We expect that the research of the participants of this conference will be useful for scientists, teachers, students and representatives of the business community.

1.1. M3E2 2021 at a glance

The **Monitoring, Modeling & Management of Emergent Economy** (M3E2, https://m3e2.ccjournals.eu/2021/) is a peer-reviewed international conference focusing on research advances and applications of nonlinear dynamics methods, econophysics and complex systems methodology of emergent economy.

The M3E2 Conference occupies contributions in all aspects of Computational Finance, Economics, Risk Management, Statistical Finance, Trading and Market Microstructure, (Deep) Machine Learning technologies and tools, paradigms and models, relevant to modern financial engineering and technological decisions in the modern age. There is urgent general need for principled changes in postclassic economy elicited by current models, tools, services, networks and IT communication.

Figure 1: M3E2 2021 logo.

M3E2 2021 topics of interest since 2019 [1, 2]:

- Complex cyberphysical systems, synergy, econophysics, economy of agents
- Dynamics of emergent markets in crisis and post-crisis period
- Economic security
- Global challenges for economic theory and practice in Europe
- Information systems and technologies in economics
- Innovation models of economic development
- Machine learning for prediction of emergent economy dynamics
- Management of the state's economic safety and economic safety of economic agents
- Methods and models of artificial intelligence in economic systems
- Modeling of hospitality sphere development
- Models of global transformations
- Monitoring, modeling and forecasting in the banking sector
- Monitoring, modeling, forecasting and preemption of crisis in socio-economic systems
- Optimal management of socio-economic processes
- Risk management models in emergent economy

This volume contains the selected and revised papers presented at M3E2 2021: 9th International Conference on Monitoring, Modeling & Management of Emergent Economy held on May 26-28, 2021 in Odessa, Ukraine.

There were 45 submissions. Each submission was reviewed by at least 3, and on the average 3.1, program committee members. 11 papers were accepted for this volume as regular papers.

1.2. M3E2 2021 reviewers

Dr. **George Abuselidze**, Professor of Economics and Business, Department of Finance, Banking and Insurance, Batumi Shota Rustaveli State University, Batumi, Georgia.

George Abuselidze, from the Batumi Shota Rustaveli State University (in 2001) and Ivane Javakhishvili Tbilisi State University, Georgia, in 2005, and a Doctor of Economics Sciences degree (Dr. habil.) from the National Academy of Sciences of Georgia, in 2005. Since 2002, he has been working in the Finance and Banking at the Ivane Javakhishvili Tbilisi State University, since 2006 - Batumi Shota Rustaveli

State University where he is currently Head department of Finance, Banking and Insurance. His research interests include Economics, Econometrics, Finance and Social Sciences (miscellaneous). He has published a number of papers in international journals and volumes in book series, is a member of editorial or/and review boards of Oeconomia Copernicana, Journal of Financial Economic Policy, International Journal of Economics and Finance, journal of Science and studies of accounting and finance: problems and perspectives, Management Studies and etc. He also played instrumental role in different prestigious internal collaborative research project with USA, Canada, Lithuania, Poland, Ukraine, Turkey and etc.

WWW: https://orcid.org/0000-0002-5834-1233
E-mail: george.abuselidze@bsu.edu.ge

Prof. Dr. **Vitalina Babenko** is Professor (Full) of International E-commerce and Hotel&Restaurant Business Department, V. N. Karazin Kharkiv National University (Ukraine), Dr. Sci. (habil.) in Economics, PhD in Technical Sciences.

Her sphere of scientific interests includes the management of innovation processes, economic-mathematical modelling, information systems and technologies. She has published around 300 papers in peer-reviewed journals and in the proceedings of conferences. She has taken part in more than 100 conferences and workshops as a Chair, Keynote Speaker, member of the Scientific Program Committee, an Organizer and Session Chair. She is Editor-in-Chief of Journal of International Relations, Economics, Country Studies, Tourism; International Journal of Economics and Management Systems and member of editorial board of some scientific journals. She is the Member of Doctoral Academic Councils in Economics, Expert of the Ministry of Education of Ukraine and Expert of Ukrainian Institute Scientific Technical Information. Council many national and international informatics associations.

WWW: http://international-relations-tourism.karazin.ua/Babenko
E-mail: vitalinababenko@karazin.ua

Dr. **Oleksandr Chernyak** was the Professor, Head of Economic Cybernetics Department, Taras Shevchenko National University of Kyiv

WWW: http://science.univ.kiev.ua/en/researchgroups/research.php?ELEMENT_ID=2594

Dr. **Hanna Danylchuk**, Associate Professor of the Department of Economics and Business Modeling, The Bohdan

Khmelnytsky National University of Cherkasy, Cherkasy, Ukraine.

Hanna Danylchuk, born in 1969, received her PhD in Economics (East European University of Economics and Management) in 2015. Since 2006 she has been working in the field of economic systems modeling at The Bohdan Khmelnytsky National University of Cherkasy. Research interests: modeling of crisis phenomena in financial markets. She has published a number of papers in international journals.

E-mail: abdanilchuk@gmail.com

Dr. **Irina Georgescu**, Lecturer of Computational Intelligence, Department of Informatics and Economic Cybernetics, Bucharest University of Economics, Bucharest, Romania.

Irina GEORGESCU holds a PhD in Economics from Turku Centre for Computer Science, Turku, Finland. Currently she is a lecturer at the Department of Economic Informatics and Cybernetics, Bucharest Academy of Economic Studies. Her research interests lie in the areas of fuzzy economics, computational intelligence and econometrics. She is the author of about 40 journal papers and 2 books published in Springer Verlag.

E-mail: irina.georgescu@csie.ase.ro

Dr. **Lidiya Guryanova**, PhD, Doctor of Economics (Habilitation in Economics), Associate Professor, professor of Economic Cybernetics Department, Simon Kuznets Kharkiv National University of Economics.

Coordinator of the Master's program "Assessment, analysis and forecasting of socio-economic processes" in the specialty 8.18010024 "Applied Economics", a member of the Ukrainian Association of Economic Cybernetics, a reviewer of the journals "Economics of Development" (Ukraine), "Mathematical Problems in Engineering" (USA), Head of Section, a member of the program, organizational and technical committees of international scientific-practical conference "Modern problems of modelling the socio-economic systems" (Ukraine, Slovenia, Poland, the USA, Bulgaria), "Innovation and information technology in the development of business and education" (Russian Federation, Ukraine, Germany, Bulgaria, France), scientific director of training of highly qualified personnel (specialty 08.00.11 – mathematical methods, models and information technology in the economy).

WWW: https://ek.hneu.edu.ua/en/professors/guryanova-l-s/

E-mail: guryanovalidiya@gmail.com

Dr. **Alexey Hostryk**, Odessa National Economic University, Ukraine.
WWW: https://orcid.org/0000-0002-5834-1233
E-mail: AlexeyGostrik@gmail.com

Dr. **Pavlo Hryhoruk**, Professor of Department of Automated Systems and Modeling in Economics of Khmelnytskyi National University, Khmelnytskyi, Ukraine.

Pavlo Hryhoruk, received a Doctor of Economic Sciences degree (Dr. habil.) from the Khmelnytskyi National University, in 2013. Since 1997, he has been working in the field of economic and mathematical modeling at the Khmelnytskyi National University, where he is currently Head of Department of Automated Systems and Modeling in Economics. Directions of studies are related to decision-making, multidimensional modeling of socio-economic systems, comprehensive assessment of economic phenomena latent characteristics, financial security, sustainable development, information technologies in education. He has published a number of papers in domestic and international journals, monographs and volumes in book series. Since December 2020 he is Editor-in-Chief of *Herald of Khmelnytskyi National University. Economic Sciences*.

E-mail: violete@ukr.net, hryhoruk@khnu.km.ua

Dr. **Serhii Hushko**, State University of Economics and Technology, Ukraine.
WWW: https://www.duet.edu.ua/en/persons/12
E-mail: gushko77@gmail.com

Dr. **Nila Khrushch**, Professor, Head of Department of Finance, Banking and Insurance of Khmelnytskyi National University, Khmelnytskyi, Ukraine.

Nila Khrushch, received a Doctor of Economic Sciences degree (Dr. habil.) from the Institute of Market Problems and Economic & Ecological Researched of National Academy of Sciences of Ukraine, Odesa in 2007. Since 1993, she has been working in the field of innovation-investment and strategic management at the Khmelnytskyi National University, where she is currently Head of Department of Finance, Banking and Insurance. Area of studies includes financial and economic research and modeling of business entities activity, financial security, sustainable development, strategic and financial management. She has published a number of papers in domestic and international journals, monographs and volumes in book series, is a member of editorial board of *Herald of Khmelnytskyi National University. Economic Sciences.*

E-mail: nila.ukr@gmail.com, khrushch@khnu.km.ua

Dr. **Liubov Kibalnyk**, Head and Professor of Department of Economics and Business Modelling, Cherkasy Bohdan Khmelnytsky National University, Cherkasy, Ukraine.

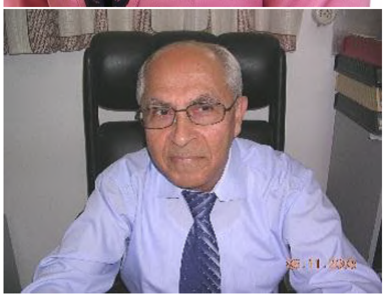
Liubov Kibalnyk, born in 1969, received a Candidate of Economic Sciences degree (Dr. phil.) from the Taras Shevchenko National University of Kyiv, Ukraine, in 2002, and a Doctor of Economic Sciences degree (Dr. habil.) from the Institute of International Relations of Taras Shevchenko National University of Kyiv, in 2015. Since 1994, she has been working in the field of economics, international economic relations and modeling at the Cherkasy Bohdan Khmelnytsky National University, where he is currently Head of Department.

She specialized in the field of modeling economic processes in the global environment. She is the author of more than 150 scientific and methodological works published in international and national editions.

WWW: http://econom-law.cdu.edu.ua/?page_id=804

E-mail: liubovkibalnyk@gmail.com

Dr. **Arnold Kiv**, Ben-Gurion University of the Negev, Israel.



Arnold Kiv received the D. Sc. (Dr. Hab.) degree in solid state physics from Tartu Institute of Physics, Tartu, Estonia, in 1978. From 1964 to 1982, he was a Senior Researcher and a Head of the Laboratory of Radiation Effects, Institute of Nuclear Physics, Academy of Sciences, Tashkent, Uzbekistan. From 1983 to 1998, he was a Head of the Department of Theoretical Physics, South-Ukrainian National Pedagogical University, Odessa, Ukraine. In 1997, he was an Invited Professor, Western Ontario University, Canada. From 1999 to the present, he is a Professor-Researcher in the Department of Materials Engineering, Ben-Gurion University of the Negev, Israel. In 1996 and 2011 he was co-Director of NATO Advanced research Workshops and an Editor of two NATO Series books. He has about 200 publications, three monographs and three Invention Certificates in the field of radiation effects in solid state electronics. His research interests include mechanisms of formation of radiation defects in solids, interaction of fast particles with materials, radiation methods in microelectronics, including computer simulation, analytical calculations and experimental studies.

PhD of Public Administration, **Oksana Kovtun**, associate professor, Department of Public Administration and Project Management, State Higher Educational Institution "University of Educational Management", Kyiv, Ukraine.

Oksana Kovtun, born in 1971, received a Candidate of Public Administration degree (Dr. phil.) from the Council for the Study of Productive Forces of Ukraine at the National Academy of Sciences of Ukraine, Ukraine, in 2010. Since 2000, she has been working in the field of public finance research, household financial behavior and mechanisms for their activation in Classic Private University (Zaporizhzhia). I am currently an associate professor at the Department of Public Administration and Project Management, State Higher Educational Institution «University of Educational Management». Shies research interests: modeling of financial markets, study of structural changes in the economy, innovations in the financial sector. She has published a number of papers in international journals and workshops.

WWW: http://umo.edu.ua/institutes/imp/struktura-institutu/kaf-upravl-proekt/sklad/kovtun-oksana-anatolijivna

E-mail: kovtun.oa71@gmail.com

Dr. **Hanna Kucherova**, Professor of economy, Department of Economy, Classic Private University, Zaporizhzhia, Ukraine.

Hanna Kucherova, born in 1983, received a Candidate of Economical Sciences degree from the Classic Private University, in 2011, and a Doctor of Economical Sciences degree (Dr. habil.) from the Classic Private University, in 2017. Since 2008, she has been working in the field of pricing, economy, tax, economy behavior at the Classic Private University of Ukraine, where he is currently professor Chair of Economics. Her research interests include modeling the country's business climate, behavior of socio-economic agents, tax consciousness and information transparency.

WWW: https://scholar.google.ru/citations?user=FhBOVExn1foC

E-mail: kucherovahanna@gmail.com

Dr. **Serhii Lehenchuk**, head of the department of management information systems and accounting, Zhytomyr Polytechnic State University, Zhytomyr, Ukraine.

Serhii Lehenchuk, born in 1981, received a Candidate of Economic Sciences degree (Dr. phil.) from the National Agrarian University, Kyiv, Ukraine, in 2006, and a Doctor of Economic Sciences degree (Dr. habil.) from the Zhytomyr State Technological University, in 2011. Since 2004, he has been working in the field of accounting and information systems at the Zhytomyr Polytechnic State University. His research interests include general accounting theory, positive accounting theory and accounting choice, intangible assets accounting, corporate and integrated reporting. He has published a number of papers in international and Ukrainian journals, is a member of editorial boards of "Economics, Management and Administration", "Problems of Theory and Methodology of Accounting, Control and Analysis", "Socio-Economic Research Bulletin", "Public Policy and Accounting".

WWW: https://ztu.edu.ua/ua/structure/faculties/fbso/bok.php

E-mail: legenchyk2014@gmail.com

Dr. **Nataliia Maksyshko**, Doctor of Economic Sciences, Professor, Head of Department of Economic Cybernetics, Professor, Zaporizhzhia National University, Zaporizhzhia, Ukraine.

Nataliia Maksyshko, received a Candidate in Physics and Mathematics Sciences degree (Dr. phil.) from the V.M. Glushkov Institute of Cybernetics of National Academy of Sciences of Ukraine, in 1990 (Mathematical modeling and computational methods), and a Doctor of Economics Sciences degree (Dr. habil.) (Economic and mathematical modeling) from the Kyiv National Economic University named after Vadym Hetman, in 2010. Since 1983, she has been working in the field of economic and mathematical modeling at the Zaporizhzhia National University, where she is currently Head of Department of Economic Cybernetics. Her research interests include economic and mathematical modeling of complex economic systems, economic dynamics, discrete optimization, information technology in economics and education. She has published a number of papers in international journals and volumes in book series, is a member of editorial board of Bulletin of Zaporizhzhia National University. Economic sciences.

E-mail: maxishko@ukr.net

Dr. **Andriy Matviychuk**, Professor of Economic and Mathematical Modeling, Kyiv National Economic University named after Vadym Hetman, Kyiv, Ukraine.

Andriy Matviychuk, born in 1978, became a Candidate of Science in Economics (PhD) by specialty Economic and Mathematical Modelling at the Technological University of Podillia (Khmelnitskiy) in 2003, and a Doctor of Economics (DSc) with a degree in Mathematical Methods, Models and

Information Technologies in Economics at the Kyiv National Economic University named after Vadym Hetman in 2008.

Since 1990 he has been working in the field of AI & ML at the Kyiv National Economic University named after Vadym Hetman, where he is currently professor of Economic and Mathematical Modeling Department, director of Institute of Modeling and Informational Technologies in Economics and CEO at KNEU Science Park. His research interests cover the field of mathematical modeling of complex systems, primarily using tools of neural networks and fuzzy logic. He is a Chief Editor of scientific and analytical journal Neuro-Fuzzy Modeling Techniques in Economics and a member of editorial boards of a number of scientific journals.

WWW: https://kneu.edu.ua/ua/depts9/k_ekon_matematychn_modeljuvannja/vykladachi_kmm/Matvijchuk.A.V/

E-mail: editor@nfmte.com

Dr. **Inese Mavlutova**, Professor (Full) at Department of Economics and Finance, BA School of Business and Finance, Latvia.

E-mail: inese.mavlutova@ba.lv

Dr. **Iveta Mietule**, Rector of Rezekne Academy of Technologies, Latvia, Professor of Economics.

Iveta Mietule was born in 1972 in Latvia. Received her PhD in Economics from University of Latvia in 2009. Member of the Association of University Professors of Latvia, professor of Rezekne Academy of Technologies, member of the Senate of RTA, expert of the Latvian Council of Science in the field of Economic and Management Sciences. Author of more than 50 publications, 3 monographs. Participant and contributor of more than 15 projects and programmes, included FRONTEX, ERDF, ESF, INTERREG, ERASMUS and the Latvian National Research Programme. International experience of organizing and participating in more than 50 events (conferences, lectures, seminars etc.).

WWW: https://orcid.org/0000-0001-7662-9866

E-mail: iveta.mietule@rta.lv, mietule@inbox.lv

Dr. **Dariusz Pawliszczy**, Mayor of Gromadka, Poland.

E-mail: pawliszczy@interia.pl

Dr. **Oleg Pursky**, Professor of Computer Science and Information Systems, Head of Department of Computer Science and Information Systems, Kyiv National University of Trade and Economics, Kyiv, Ukraine.

Oleg Pursky, born in 1967, received a Candidate of Sciences in Physics and Mathematics degree (Dr. phil.) from the Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, in 2001, and a Doctor of Sciences in Physics and Mathematics degree (Dr. habil.) from the Taras Shevchenko National University of Kyiv, Ukraine, in 2010.

His research interests include informational systems development, computer simulation and modeling of socio-economic systems. He has published a number of papers in international journals, monographs and volumes in book series, is a member of editorial board of International Journal of Economic Theory and Application, reviewer of scientific journals International Journal of Modern Physics (B) and Heat Transfer and certified Data Science&Machine Learning specialist. He is a member of Scientific Council section of Ukrainian Ministry of Education and Science on the specialty "Informatics and Cybernetics". Currently, he is working as a Head of Department of Computer Science and Information Systems, Kyiv National University of Trade and Economics.

WWW: https://knute.edu.ua/blog/read/?pid=12695&uk
E-mail: Pursky_O@ukr.net

Dr. **Sultan Ramazanov**, Kyiv National Economic University named after Vadym Hetman, Ukraine.

Born 12/13/1949. Mathematician, cybernetic, environmental economist; design specialist and development of integrated management information systems complex ecological-economic objects with elements of artificial intelligence. Professor, Doctor of Engineering, Doctor of Economics Science, Honored Worker of Science and Technology of Ukraine. Honored Worker of Education of Ukraine. Excellence in education of Ukraine. Professor of the Department "Information systems in the economy" of Kyiv National Economic University named after Vadym Hetman. Professor Emeritus of East Ukrainian National University named after V. Dahl and Poltava University of Economics and Trade. Prepared 1 DSc and 14 PhDs (7 scientific fields). He is the author of 495 scientific and methodological works, including 41 monographs and 14 textbooks. Expert of the Ministry of Education and Science of Ukraine ("Informatics and Cybernetics"). Academician of Academies: International Academy of Informatics, The International Academy of Environmental Sciences and Life Safety, The Academy of Technological Sciences of Ukraine, the Academy of Economic Sciences of Ukraine, Transport Academy of Ukraine. Total work experience in Ukrainian universities - 47 years (since 1973). 28 years – head of the department and at the same time 13 years – dean of the faculty (EUNU named after V. Dahl, Luhansk and Severodonetsk) ([]from 1973 to 2014). Since 2014 - Professor of Departments at PUET (Poltava) and KNEU (Kyiv).

E-mail: sramazanov@i.ua, sultan.ramazanov@kneu.edu.ua

Dr. **Serhiy Semerikov**, Professor of Computer Science and Educational technology, Kryvyi Rih State Pedagogical University, Ukraine.

Serhiy Semerikov is professor of Department of Computer Science and Applied Mathematics at Kryvyi Rih State Pedagogical University. He got both PhD and DSc in education (informatics) from the National Pedagogical Dragomanov University in 2001 and 2009, respectively. The main directions of Dr. Semerikov' research is methods of learning and educational technology.

WWW: https://kdpu.edu.ua/semerikov/

E-mail: semerikov@gmail.com

Dr. **Nadiia Shmygol**, Professor of Management Department, National University "Zaporizhzhia Polytechnic", Zaporizhzhia, Ukraine.

Nadiia Shmygol, born in Pavlodar city in Kazakhstan 1972. In 2005 defended a thesis and obtained a diploma of Candidate of Sciences (comparable to the academic degree of Doctor of Philosophy, Ph.D.) Economics in Economic and Mathematical Modeling. In 2013 defended a thesis and obtained a diploma of Doctor of Sciences in Economy and Management of enterprises. Certificate of professor of Accounting and Audit Department (2014). Author of over 200 scientific and practical publications in the field of economics and management.

Sphere of scientific interests: management accounting, CSR, sustainable development, circular economy, sustainable marketing, resource-efficient production, decision-making methods, environmental management.

WWW: https://zp.edu.ua/nadiya-mikolayivna-shmigol

E-mail: nadezdashm@gmail.com

Dr. **Kateryna Shymanska**, Head of the Department, Department of Digital Economy and International Economic Relations, Zhytomyr Polytechnic State University, Zhytomyr, Ukraine.

Kateryna Shymanska, born in 1985, received a Candidate of Economic Sciences degree (Dr. phil.) from the Zhytomyr State Technological University, Ukraine, in 2010, and a Doctor of Economic Sciences degree (Dr. habil.) from the Vasyl'

Stus Donetsk National University, in 2019. Since 2014, she has been working in the field of international economic relations at the Zhytomyr Polytechnic State University, where she is currently Head of the Department of Digital Economy and International Economic Relations. Her research interests include international migration consequences and regulation, but also international trade and the challenges of the digital economy. She has published a number of papers on the specified problems.

WWW: https://ztu.edu.ua/ua/structure/faculties/fbso/kme.php

E-mail: kv.shymanska@gmail.com

Vladimir N. Soloviev received the D. Sc. (Dr. Hab.) degree in solid state physics from Institute of Physics of the National Academy of Sciences of Ukraine, in 1993. From 1992 to 2000 and from 2016 to the present head of the Department of Informatics and Applied Mathematics of Kryvyi Rih State Pedagogical University. In the period from 2000 to 2016, he carry out research on critical and crisis phenomena in the financial markets at various universities in Kyiv, Cherkasy and Kryvyi Rih. He has about 300 publications in the field of solid state physics, complex systems and quantitative methods of constructing precursors of crisis phenomena in systems of different nature.

Dr. **Victoria Solovieva**, Associate Professor, Head of Department, State University of Economics and Technology, Ukraine.

She graduated from the Faculty of Physics and Mathematics of the Kryvyi Rih Pedagogical Institute. She has a Ph.D., specializing in Economic and Mathematical Modeling. He has about 130 publications in the field of complex systems and quantitative methods for constructing precursors of crisis phenomena in systems of various nature.

E-mail: vikasolovieva2027@gmail.com

Dr. **Galyna Velykoivanenko**, Head of the Economic and Mathematical Modeling Department, Kyiv National Economic University named after Vadym Hetman, Ukraine.

E-mail: ivanenko@kneu.edu.ua

Dr. **Eugene Yakub**, Professor of Economic Information and Computer Technology, Department of Economic Cybernetics and Information Technology, Odessa National Economic University, Odessa, Ukraine.

Eugene Yakub, born in 1946, received a Candidate of Technical Sciences degree (Dr.Phil.) from the Odessa Technological Institute of Refrigeration, USSR, in 1973, and a Doctor of Physical and Mathematical Sciences degree (Dr. habil.) from the Institute of High Temperatures of the Academy of Sciences of USSR, in 1990. Since 1977, he has been working in the field of mathematical and computer modelling at the Odessa National Economic University, where he is currently head of Department of Economic Cybernetics and Information Technology. His research interests include modelling and computer simulation of various complex natural and economic systems. He has published a number of papers in international journals and monographs, is a member of several Academic Councils.

WWW: http://oneu.edu.ua/ru/yakub-yevgen-solomonovich
E-mail: yakub@oneu.edu.ua

Oleh Yatsiuk, Department of Economics and Management Theory, Ivano-Frankivsk National Technical University of Oil and Gas, Ivano-Frankivsk, Ukraine.

Oleh Yatsiuk, born in 1978, received a Master degree in Economics from the Ivano-Frankivsk National Technical University of Oil and Gas, in 2000. Since 2001, he has been working in the field of enterprise' financial sanation at the Ivano-Frankivsk National Technical University of Oil and Gas. His research interests include finances of the enterprise, anti-crisis management and HR management. He has more than 200 published scientific and educational works, including monographs, textbooks, papers in domestic and international journals.

WWW: https://nung.edu.ua/person/yacyuk-oleg-stepanovich-0
E-mail: olegstya@gmail.com

Dr. **Nataliia Zachosova**, Professor of Management and Economic Security Department, Bohdan Khmelnytsky National University of Cherkasy, Cherkasy, Ukraine.

Nataliia Zachosova, born in 1986, received a Candidate of Economic Sciences degree (Dr. phil.) in 2011, and a Doctor of Economic Sciences degree (Dr. habil.) in 2017. In 2018 she received the Scholarship of the Cabinet of Ministers of Ukraine for Young Scientists (and in 2020 she got it once again) and in 2019 – the Nominal Scholarship of the Verkhovna Rada of Ukraine for the Most Talented Young Scientists. Since 2014, she has been working in the field of financial and economic security management at the Bohdan Khmelnytsky National University of Cherkasy, where she is currently Professor and the Chairman of the Council of Young Scientists. Her research interests include security oriented management, strategic and personnel management. She has published a number of papers in international journals and volumes in book series, is a member of editorial boards of Bulletin of the Cherkasy Bohdan Khmelnytsky National University, Series "Economic Sciences".

WWW: https://orcid.org/0000-0001-8469-3681

E-mail: natazachosova@gmail.com

Dr. **Pavel Zakharchenko**, Professor of Economic Cybernetics, Head of Department of Economic Cybernetics and Finances, Berdyansk State Pedagogical University, Berdyansk, Ukraine.

Pavel Zakharchenko, born in 1957, received a Candidate of Technical Sciences (Dr. phil.) from the Kharkov Institute of Radio Electronics, USSR, in 1982, and a Doctor of Economical Sciences degree (Dr. habil.) from the Kiev National Economic University of Ukraine, in 2010. Since 2000, he has been working in the field of finance and economic cybernetics at the Berdyansk State Pedagogical University, where he is currently Head of Department of Economic Cybernetics and Finances. His research interests include health economics models, chaos and catastrophe theory. He has published a number of articles in international journals and volumes in book series, and is a member of the editorial board of scientific journals.

WWW: https://bdpu.org.ua/en/faculties/gef/structure-gef/kaf-fin/composition-kaf-fin/zaharchenko/

E-mail: pvzz1957@gmail.com

2. Articles overview

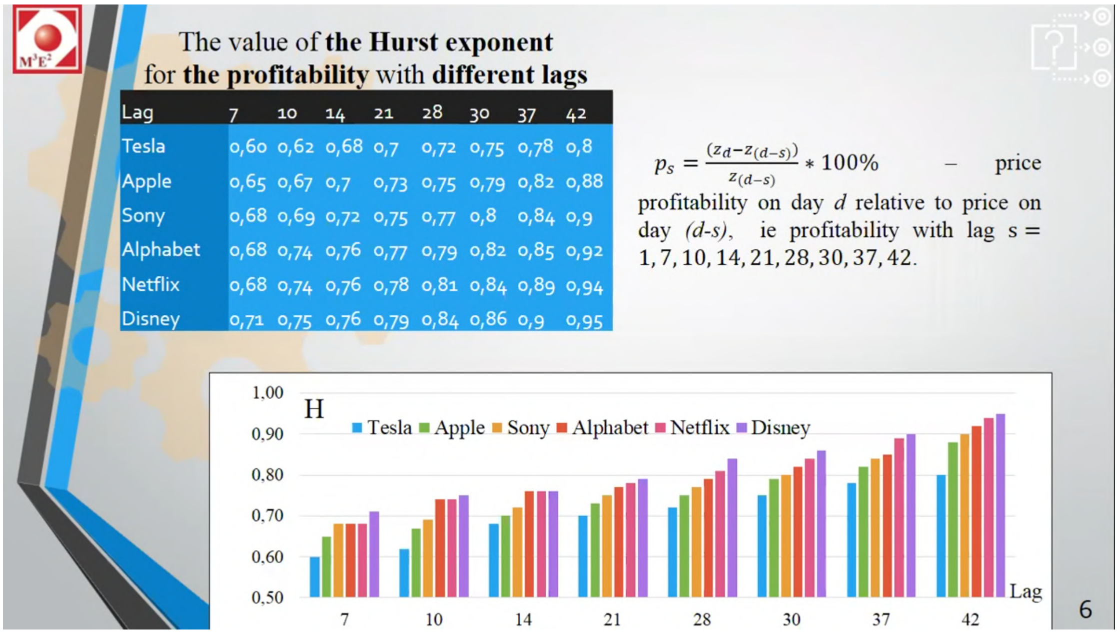
The aim of the article "Innovative behavior of bitcoin market agents during COVID-19: recurrence analysis" [3] by Hanna Yu. Kucherova, Vita O. Los, Dmytro V. Ocheretin, Olha V. Bilska and Evgenia V. Makazan (figure 2) is to study the series of the dynamics of the price of bitcoin and the frequency of online requests for bitcoin as an indicator of the behavior of agents of the digital economy using the methods of qualitative recurrent analysis. The types of constructed time series plots of the price of bitcoin and the frequency of requests for bitcoin are defined as drift with a superimposed linearly gradually increasing sequence, which indicates the unpredictability of the behavior of digital economy agents with a gradual stabilization in new quality trend. The scientific novelty of the research results lies in the proven connection between the series of bitcoin price dynamics and the frequency of online requests for bitcoin, tracking changes in the behavior of digital economy agents before and after the introduction of quarantine restrictions. The procedure for conducting a qualitative recurrence analysis of the series of dynamics is generalized, which takes into account the specifics of the formation of the frequency of online requests for bitcoin, the price and the behavioral aspect of its formation. The practical value lies in defining the characterization of the behavior model of digital economy agents under conditions of quarantine restrictions. The behavior of digital economy agents in the context of COVID-19 requires further research, in particular, using cross-recurrent analysis methods.

This article highlights further research by the authors, begun in [4, 5, 6, 7, 8, 9, 10].

Figure 2: Presentation of paper [3].

The article “Comparative analysis of the stock quotes dynamics for IT and the entertainment industry companies based on the characteristics of memory depth” [11] by Nataliia K. Maksyshko and Oksana V. Vasylieva (figure 3) is devoted to the study and comparative analysis of the stock quotes dynamics for the world’s leading companies in the IT sector and the entertainment industry. Today, these areas are developing the fastest and most powerful, which attracts the attention of investors around the world. This is due to the rapid development of digital communication technologies, the growth of intellectualization and individualization of goods and services, and so on. These spheres have strong development potential, but the question to how their companies’ stock quotes respond to the impact of such a natural but crisis phenomenon as the COVID-19 pandemic remains open. Based on the nonlinear paradigm of the financial markets dynamics, the paper considers and conducts a comprehensive fractal analysis of the quotations dynamics for six leading companies (Apple Inc., Tesla Inc., Alphabet Inc., The Walt Disney Company, Sony Corporation, Netflix) in this area before and during the COVID-19 pandemic. As a result of the application of the rescaled range analysis (R/S analysis), the presence of the persistence property and long-term memory in the stock quotes dynamics for all companies and its absence in their time series of profitability was confirmed. The application of the method of sequential R/S analysis made it possible to construct fuzzy sets of memory depths for the considered time series and to deepen the analysis of the dynamics due to the quantitative characteristics calculated on their basis. Taking into account the characteristics of memory depth in the dynamics of quotations made it possible to conduct a comparative analysis of the dynamics, both under the influence of the natural crisis situation and in terms of investing in different terms. The peculiarities of the delayed profitability dynamics of quotations for each of the companies are also taken into consideration and compared. The developed recommendations can be used in investment activities in the stock market.

This article highlights further research by the authors, begun in [12, 13, 14].



Lag	7	10	14	21	28	30	37	42
Tesla	0,60	0,62	0,68	0,7	0,72	0,75	0,78	0,8
Apple	0,65	0,67	0,7	0,73	0,75	0,79	0,82	0,88
Sony	0,68	0,69	0,72	0,75	0,77	0,8	0,84	0,9
Alphabet	0,68	0,74	0,76	0,77	0,79	0,82	0,85	0,92
Netflix	0,68	0,74	0,76	0,78	0,81	0,84	0,89	0,94
Disney	0,71	0,75	0,76	0,79	0,84	0,86	0,9	0,95

Figure 3: Presentation of paper [11].

Cryptocurrencies refer to a type of digital asset that uses distributed ledger, or blockchain technology to enable a secure transaction. Like other financial assets, they show signs of complex systems built from a large number of nonlinearly interacting constituents, which exhibits collective behavior and, due to an exchange of energy or information with the environment, can easily modify its internal structure and patterns of activity. The article "Econophysics of cryptocurrency crashes: a systematic review" [15] by Andrii O. Bielinskyi (figure 4), Oleksandr A. Serdyuk, Serhiy O. Semerikov and Vladimir N. Soloviev review the econophysics analysis methods and models adopted in or invented for financial time series and their subtle properties, which are applicable to time series in other disciplines. Quantitative measures of complexity have been proposed, classified, and adapted to the cryptocurrency market. Their behavior in the face of critical events and known cryptocurrency market crashes has been analyzed. It has been shown that most of these measures behave characteristically in the periods preceding the critical event. Therefore, it is possible to build indicators-precursors of crisis phenomena in the cryptocurrency market.

This article highlights further research by the authors, begun in [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33].

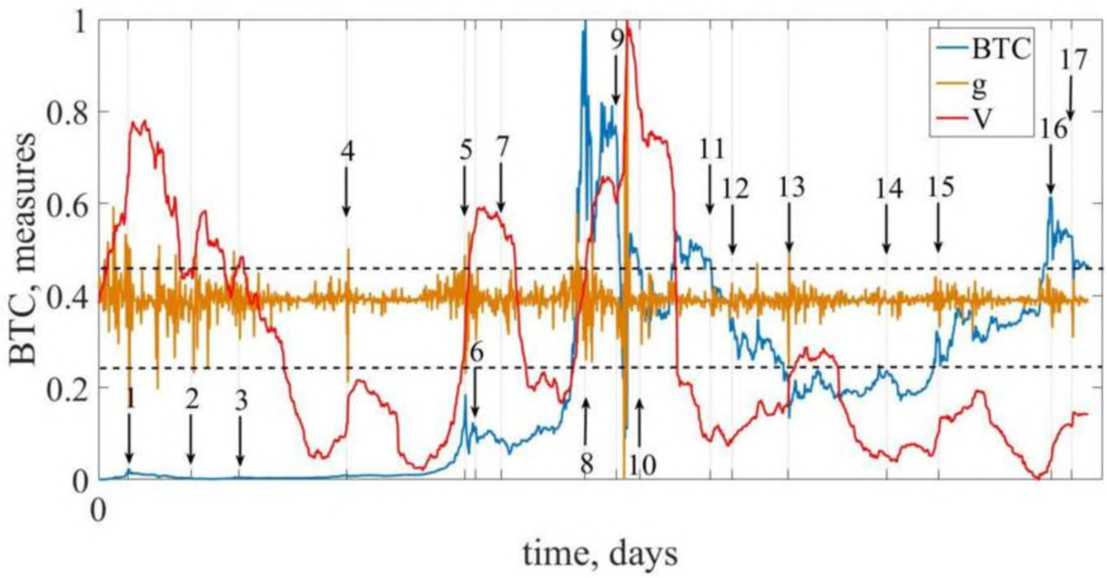


Figure 4: Presentation of paper [15].

The focus of the article "Irreversibility of financial time series: a case of crisis" [34] by Andrii O. Bielinskyi, Serhii V. Hushko (figure 5), Andriy V. Matviychuk, Oleksandr A. Serdyuk, Serhiy O. Semerikov and Vladimir N. Soloviev to measure the varying irreversibility of stock markets. A fundamental idea of this study is that financial systems are complex and nonlinear systems that are presented to be non-Gaussian fractal and chaotic. Their complexity and different aspects of nonlinear properties, such as time irreversibility, vary over time and for a long-range of scales. Therefore, this work presents approaches to measure the complexity and irreversibility of the time series. To the presented methods authors include Guzik's index, Porta's index, Costa's index, based on complex networks measures, Multiscale time irreversibility index and based on permutation patterns measures. This study presents that the corresponding measures can be used as indicators or indicator-precursors of crisis states in stock markets.

This article highlights further research by the authors, begun in [35, 36, 37].

The article "Big Data based marketing forecasting" [38] by Sergey M. Ivanov and Mykola M. Ivanov (figure 6) discusses the use of big data as a tool to increase data transfer speed while providing access to multidimensional data in the process of forecasting product sales in the market. In this paper discusses modern big data tools that use the MapReduce model. The big data presented in this article is a single, centralized source of information across your entire domain. In the paper also proposes the structure of a marketing analytics system that includes many databases in which transactions are processed in real time. For marketing forecasting of multidimensional data in Matlab, a neural network is considered and built. For training and building a network, it is proposed to construct a matrix of input data for presentation in a neural network and a matrix of target data that determine the output statistical information.

Figure 5: Presentation of paper [34].

Input and output data in the neural network is presented in the form of a 5x10 matrix, which represents static information about 10 products for five days of the week. The application of the Levenberg-Marquardt algorithm for training a neural network is considered. The results of the neural network training process in Matlab are also presented. The obtained forecasting results are given, which allows us to conclude about the advantages of a neural network in multivariate forecasting in real time.

This article highlights further research by the authors, begun in [39, 40, 41].

The article "Fuzzy model for complex risk assessment of an enterprise investment project" [42] by Inna I. Chaikovska, Pavlo M. Hryhoruk and Maksym Yu. Chaikovskyi (figure 7) proposes an economic-mathematical model for determining a comprehensive risk assessment of the investment project of the enterprise which are based on the approaches of A. Nedosekin. The model is built using fuzzy logic and takes into account the probability of occurrence of each of the identified risks and the level of impact of each of them on the project. The probability of risk is set by experts in the form of points and converted into linguistic terms, and the level of influence of each of them on the project – the ratio of benefits and is determined using Fishburne scales. The proposed Project Risk Model consists of the following stages: formation of initial data using expert opinions; construction of a hierarchical project risk tree; determination of weight coefficients (Fishburne weights) of project risks; selection and description of membership function and linguistic variables; conversion of input data provided by experts from a score scale into linguistic terms; recognition of qualitative input data on a linguistic scale; determination of a complex indicator of investment project risks; interpretation of a complex indicator. The developed model allows managing the risks of the project to maximize the probability of its successful implementation, to compare alternative projects and choose less risky, to minimize the level of unforeseen costs of the project.

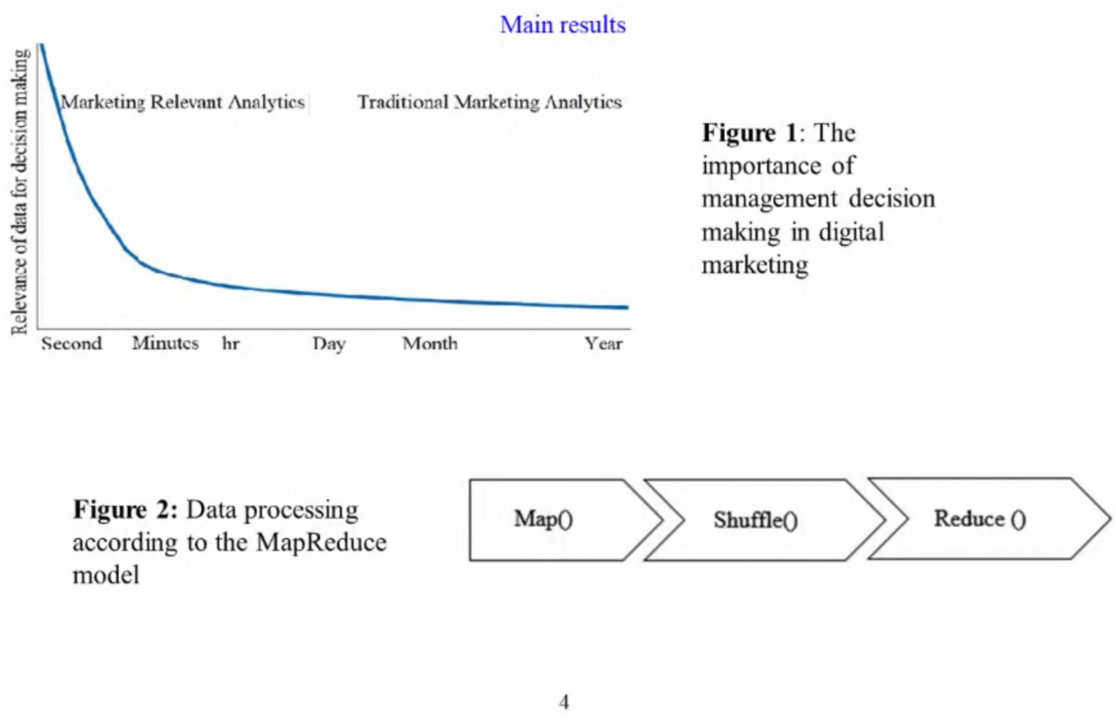


Figure 6: Presentation of paper [38].

This article highlights further research by the authors, begun in [43, 44, 45, 46].

The article "Modeling structural changes in the regional economic development of Ukraine during the COVID-19 pandemic" [47] by Pavlo M. Hryhoruk (figure 8), Nila A. Khrushch and Svitlana S. Grygoruk investigates the issues of evaluating structural changes in the regions' economic development based on the comprehensive index assessment technology. The impact of the COVID-19 pandemic on regional development and changes in the regional structure is considered. The authors propose the use of block convolution to design a comprehensive index based on a set of metric initial indicators that characterize the regions' economic development. Grouping the set of initial indicators is carried out based on the method of an extreme grouping of parameters and the method of principal components. A weighted linear additive convolution was used to develop partial composite indices and an economic development comprehensive index. The practical approbation was carried out for the regions of Ukraine according to the data of 9 months of 2019 and the same period of 2020. To establish the regions' structure, authors used the division of the comprehensive index values into intervals and further distributing regions into classes according to the level of economic development. There is a general decrease in the value of the integrated indicator in 2020, caused by the impact of the COVID-19 pandemic. However, no significant changes in the structure of the regions were detected, which indicates an equally negative impact of the pandemic for all regions of Ukraine.

This article highlights further research by the authors, begun in [48].

The article "The use of genetic algorithms for multicriteria optimization of the oil and gas

Figure 7: Presentation of paper [42].

enterprises financial stability" [49] by Marta V. Shkvaryliuk, Liliana T. Horal, Inesa M. Khvostina, Natalia I. Yashcheritsyna and Vira I. Shiyko (figure 9) considers the problem of optimizing the financial condition of oil and gas companies. The offered methods of optimization of a financial condition by scientists from different countries are investigated. It is determined that the financial condition of the enterprise depends on the effectiveness of the risk management system of enterprises. It is proved that the enterprises of the oil and gas complex need to develop a system for risk management to ensure the appropriate financial condition. The financial condition is estimated according to the system of certain financial indicators, the integrated indicator of financial condition assessment is constructed using the method of taxonomy. According to the results of the calculation of the integrated indicator, it is concluded that this indicator does not have a stable trend. On the basis of the conducted researches it is offered to carry out optimization of an integral indicator of a financial condition with use of genetic algorithm in the Matlab environment. Based on the obtained results, recommendations of the management of the researched enterprises on increase of management efficiency are given.

This article highlights further research by the authors, begun in [50, 51, 52, 53, 54, 55].

The article "Fuzzy modelling of the country's migration attractiveness" [56] by Hanna B. Danylchuk, Liubov O. Kibalnyk (figure 10), Oksana A. Kovtun, Oleg I. Pursky and Zenon Sta-

Figure 8: Presentation of paper [47].

chowiak deals with the analysis the current state of migration in the context of globalization and identifies the most important corridors for the labour movement. The main donor countries of migrants are developing countries, with low socio-economic indicators, difficult environmental conditions and high levels of poverty. According to forecasts, the most migratory flows will take place in the countries of North America and in Europe, which is due to rising trends in unemployment in the countries of the "third world" and the demand for cheap labour, changes in the structure of the economies of developed countries, changes in labour market demand. The main world regional corridors in 1990–2019 have been identified through statistical analysis. And their growing and declining trends. The need to use economic and mathematical modelling techniques to analyse and determine the migration attractiveness of recipient countries in an uncertain environment has been substantiated. It has been shown that fuzzy logic tools are the most effective in this case. Based on the results of the simulation using the Mamdani method, the world's attractiveness rating for migration is calculated, which with a "high" thermo leads such countries as Italy, France, United Arab Emirates. The findings suggest that migrants are attracted by countries with the lowest inflation rates, high and average GDP per capita and average or low taxation levels.

This article highlights further research by the authors, begun in [58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69].

In the article "Computational method determining integral risk indicators of regional socio-economic development" [57], Oleg I. Pursky, Tetiana V. Dubovyk, Iryna O. Buchatska, Iryna

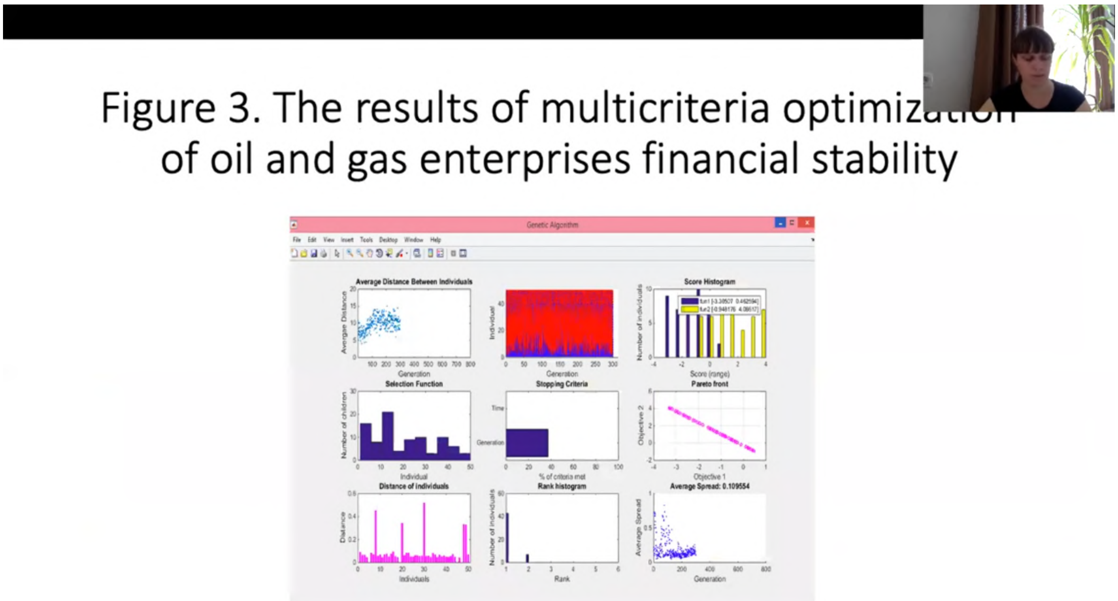


Figure 9: Presentation of paper [49].

Figure 10: Presentation of paper [56].

S. Lutsenko and Hanna B. Danylchuk (figure 11) present the computational method for risk assessment of the socio-economic development of regions. An attempt has been made to develop a method for the determination of integral risk indicators of socio-economic development based on the joint use of the methods of factor analysis and expert evaluation. This approach has

Figure 11: Hanna B. Danylchuk, Vita O. Los, Hanna Yu. Kucherova and Liubov O. Kibalnyk after presentation of paper [57].

increased the reliability of the calculations and made it possible to analyze the influence of socio-economic indicators on the risk level of socio-economic development. The integral risk indicator shows the effect of the inconsistency in the level of factor provision on the socio-economic development of the *j*-th region (district) in comparison with the general situation in the country (regions). The closer the value of integral risk indicator is to 1, the higher the level of risk in this region. Using Kyiv region districts as an example, the process of risk assessment for regional socio-economic development has been considered. The results obtained in this investigation demonstrate that the presented computational method solves the problem of formalization of risk assessment for the socio-economic development of regions.

The article "Modelling the logistics system of an enterprise producing two type of goods" [70] by Roman V. Ivanov (figure 12), Yuriy V. Sherstennikov, Vasyl M. Porokhnya and Tetyana V. Grynko is devoted to solving the scientific problem of optimizing the retail trade in the production and sale of two types of products, taking into account the change in potential demand for products. The economic and mathematical model of the production activity of the enterprise was developed taking into account logistics and market demand. The logistics scheme takes into account all the main links of the logistics system, as well as the connections between them. The considered scheme makes it possible to take into account the diversification

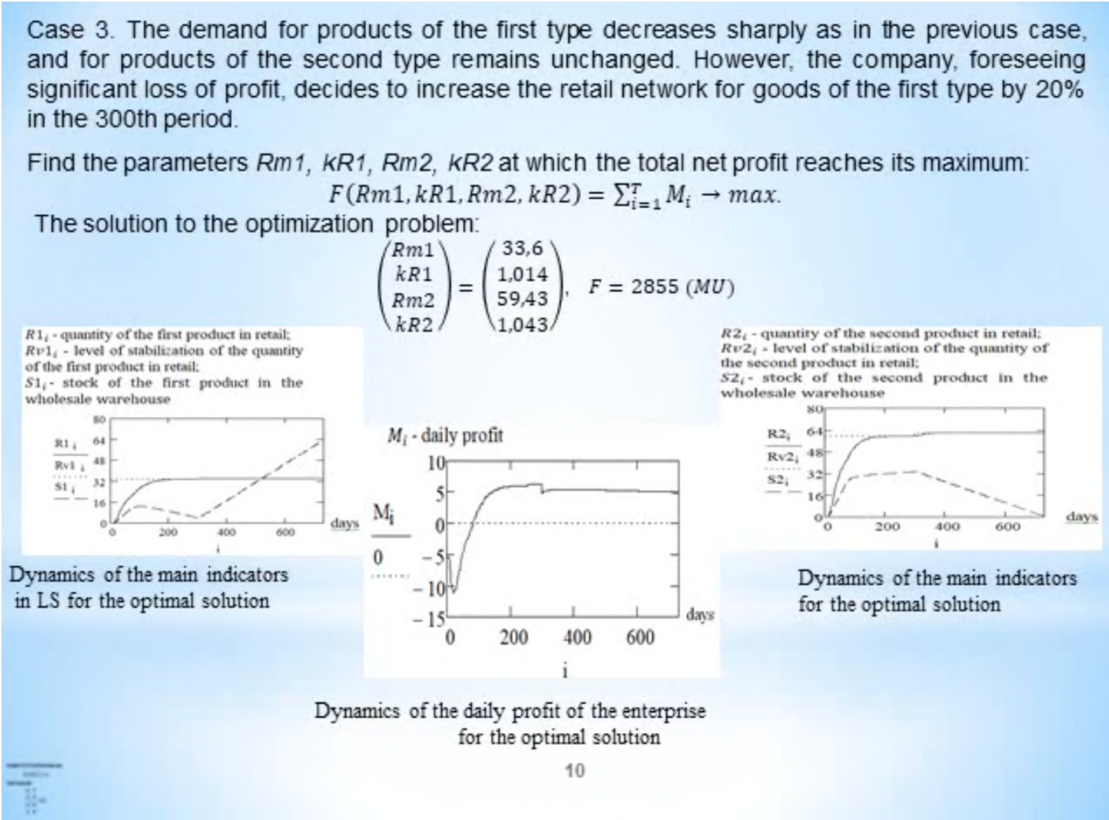


Figure 12: Presentation of paper [70].

of products manufactured by the enterprise. The mathematical model is designed for discrete time. A numerical optimization method has been developed for this mathematical model. The optimal solutions for several cases are found and investigated. The dynamics of the main characteristics of drugs was calculated for all considered cases. A comparative analysis of economic efficiency for the studied cases has been performed. The economic efficiency of retail network optimization is proved.

3. Conclusion and outlook

The vision of the M3E2 2021 is provides a premier interdisciplinary platform for researchers, practitioners and educators to present and discuss the most recent innovations, trends, and concerns as well as practical challenges encountered and solutions adopted in the fields of emergent economy.

The conference has successfully performing forum to transferring and discussing research result among the researcher, students, government, private sector or industries. Participants and presenters from several countries such as Belarus, China, Czechia, Kazakhstan, Moldova, Poland and Ukraine have attended the conference in-person and online to share their significant

Figure 13: Have a nice conference in Odessa, Ukraine!

contribution in research related to Monitoring, Modeling & Management of Emergent Economy.

We are thankful to all the authors who submitted papers and the delegates for their participation and their interest in M3E2 as a platform to share their ideas and innovation. Also, we are also thankful to all the program committee members for providing continuous guidance and efforts taken by peer reviewers contributed to improve the quality of papers provided constructive critical comments, improvements and corrections to the authors are gratefully appreciated for their contribution to the success of the conference. Moreover, we would like to thank the developers and other professional staff of *Not So Easy Science Education* platform (https://notso.easyscience.education), who made it possible for us to use the resources of this excellent and comprehensive conference management system, from the call of papers and inviting reviewers, to handling paper submissions, communicating with the authors etc.

We are looking forward to excellent presentations and fruitful discussions, which will broaden our professional horizons. We hope all participants enjoy this conference and meet again in more friendly, hilarious, and happiness of further M3E2 2022.

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Innovative behavior of bitcoin market agents during COVID-19: recurrence analysis

Hanna Yu. Kucherova[1], Vita O. Los[2], Dmytro V. Ocheretin[2], Olha V. Bilska[1] and Evgenia V. Makazan[2]

[1]*Classic Private University, 70B Zhukovsky Str., Zaporizhzhia, 69002, Ukraine*

[2]*Zaporizhzhia National University, 66 Zhukovsky Str., Zaporizhzhia, 69600, Ukraine*

Abstract

The relevance of the research subject is explained by a fundamental change in the conditions of existence and development of agents of the digital economy, limited knowledge about their behavior under conditions of quarantine restrictions. The aim of the research is to study the series of the dynamics of the price of bitcoin and the frequency of online requests for bitcoin as an indicator of the behavior of agents of the digital economy using the methods of qualitative recurrent analysis. The types of constructed time series plots of the price of bitcoin and the frequency of requests for bitcoin are defined as drift with a superimposed linearly gradually increasing sequence, which indicates the unpredictability of the behavior of digital economy agents with a gradual stabilization in new quality trend. The scientific novelty of the research results lies in the proven connection between the series of bitcoin price dynamics and the frequency of online requests for bitcoin, tracking changes in the behavior of digital economy agents before and after the introduction of quarantine restrictions. The procedure for conducting a qualitative recurrence analysis of the series of dynamics is generalized, which takes into account the specifics of the formation of the frequency of online requests for bitcoin, the price and the behavioral aspect of its formation. The practical value lies in defining the characterization of the behavior model of digital economy agents under conditions of quarantine restrictions. The behavior of digital economy agents in the context of COVID-19 requires further research, in particular, using cross-recurrent analysis methods.

Keywords

COVID-19, cross-recurrent analysis, agents of the digital economy, bitcoin market

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kucherovahanna@gmail.com (H. Yu. Kucherova); vitalos.2704@gmail.com (V. O. Los); odvisua@gmail.com (D. V. Ocheretin); lexusol1987@gmail.com (O. V. Bilska); e.v.makazan@gmail.com (E. V. Makazan)

https://scholar.google.com.ua/citations?user=FhBOVExn1foC (H. Yu. Kucherova); https://sites.znu.edu.ua/cms/index.php?action=news/view_details&news_id=37232&lang=eng&news_code=los-vita-oleksiyivna (V. O. Los); http://sites.znu.edu.ua/cms/index.php?action=news/view_details&news_id=37234&lang=eng&news_code=ocheretin-dmitro-valerijovich (D. V. Ocheretin); https://scholar.google.com.ua/citations?user=FsfgkC0AAAAJ (O. V. Bilska); https://sites.znu.edu.ua/cms/index.php?action=news/view_details&news_id=38742&lang=ukr&news_code=makazan---vgeniya-vasilivna (E. V. Makazan)

0000-0002-8635-6758 (H. Yu. Kucherova); 0000-0002-7932-5232 (V. O. Los); 0000-0001-6705-6381 (D. V. Ocheretin); 0000-0002-0940-8294 (O. V. Bilska); 0000-0001-5855-0476 (E. V. Makazan)

1. Introduction

It is difficult to find a clear generalization of the processes that have begun and are currently taking place in the countries of the world in the COVID pandemic [1, 2, 3]. Unpreparedness for new risks of this type was demonstrated by both individuals and legal entities, authorities and society as a whole. And this is expected, since the collapse of development and interaction of individual sectors of the socio-economic environment has been forming for decades. The unsystematic nature of the government's programmatic actions and the devaluation of the priority issues of ensuring the basic conditions for the survival of society and business have led to the fact that we have become hostages of our own impotence in providing the necessary resistance to the expansion of the disease and the negative socio-economic consequences of the pandemic.

New living conditions draw attention directly to the model of our behavior and in the digital market, determine that the issues of adhering to the security of interactions and the possibility of development within the new established boundaries are priority basic goals for each of us. The established models of the existence of forms and ways of life in the countries of the world have confirmed the need to reduce them, reorient the volumes of consumption, production, change business models, ways of interaction, and so on. New challenges and updated values have actualized the demand for specific goods and services that provide a solution to complex issues of socio-economic security for each in pandemic, have led to fundamental changes in the behavior of entities in all online and offline business markets. Today, timely tracking of changes in behavior in the markets contributes to the formation of a new quality of management, quick adaptation of business, changes in the basic principles of interaction and functioning of subjects of all spheres, clarification of current trends and prediction of the formation of new trends in key indicators.

An immediate rethinking of the socio-economic behavior of everyone is fundamental; the adoption of its new restrictions should take place both at the level of the subjects' consciousness and taking into account the limiting security measures. A business that has been developing offline as a priority, being partially present in the online space, while ignoring the rapid development of Internet technologies, is now unable to overcome the financial and economic losses that were incurred as a result of quarantine restrictions. Therefore, it is logical to actualize the movement of most of the business to the online space, more active use of modern digital technologies and tools.

The parameters of consumer and business requests in the context of the pandemic have undergone fundamental changes both in structure and in quality, in the way of use. Unfortunately, modern socio-economic instruments are not able to describe the general picture of changes in demand and the behavior of economy agents, to demonstrate and reliably predict the trends of current changes in key instruments, the dynamics of which forms the profitability of the markets. Because the external environment during the pandemic has become too unpredictable. Thus, tools such as Google Trend have become useful, where the frequency of queries for keywords makes it possible to track trends in the interests of subjects of the digital market, and not only digital market in real time.

The frequency of online queries as an integrated indicator of information retrieval and the activity of economy agents determines the sphere of consumer interest, which is described by a tuple of parameters $\langle C_S, M, R_F, t\rangle$, where C_S – semantic search characteristics (semantic core), M – meaning of the semantic search, R_F – business activity of agents (frequency of requests), t – time period.

The behavioral model of economy agents is formed in such a way that with an increase in the subject's interest in the target search area, the frequency of online queries for keywords increases. If the agents' interest is not cognitive, but corresponds to the implementation of their strategy in the market, then an increase in the frequency of requests affects the price parameters of target instruments in the direction of their values increase. Thus, an upward trend in the dynamics of time series of key indicators is formed, followed by market monitoring, the results of which arouse interest in relevant modern tools.

This direct cyclical pattern of interdependence of interests in tools and the frequency of corresponding online requests requires system monitoring of their dynamics and an explanation of their changes in different periods of time, which will make it possible to determine or establish a model of agent behavior, factors influencing it and predict its development in new conditions of socio-economic system in offline and online.

Thus, the purpose of the research is the behavior of agents of the digital economy, which is described using the rate of requests for the bitcoin rate in the context of COVID-19. The object of the research is the time series: requests "bitcoin" in English and Russian of Ukrainian users of the digital market, and the price of bitcoin. The subject of research is the methods of nonlinear dynamics.

2. Related works

A significant number of scientific and practical works are devoted to the research of time series of cryptocurrencies, carried out by methods of machine learning and forecasting [4, 5], nonlinear autoregression [6], binomial logistic regression [7], recurrent neural network, ARIMA model [8], Bayesian neural networks [9], theory of complex systems [10, 11, 12, 13, 14], fractal analysis [10, 15, 16, 17, 18, 19, 20] and others. However, the identification of the behavior of digital economy agents requires a search for specific parameters that would uniquely determine it. The search for such parameters is difficult, since their characterized by weak structuring and significant subjectivity. So, to date, the results of the influence of social networks on the dynamics of the indicators of the digital market [8] have been obtained, including a quantitative relationship between social networks and the bitcoin price [15, 16, 17], a connection between the parameters of bitcoin and online searches in Google has been confirmed [19], Google queries and exchange rates [19]. Based on the results of the research, the authors confirm the thesis that the indicators of online search and social networks really determine the dynamics of the rate of cryptocurrencies, therefore, this direction requires further research. Researchers [4, 8, 9, 21, 22] have proved that the complexity of the processes, the peculiarities of the formation time series of bitcoin and the frequency of requests for bitcoin require the use of the methodology of nonlinear dynamics, the methods of which are actively used in the study of processes during the manifestation of crisis phenomena [6, 10], which in fully complies with the conditions of today. However, a fairly powerful toolkit of nonlinear dynamics only partially used in the study of the behavior of economy agents in the online information space, which proves the timeliness of this study.

3. Matherials

According to Google Trends data [23], the search query “bitcoin” is characterized by a simple semantic core, which serves as the basis for the formation of a more complex, refined online information search. In particular, online information requests such as “bitcoin halving”, “price bitcoin”, “what is bitcoin”, “bitcoin course”, “bitcoin dollar”, “bitcoin price usd”, “freebitcoin” have shown the maximum popularity in the online environment over the past year. Interest in bitcoin is not evenly distributed in Ukraine, most often in Kyiv, Kyiv region, Kharkiv, Odesa and Chernivtsi regions are interested in its price and characteristics. Google Trends the values of the frequency of requests in the Ukrainian language as insufficient in importance. Google Trends noted that the frequency of Bitcoin requests was very popular in Russian and English. At the same time, the dynamics of the frequency of requests in the Ukrainian language showed a similarity of the dynamics of the frequency of requests in Russian. The behavior of market agents in terms of the frequency of Bitcoin requests in Ukrainian the same as the frequency of Bitcoin requests in Russian and English. The dynamics of the studied time series is shown in figure 1.

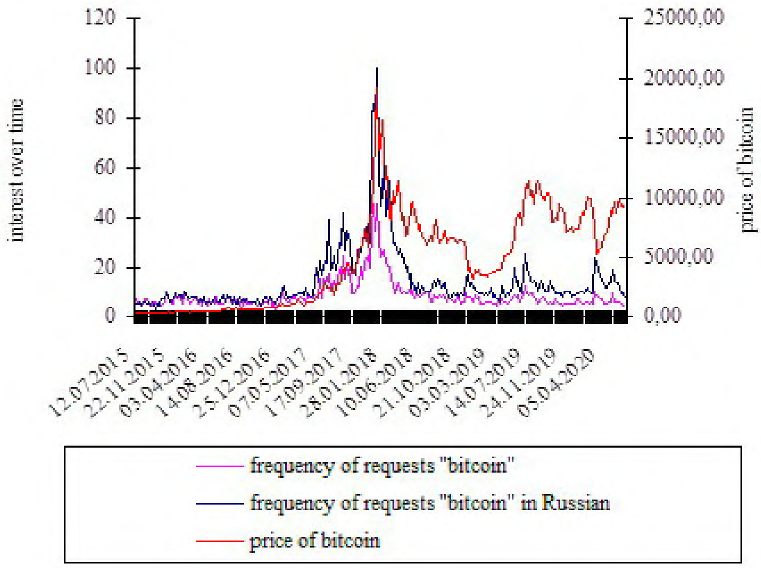


Figure 1: Frequency of requests “bitcoin”, “bitcoin” in Russian on Google in Ukraine.

Figure 1 shows that the dynamics of indicators is characterized by a high value of volatility. The dynamic series of the frequency of requests “bitcoin” is described by a 70.58% coefficient of variation, “bitcoin” in Russian – 90.48%, and the price of bitcoin – 83.43%. In addition, the trend in the frequency of requests for bitcoin in English and Russian are similar, but differ in the values of the number of requests, which is quite logical, taking into account the dominant languages in the country.

The maximum value of the bitcoin price for the study period was observed on December 17, 2017 and amounted to $19140.8 , the minimum – on August 23, 2015 ($228.17). However, the maximum value of the frequency of requests "bitcoin" (46 points) and "bitcoin" in Russian (100 points) was also reached on December 17, 2017. The minimum value of the series of dynamics of the frequency of requests for "bitcoin" of Russian at the level of 3 points was observed on September 27, 2015 and August 07, 2016, and the series of dynamics of the frequency of requests for "bitcoin" on December 11, 2016.

The maximum increase in the price of bitcoin [24] for the study period was recorded at the level of 41.5% (July 23, 2017), that is, the price increased by 800.6 dollars relative to the previous period (July 16, 2017). The maximum decrease in the price of bitcoin was observed precisely at the beginning of quarantine measures in March 2020 (March 15, 2020). Bitcoin price fell by 33.5% or $2,715.8 relative to the previous period (March 08, 2020). In general, on average, over the period under study, the price of bitcoin increased by 1.9% weekly. So, in June 2020, the price of bitcoin increased 29.4 times compared to the same period in 2015.

Studying individual periods of the time series of dynamics, it was found that the maximum values of the price of bitcoin and the frequency of online requests for bitcoin either coincide in the period, or are very close in time. Also, sharp trends in the price of bitcoin are accompanied by a surge in interest in it, even if the price trend is decreasing. Whereas the stable fluctuations of the bitcoin price within the boundaries of a certain corridor, to a lesser extent, but still of interest to economy agents.

It is proposed to eliminate the unpredictability of the studied time series of dynamics using recurrent analysis, the implementation of which will make it possible to characterize the behavior of digital economy agents in new conditions of existence and development.

4. Methodology

To study the time series, the authors proposed a methodology based on the method of recurrence analysis for analyzing the behavior of digital economy agents in the context of COVID-19.

The input time series for the analysis were generated using data from Google Trends [23] and InvestFunds [24] for the period from July 12, 2015 to June 28, 2020. Authors investigate three time series, namely: the frequency of online requests for bitcoin in Ukraine on English and Russian, and the price of bitcoin. After the formation of the database, the behavior of the dynamics of the studied time series should be analyzed in order to establish the type of behavior and the presence of a trend or random behavior of the series.

To apply the proposed methodology, the data values of the initial series were normalized in the interval [0, 1] by the formula

$$x_{norm} = \frac{x_i - x_{\min}}{x_{\max} - x_{\min}}, \tag{1}$$

where x_{norm} – normalized time series value;

$x_{\max}$, $x_{\min}$ – maximum and minimum value of the time series, respectively.

After the formation of the set of initial data, the next step is the choice of the reasonable delay of the time series (G). Choosing G should take into account the following [19]:

- G must be large enough so that the value $x(t)$ different from the value $x(t+G)$;
- but if G is too large, then at time $(t+G)$ the system will lose information about what happened at time t.

Taking into account the above, the authors define the function of mutual information (AMI) S for the time series that analyzed, taking into account nonlinear correlations [25]. The time series delay time G corresponds to the first local minimum of the mutual information function (AMI). The time series reasonable delay time was calculated in the R environment using the tseriesChaos library.

To determine the dimension of the time series, the authors used the false nearest neighbor method given in the work [26]. This method is based on the assumption that in a correctly constructed attractor the neighboring points of the phase trajectory remain very close at the following iterations. If the nearest points diverge, then they are called false nearest neighbors. The problem is to choose a dimension of the phase space r at which the fraction of points with false neighbors is minimized. The calculation of false neighbors was calculated in the R environment using the fractal library.

The next step is to build a recurrence plot based on the calculated space dimension and time delay parameters. Recurrence plot is the projection of m-dimensional pseudophase space onto a surface. Let the point x_i correspond to the point of the phase trajectory $x(t)$ that describes the dynamical system in the m-dimensional space at the time $t = i$ $(i = 1, \dots, N)$, then the recurrence plot is an array of points in which a nonzero element with coordinates (i, j) corresponds to the case when the distance between x_i and x_j is less than γ:

$$RP_{i,j} = \theta\left(\gamma - \|x_i - x_j\|\right)\ x_i,\ x_j\ \in R^m\,(i, j = 1, \dots, N), \tag{2}$$

where γ – the size of the neighborhood of the point x_i,

$\|x_i - x_j\|$ – distance between points,

$\theta(\cdot)$ – Heaviside function.

The next stage of the proposed methodology is analysis of the measures of recurrence plot. This analysis makes it possible to determine the measures of complexity of the structures of recurrence plot, such as: recurrence rate, percent recurrence, percent determinism, average diagonal line length, maximum diagonal line length [17, 27, 28]. Let's consider these measures in more detail. Recurrence rate (RR) shows the density of recurrent points, that is, it characterizes the probability of repetition of system states:

$$\mathrm{RR} = \frac{1}{N^2}\sum_{i,\,j=1}^{N} R_{i,j}, \tag{3}$$

where N – number of points on the phase space trajectory.

Percent recurrence ($\%REC$) displays a decrease in the regularity of the system's behavior and is used to analyze the dynamic structure of a time series:

$$\%REC = \frac{\sum\limits_{l=1}^{N} lP(l)}{N} \cdot 100. \tag{4}$$

If the percent recurrence value is less than 1%, then it is said that there is a regular, clearly defined dynamics of the time series behavior. For the regularity of the system's behaviors percent recurrence value from 1% to 5%. If the value of the percent recurrence lies in the range from 5% to 20% or more, then this indicates that the dynamics of the time series behavior is not regular and noisy interevent-type data. Depending on the parameters of the delay and the dimension of the phase space, the following average values of the percent recurrence can be obtained: a low ($\%REC \approx$ 1% to 3%), moderate ($\%REC \approx$ 5% to 10%), and high ($\%REC \approx$ 15% to 20%) [29].

Percent determinism ($\%DET$) considers the diagonal lines of the recurrence plot. The frequency distribution of the lengths of the diagonal lines in the recurrence plot can be written as follows: $P^{\gamma}(l) = \{l_i,\ 1, ..., N_l\}$, where l_i – i-th diagonal line length, N_l – the number of diagonal lines (each line is counted only once). If the time series is stochastic, then the diagonal lines of the recurrence diagram are very short or completely absent. And deterministic time series are characterized by long diagonals and a small number of separate recurrent points.Percent determinism ($\%DET$) is defined using this formula:

$$\%DET = \frac{\sum\limits_{l=l_{\min}}^{N} lP(l)}{\sum\limits_{l=l_1}^{N} lP(l)} \cdot 100. \tag{5}$$

Percent determinism characterizes the level of predictability of the time series.

Diagonal structures show the time during which a section of a trajectory comes close enough to another section of a trajectory. Thus, these lines allow conclusions to be drawn regarding the divergence of the trajectory elements. These indicators are average diagonal line length (ADL) and maximum diagonal line length (MDL).

Average diagonal line length (ADL) characterizes the average time during which two sections of the trajectory pass close to each other, and can be considered as the average predictability time of the series. This measure is calculated using the formula:

$$ADL = \frac{\sum\limits_{l=l_{\min}}^{N} lP(l)}{\sum\limits_{l=l_{\min}}^{N} P(l)}. \tag{6}$$

Respectively, maximum diagonal line length (MDL) characterizes the length of the trend and is defined as:

$$MDL = \max\left(\{l_i;\ i = 1, ...,\ N_l\}\right). \tag{7}$$

Based on the analysis of the statistical characteristics of the recurrence plot, it is possible to determine the presence of homogeneous processes with independent random values, processes with slowly varying parameters, periodic or oscillating processes that correspond to nonlinear systems. Thus, the analysis of the recurrence plot allows one to evaluate the characteristics of a nonlinear object on relatively short time series, which makes it possible to make decisions regarding the object's control. The measures of the recurrence plot was calculated in the R environment using the nonlinearTseries library.

5. Results

Based on the results of the calculations, recurrence plots were obtained, the topological analysis of which makes it possible to determine the structure, type, changes in the behavior of the research object, the boundaries of phase transitions, and to establish the sensitivity of quantitative measures.

Recurrence quantification analysis (RQA) can be used not only to quantify the dynamics of a whole time-series, but also to study changes in the dynamics of a time series. Windowed RQA is potentially a very powerful tool for detecting changes in subsets of the whole time-series. To conduct a better study of the temporal structure, the dynamic series of the bitcoin request frequency (in English) and time-series of price of bitcoin were divided into four periods, the length of each of which was 65 observations: A) July 12, 2015 – October 02, 2016, B) October 09, 2016 – December 31, 2017, C) January 07, 2018 – March 31, 2019, D) April 07,2019 – June 28, 2020. Recurrence plot of the behavior of digital economy agents were constructed and RQA was calculated for each subset of the time series (adjacent non-overlapping windows of 65 data points) (figure 2).

Gradual changes in the parameters of the behavior of digital economy agents are clearly visible, in particular during period B (October 09, 2016 – December 31, 2017), where the drift of the attractor was revealed (white lower and upper corners of the plot, a diagonal line) and the emergence of a new structure during the period of influence of the consequences COVID-2019 in period D (April 07,2019 – June 28, 2020).

Analyzed period on the recurrence plot is displayed by contrasting areas and stripes, which is explained by sharp changes in values, randomness, rarity, in general, stochastic behavior of digital market entities, unpredictability of changes in interest in it.

However, separately placed random recurrent points are not the result of randomness, their presence proves that interest in the price of bitcoin is simply not stable over time and can be caused by significant fluctuations. The measures of the recurrence plot are investigated such as: percent recurrence (*%REC*), percent determinism (*%DET*), average diagonal line length (*ADL*), maximum diagonal line length (*MDL*) (table 1).

Table 1
Recurrence Quantification Analysis of the frequency of requests "bitcoin"

Measures of the recurrence plot	1 plot (A)	2 plot (B)	3 plot (C)	4 plot (D)
%REC	71.46	9.83	54.67	57.93
%DET	93.78	73.87	93.86	89.60
ADL	4.80	3.97	5.74	3.12
MDL	35.00	6.00	39.00	27.00

The change in the *%REC* value from 71.46% to 57.93% in the periods under study proves that the behavior of digital market entities is unstable, noisy interevent-type data. The lowest value

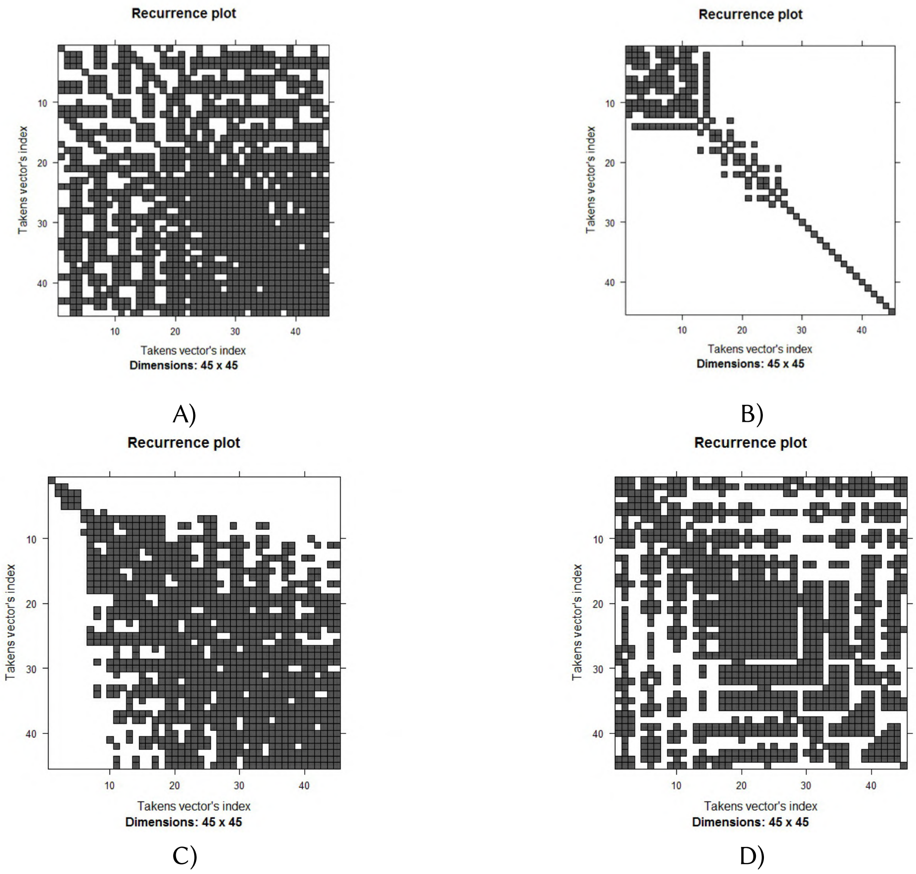


Figure 2: Recurrence plot of the behavior of digital economy agents (frequency of online requests "bitcoin" according to Google Trends in Ukraine): A) July 12, 2015 – October 02, 2016, B) October 09, 2016 – December 31, 2017, C) January 07, 2018 – March 31, 2019, D) April 07,2019 – June 28, 2020.

of *%REC* (9.83%) was recorded in period B, when the trajectory of behavior reaches a new level of development of processes. Starting from period C, the *%REC* value increases significantly and fluctuates within the range of 54-57%. The obtained values indicate an increase in irregularity and the presence of noisy interevent-type data in this time series. Percent determinism (*%DET*) ranges from 73-93%. In addition, in periods A and C the measure value is almost the same, while in period D it slightly decreased to 89.60%.

Average trend predictability time (*ADL*) and maximum diagonal line length (*MDL*) are lowest in periods B and D. Want in period D the *MDL* value reaches 27 points, which is 4.5 times more than the same value in period B. Thus, the trajectory of the behavior of digital economy agents demonstrates a phase transition to another level of development in period B and the formation of a new, approximate model to the existing one, but with new qualitative parameters in period D.

At the same time, the recurrence plots for the corresponding periods of the bitcoin price are also characterized by the attractor drift. However, the dynamics of the indicator adheres to the original trajectory from September 10, 2016 and takes into account the changes in processes during the D period as a result of the impact of COVID-2019 (figure 3).

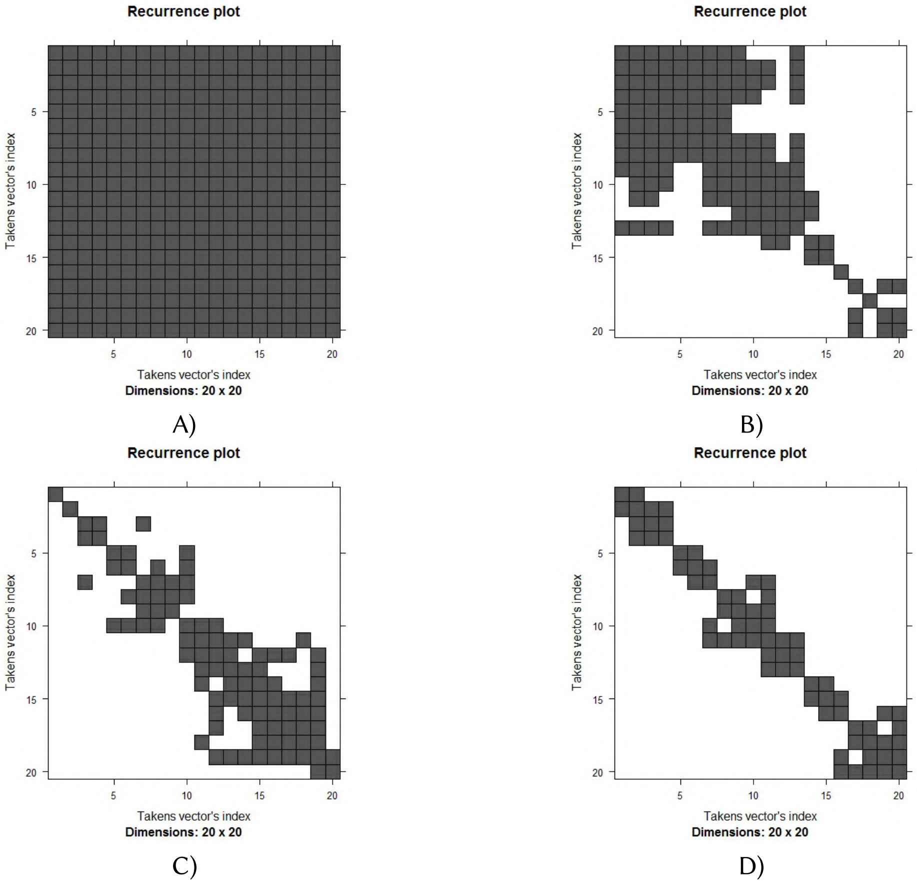


Figure 3: Recurrence plot of the price of bitcoin: A) July 12, 2015 – October 02, 2016, B) October 09, 2016 – December 31, 2017, C) January 07, 2018 – March 31, 2019, D) April 07,2019 – June 28, 2020.

Despite the powerful volatility and extreme growth in the price of bitcoin (in particular in 2018), the symmetry of plots proves the constancy of the formation of bitcoin price over time, the tendency of points to a given trajectory, but with the inherent randomness of qualitative values. The measures of the recurrence plot for price of bitcoin are in table 2.

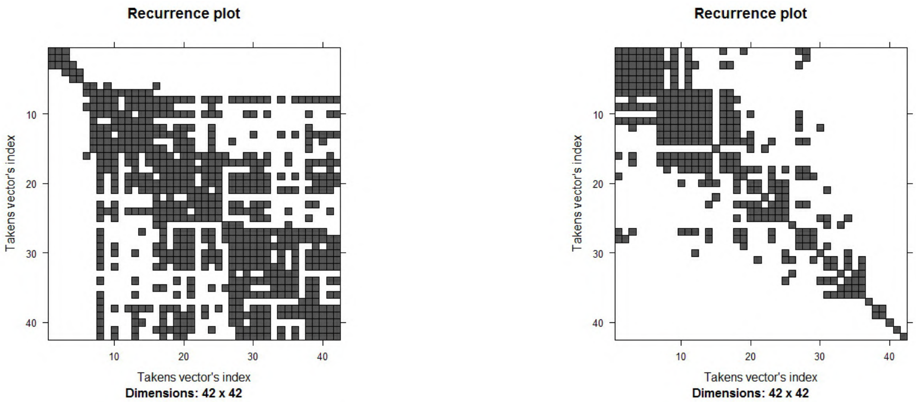
Table 2
Recurrence Quantification Analysis of the price of bitcoin

Measures of the recurrence plot	1 plot (A)	2 plot (B)	3 plot (C)	4 plot (D)
%REC	1	37.5	24.5	17.5
%DET	99.5	82.67	77.55	82.86
ADL	19	12	10	5
MDL	10.76	6.53	4.47	3.87

The *%REC* value of the series of bitcoin price dynamics is lower than the value of the similar measure of the frequency of requests for bitcoin, which proves the more constant behavior of digital economy agents. However, in period A, a time series of bitcoin price dynamics were characterized by regularity and clear definition of the dynamics of changes in the time series ($\%REC = 1\%$).

The indicators of predictability of the time series (*%DET*) are similar with the dynamics of changes in requests for bitcoin in periods B and D, but differ significantly in values. In particular, the lowest values of the measure were recorded in period C.

However, the time series of bitcoin price dynamics are characterized by a high level of unpredictability, the average predictability time significantly decreased from 19 points in period A to 5 points in period D. The length of the trend decreased from 10.76 points in period A to 3.87 points in period D. Another result was obtained from the data of recurrence plot of agents of the digital economy for the indicator of the frequency of requests "bitcoin" in Russian (figure 4, figure 5). And this is logical, since Ukraine is a bilingual country (Ukrainian and Russian languages).

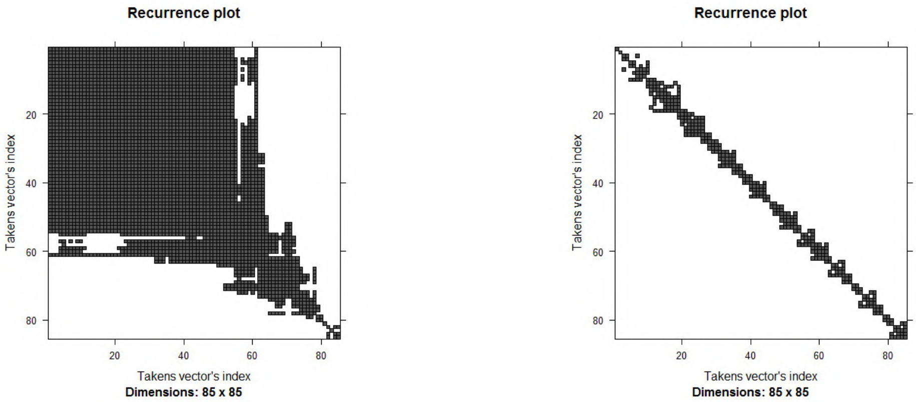


A) B)

Figure 4: Recurrence plot of the behavior of digital economy agents (frequency of online requests "bitcoin" in Russian according to Google Trends in Ukraine): A) July 12, 2015 – December 31, 2017, B) January 07, 2018 – June 28, 2020.

In figure 5, period A clearly shows the drift of the attractor, the formation of a strong trend in the price of bitcoin, which is displayed in the form of an arrow. The formation of a trend is accompanied by an increase in interest in price changes, the frequency of requests is not stable in time for the frequency and value (figure 4, A).

Black stripes characterize nonstationarity of behavior, which means the formation of a transitional period. Periodic patterns characterize the cyclical nature of certain changes in interest in the price of bitcoin, the distance between which determines the period. Black dots, which are repeated in isolation, characterize a random interest in the price of bitcoin, its rapid changes (figure 4, A). The formation of the price trend, which is presented on the recurrence plot for period B (figure 5, B), is characterized by a certain stationarity of fluctuations within the accepted range of values. The fading of bitcoin price fluctuations is accompanied by the



A) B)

Figure 5: Recurrence plot of the price of bitcoin: A) July 12, 2015 – December 31, 2017, B) January 07, 2018 – June 28, 2020.

systematization of interest in it, streamlining the behavior of digital economy agents, reducing it to the usual monitoring of prices (figure 4, B). The measures of the recurrence plot for price of bitcoin and frequency of online requests "bitcoin" in Russian are in table 3.

Table 3
Recurrence Quantification Analysis of the price of bitcoin and frequency of online requests "bitcoin" in Russian

Measures of the recurrence plot	Price of bitcoin		Frequency of online requests "bitcoin" in Russian	
	1 plot (A)	2 plot (B)	1 plot (A)	2 plot (B)
%REC	54.7	5.41	20.07	39.45
%DET	98	82.6	71.19	74.71
ADL	19.1	5.7	4	3,9
MDL	77	35	13	19

The constancy and regularity of the bitcoin price dynamics significantly increases in period B compared to period A, the *%REC* value changed from 54.71% to 5.41%. However, the percent determinism (*%DET*) is reduced from 97.97% to 82.61% in the corresponding periods. Over the selected periods, the average predictability time decreased from 19.08 points to 5.7 points, and the trend length decreased by half – from 77 points to 35 points. There is a change in the phase space, trends in general. Other trends show metrics for the time series of bitcoin requests.

The percent determinism (*%DET*) is decreasing, from 97.97% to 82.61% in the corresponding periods. The average predictability level slightly decreased from 4 points to 3.9 points, and the trend length increased from 13 points to 19 points, which confirms the formation of a clear trend and a strong trend.

6. Conclusions

As a result of the research, the relationship between the time series of the price of bitcoin and the frequency of online requests for bitcoin as an indicator that characterizes the behavior of agents of the digital economy. The behavior differs were confirmed in the periods before and after the introduction of the COVID-19 quarantine restrictions. An increase in the activity and internal disturbances in the behavior of agents of the digital economy during periods of significant volatility in the price of bitcoin and, on the contrary, its decrease during a period of stabilization and insignificant fluctuations in a certain price band was confirmed. This pattern is explained by the decentralization of the digital market tools, the complexity of the processes of internal self-organization of agents, the strategies they have chosen in the digital market, their role, emotional-volitional, behavioral and cognitive features that are uncommonly manifested during quarantine restrictions. The approach and procedure for the implementation of the Recurrence Quantification Analysis (RQA) of the time series of the price of bitcoin and the frequency of online requests about it are determined. The investigated recurrence plots of the bitcoin price and the frequency of online requests for bitcoin indicate the unpredictability of the behavior of digital economy agents with a pronounced linear trend. As a result of the research of windowed RQA, the authors came to the conclusion that the behavior of the agents of the digital economy (the frequency of requests for "bitcoin" in English) showed a change in trend precisely in the period October 09, 2016 – March 31, 2019, where the drift of the attractor and the emergence of a new structure were revealed. During the period of COVID-2019 quarantine restrictions, the behavior of economy agents is characterized by an increased level of stochastic, although interest in the price of bitcoin is unstable over time, but can be caused by significant fluctuations. A qualitative analysis of recurrence plots of the behavior of digital economy agents confirmed the formation of a new phase transition and destabilization during the period of quarantine restrictions, which forms a behavior model similar to the previous one, but with new qualitative parameters. The bitcoin price is characterized by the drift of the attractor, the constancy of price formation over time, adherence to the initial trajectory and the tendency of points towards it with a characteristic randomness of qualitative values, which is explained by changes in socio-economic processes, in particular as a result of the impact of COVID-2019. However, the time series of the bitcoin price and the frequency of requests "bitcoin" in Russian are qualitatively different from the similar series of the frequency of requests "bitcoin" in English for Ukrainian online-user. The investigated series of bitcoin price dynamics over two periods also demonstrates the drift of the attractor and the formation of a strong trend, which is accompanied by an increase and stabilization of interest in price fluctuations. The frequency of online requests is unstable over time only during the period July 12, 2015 – December 31, 2017. The non-stationarity of the behavior of economy agents was confirmed, the formation of a transitional period with a characteristic alternation of cyclical and random changes in interest in the bitcoin price in the period July 12, 2015 – December 31, 2017 was recorded. Whereas the period January 07, 2018 – June 28, 2020 is characterized by a certain stationarity of fluctuations, which is accompanied by the systematization of interest in the price of bitcoin, streamlining the behavior of agents of the digital economy, reducing it to the usual monitoring of prices. Thus, the behavior of digital economy agents in the context of COVID-2019 has generated a new trend with updated qualitative indicators, which requires further research, including using cross-recurrent analysis methods.

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Comparative analysis of the stock quotes dynamics for IT and the entertainment industry companies based on the characteristics of memory depth

Nataliia K. Maksyshko, Oksana V. Vasylieva

Zaporizhzhia National University, 66 Zhukovsky Str., Zaporizhzhia, 69600, Ukraine

Abstract

The article is devoted to the study and comparative analysis of the stock quotes dynamics for the world's leading companies in the IT sector and the entertainment industry. Today, these areas are developing the fastest and most powerful, which attracts the attention of investors around the world. This is due to the rapid development of digital communication technologies, the growth of intellectualization and individualization of goods and services, and so on. These spheres have strong development potential, but the question to how their companies' stock quotes respond to the impact of such a natural but crisis phenomenon as the COVID-19 pandemic remains open. Based on the nonlinear paradigm of the financial markets dynamics, the paper considers and conducts a comprehensive fractal analysis of the quotations dynamics for six leading companies (Apple Inc., Tesla Inc., Alphabet Inc., The Walt Disney Company, Sony Corporation, Netflix) in this area before and during the COVID-19 pandemic. As a result of the application of the rescaled range analysis (R/S analysis), the presence of the persistence property and long-term memory in the stock quotes dynamics for all companies and its absence in their time series of profitability was confirmed. The application of the method of sequential R/S analysis made it possible to construct fuzzy sets of memory depths for the considered time series and to deepen the analysis of the dynamics due to the quantitative characteristics calculated on their basis. Taking into account the characteristics of memory depth in the dynamics of quotations made it possible to conduct a comparative analysis of the dynamics, both under the influence of the natural crisis situation and in terms of investing in different terms. The peculiarities of the delayed profitability dynamics of quotations for each of the companies are also taken into consideration and compared. The developed recommendations can be used in investment activities in the stock market.

Keywords

sequential R/S analysis, fuzzy sets, stock market

1. Introduction

Trading in financial instruments on stock exchanges is increasingly becoming a source of income for various investors. At the same time, investors face the problem of choosing financial instruments in which to invest. For the investor, effective management of their financial

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maxishko@ukr.net (N. K. Maksyshko); oksanabay@ukr.net (O. V. Vasylieva)
http://sites.znu.edu.ua/cms/index.php?action=news/view_details&news_id=37228&lang=eng&news_code=maksishko-nataliya-kostyantinivna (N. K. Maksyshko); https://sites.znu.edu.ua/cms/index.php?action=news/view_details&news_id=37230&lang=eng&news_code=vasil--va-oksana-volodimirivna (O. V. Vasylieva)
0000-0002-0473-7195 (N. K. Maksyshko); 0000-0002-6332-2707 (O. V. Vasylieva)

resources now – means to get additional benefits in the future. But in order to get these benefits, they need to compare financial instruments and choose the most profitable and least risky among them.

The issue of analyzing the stock markets dynamics in order to develop practical recommendations for the investor is not new, but it remains relevant and extremely important [1]. Development of investment strategy and awareness of prospects and risks of specific investment instruments is the key to successful investment activities.

For comparative analysis, traditionally, statistical characteristics of dynamics are used. Traditionally, statistical methods have been used to confirm the efficient market hypothesis [2]. For example, in [3]: Kolmogrov-Smirnov and Shapiro-Wilk tests; a run test and an auto-correlation test are used to check the randomness and the normality of the data. In [4] there is applied a rolling variance ratio test; in [5] European stock markets are tested by a runs test and joint variance ratio tests. In the study [4] the French stock market is considered to change its properties from efficient to adaptive.

An alternative theory of financial markets is the fractal market hypothesis [6], which considers markets as complex nonlinear systems in which randomness and hidden patterns interact, resulting in the dynamics of financial instruments has a fractal nature and the property of persistence (the presence of long-term memory). Methods of nonlinear dynamics are considered as relevant tools of its research. Therefore, recently more and more attention is paid to the study and application of these methods.

The most popular method of studying the fractal properties of dynamics is the Hurst exponent (R/S analysis) [6]. The paper [7] provides an excursion into the research of scientists on the application of the Hurst exponent to analyze the dynamics of capital markets.

Along with the Hurst exponent for the study of financial markets and, in particular, stock markets, the following indicators are used: Lyapunov exponent (indicator of nonlinear dynamics) to diagnose the crash of stock markets [8], Shannon information entropy [9, 10], Renyi entropy [10], the Hurst-Holder exponent [11], local Whittle estimator [12]. The presence of fractal properties in the dynamics of financial markets is also investigated by calculating the Hausdorff dimension and applying the Mittag-Leffler functions [13].

In particular, in the works [14, 15], a method of sequential R/S analysis is proposed, which allows not only to establish the presence of long-term memory in the time series, but also to evaluate its quantitative and qualitative characteristics. The authors show that for the dynamics of different financial instruments that have long-term memory, memory characteristics may differ.

The authors also consider the use of different tools of nonlinear dynamics in terms of different stock markets and their segments. For example, in [16, 17] – the stock markets of individual countries are considered, in [18] – quotations of shares of certain companies, in [19] – multifractality the autocorrelations in stock portfolio returns is studied and other.

The research and comparison object of this work is the dynamics of shares quotations of leading companies in the IT sector and the entertainment industry. Today, these areas are developing the fastest and most powerful, which attracts the attention of investors around the world. This is due to the rapid development of digital communication technologies, the growth of intellectualization and individualization of goods and services, and so on. These areas have strong development potential, but the question of how stock quotes in this area respond, for example, to the impact of such a natural but crisis phenomenon as the COVID-19 pandemic remains open.

The hypothesis of this work is that the characteristics of long-term memory can also be used for comparative analysis and development of recommendations for investors operating in the stock market. The purpose of this work is to study the characteristics of the memory depth for the stock quotes dynamics of selected companies, their stability under the influence of such a crisis as the global COVID-19 pandemic, conducting a comparative analysis based on them and developing recommendations for investors.

2. Materials, methods and results

2.1. Materials and methods

Since the behavior of stock prices in the stock markets is mostly not normally distributed or close to normal distribution [6], after testing for deterministic chaos and proof of its fractal nature, it is advisable to use fractal analysis methods for its study. Such methods include the method of the Rescaled range or R/S analysis of Hurst (denote it A_1) [6] and the method of sequential R/S analysis (A_2) [14].

Let the time series (TS) X be considered. The result of applying method A_1 is to determine the Hurst exponent (H) and check its significance based on the application of the mixing test.

The value of the Hurst exponent $H \in [0; 1]$ determines the presence of certain properties of the dynamics: the value of $H \in (0; 0.5)$ corresponds to the antipersistent or ergodic TS; the value of $H = 0.5$ (and in its vicinity) indicates a random TS, in which events are random and uncorrelated, the present does not affect the future; the value of $H \in (0.5; 1]$ indicates that the TS is persistent or trend-resistant, characterized by the presence of long-term memory. The closer the value of H to 1, the more correlated the levels of the series.

Recall that the essence of the method of the Rescaled range exponent of Hurst A_1 is to construct the R/S trajectory for the TS X and determine the angle of the linear trend, built on its (R/S trajectory) starting points. At some value of $k = k_0$ R/S trajectory changes its slope quite sharply, i.e. at the point (x_{k_0}, y_{k_0}) the trajectory receives a significant in absolute value negative gain $\delta_k = y_{(k+1)} - y_k$ – there is a break from the trend and there is no return to the previous trend. It is assumed that at the point k_0 the effect of “long-term memory of the beginning of the series” dissipates. That is, the “breakdown of the trend” demonstrates the loss of memory about the initial conditions, and also signals (possibly with a lag, ie with some delay) the exhaustion of the cycle or quasi-cycle contained in the initial segment of this TS. According to [6], we adhere to the statement that after the end of the cycle (quasi-cycle) the memory about the initial conditions is lost, ie the long-term correlation of the following observations with respect to the initial one is lost. However, based on the peculiarities of the construction of the R/S-trajectory [6], the method of the R/S analysis of Hurst A_1 provides only (statistically) the average characteristic of the trend stability of the TS X as a whole and does not take into account the changing dynamics during the whole observation period.

However, the method of sequential R/S analysis A_2 [14] by modifying the scheme of construction of R/S-trajectory, allows to take into account the changing nature of the dynamics, to identify many cycles (quasi-cycles) that are characteristic of the studied TS, and ensure a more detailed assessment of the memory depth from the beginning of this TS.

Performing an iterative procedure (method A_3) using method A_2 and detecting the point of memory loss at the beginning of the time series for a set of nested segments $X = X^0 \supset X^1 \supset \ldots \supset X^r \supset X^{n-3}$(a family of time series differing by the starting point) allows to estimate the memory depth as a fuzzy set "memory depth of the TS as a whole" [15]. Note that the transition from a "clear" estimate (based on probability and requiring statistical significance) to a fuzzy estimate (non-additive measure) is due to the availability of data for which these requirements are not met.

According to [14, 15] the concept of "memory depth of TS X" is defined as a discrete fuzzy set (FS)

$$L(X) = \{(l_i, \mu_L(l_i)); l_i \in N\}, \mu_L : N \longrightarrow [0; 1], \tag{1}$$

where $N = \{l_i, i = 1, 2, ...\}$ – natural numbers set – set of basic values for memory depth,

$\mu_L(l_i) = \mu(l_i)$ – the value of the membership function, which determines the degree of belonging of a natural number l_i ("depth l_i") to a fuzzy set $L(X)$. The function $\mu_L(l_i)$ displays the base value l_i in the interval [0; 1] and displays the degree of possibility (confidence measure) in relation to the membership of the element l_i fuzzy set of memory depth $L(X)$.

The carrier of the fuzzy set $L(X)$ is the set "supp" $L(X) = L^0 = \{l_i \in N, i = 1, 2, ... : \mu_L(l_i) > 0\}$. Therefore, we finally consider the fuzzy set of memory depth of the time series X as a whole in the form

$$L(X) = \left\{(l, \mu(l)), l \in L^0\right\}. \tag{2}$$

Important characteristics based on the use of fuzzy memory set (2) are given in [15]. In this paper, for a comparative analysis of price dynamics, the following characteristics are used: l_{max} – the greatest value of the memory depth encountered:

$$l_{max} = \max_{l \in L^0} l, \tag{3}$$

l_{mc} – the memory depth value that has the largest value of the membership function $\mu(l)$:

$$\mu(l_{mc}) = \max_{l \in L^0} \mu(l), \tag{4}$$

l_{gc} – the gravity center of the fuzzy set $L(X)$ as a whole – is obtained using the defuzzification procedure:

$$l_{gc} = \frac{\sum_{l \in L^0}[l \cdot \mu(l)]}{\sum_{l \in L^0} \mu(l)}, \tag{5}$$

H_{entr_L} – information entropy indicator for the fuzzy set of memory depth $L(X)$ – used to assess the uncertainty degree regarding the variety of TS's behavior variants:

$$H_{entr_L} = -\sum_{l \in L^0} \mu(l) * \log_2 \mu(l). \tag{6}$$

Note that the scale for estimating the degree of uncertainty is considered to be ordinal by type.

Thus, the use of the selected considered characteristics of the memory depth for time series allows to deepen the comparative analysis of the stock prices dynamics for companies in the IT sector and the entertainment industry.

2.2. Stock market overview and input data

Today in the modern stock market there are separate sectors, which on the basis of information from sites [20, 21], are distributed according to the level of volatility.

The names of the companies for reduction are given according to their stock tickers.

Very high volatility: energy sector – oil (PZE, EC, HOM), gas (GAS, WGR, SPH), coal (BTU, ACI, MEE), alternative fuel (PTOI, GRPEF, BLDP); Industrial sector – industrial electronics (EDIG, SIRC, RGSE), industrial products (BOOM, PLPC, TPIC), machinery (SOHVY, CTEQF, PERT), aviation and space (Boeing, Generaldynamics, Tesla).

High volatility: Basic resources and materials – gold (HGLC, RMRK, NSRPF), metals (ACH, USNZY, GGIFF), chemical products (SKVI, LPAD, NL), forest resources and paper production (AZFL, NKSJF, FBI); Financial services – brokers (GS, LEH, BSC), banks (C, USB, NYB), insurance companies (AIG, XL, JH), management companies (WM, JNS, JNC), stock exchanges (CME, NYX, ICE), mortgage (FNM, FRE, AHM); Media / Entertainment – Worldflix Inc (WRFX), Dolby Laboratories (DLB), Disney (DIS), New-York times (NYT), Netflix (NFLX).

Average volatility: Retail and wholesale – clothing (DEST, BGI, ANF), food (WDRP, CHEF, GPDB), medicines (THCBF, HEWA), household goods (UPPR, RH); Medicine, pharmaceuticals, health care – biotechnology (VKTXW, DMTX, GNLKQ), pharmaceuticals (GMBL, MNZO), honey equipment (ARYC, SMLR, PLSE); Technology sector – Apple (APPL), Nvidia (NVDA), China Intelligence Information Systems (IICN), Genesis Electronics Group (GEGI), Microsoft (MSFT), Google (GOOG); Leisure / Restaurants / Tourism – casino (ERI, NNSR, SGMS), hotel and restaurant business (WTBDY, BCCI, UPZC), travel agencies (ACGX, AIOM, BDGN); Automotive sector – General motors (GM), Tesla Inc (TSLA), Harley Davidson (HDL).

Below average volatility: Telecommunication sector – wire communication (LICT, NULM, OTEL), wireless communication (TALK, NTL, MFOYY).

In 2020, the media and entertainment sector, especially HBO, Disney+ and Netflix, the pharmaceutical and medical sector, and the technology sector, thanks to Tesla, became the most popular in the world among investors.

Among investors, the most relevant financial instrument are shares, which are a security that certifies the participation of its owner in the formation of the authorized capital of the company and gives the right to receive a share of its profits in the form of dividends and accumulated capital.

Stock investing strategies are based on the purpose:

– receive a fixed income immediately. To do this, shares with the maximum dividends should be bought. In most cases, these are preferred shares;
– buy shares on the electronic exchange platform and wait until their value increases to sell profitably. In this case, you need to choose stocks with maximum growth potential. Experienced financiers prefer such an investment strategy, as after a steady rise in shares, the proceeds from the sale will significantly exceed the size of any dividends.

However, investing in stocks has both advantages and disadvantages. The advantages include reliability, protection against fraud, small amounts at the start of the investment, acceptable liquidity (quite high) and reliability. The disadvantages of stocks can be described as the complexity of investing, because any investment requires an understanding of the process and nuances.

Another disadvantage is the inaccessibility of investing for most Ukrainian investors, despite the fact that although almost everyone can open a brokerage account technically, not everyone will be able to put a serious amount on it. The requirements of financial monitoring and the low level of tax culture lead to shadowing of revenues. Because of this, many Ukrainians simply cannot invest in this way.

And the last disadvantage, but very significant – is the risk of losing the value of the asset. Securities can lose quite sharply in price. If, except in case of force majeure, this is not typical for real estate and individual business, then for stocks it is a reality. Unsuccessful financial statements and securities lose in value. Unsuccessful actions of the government and the price of the financial instrument falls. At the same time, the value of securities may not be restored.

Therefore, to choose a financial instrument, both an experienced investor and a beginner need to conduct a thorough analysis and choose the best instruments with minimal risk to themselves.

Today, the IT industry in quarantine is developing the fastest, modernizing and finding ways to move production to remote work. In second place in terms of development is the field of cinema. Many cinema companies are creating online broadcasting services for the library of films and TV programs to compete between companies and attract more consumers.

Financial instruments were selected for comparative analysis – shares of the six most famous companies in the world in the IT sphere and entertainment industry.

Apple Inc. — American technology company with an office in Cupertino (California), which designs and develops consumer electronics, software and online services [22].

Tesla Inc. – American car company – startup from Silicon Valley. Focused on the design, manufacture and sale of electric vehicles and their components. The main production facility is the Tesla plant [23].

Alphabet Inc. — an international conglomerate of companies created on October 2, 2015 by American programmers and entrepreneurs Larry Page and Sergey Brin, which includes Google and other companies they owned directly or through Google [24].

The Walt Disney Company — one of the largest corporations in the entertainment industry in the world. Founded on October 16, 1923 by brothers Walter and Roy Disney as a small animation studio, as of June 2015 it is one of the largest Hollywood studios, the owner of 11 theme parks and two water parks, as well as several television and radio networks, including American Television and Radio ABC [25].

Netflix Inc. — an American media service provider and production company. As of April 2020, Netflix has 182 million subscribers worldwide, of which 69 million are in the United States. Netflix is available in all countries and regions except mainland China (due to local restrictions), Iran, Syria, North Korea and the Autonomous Republic of Crimea (due to US sanctions). The company also has offices in Brazil, the Netherlands, India, Japan and South Korea. Netflix is a member of the American Film Association [26].

Sony Corporation is one of the world's largest media companies. Sony manufactures consumer and professional electronics and other high-tech products. In addition, Sony is one of the world's largest media companies, with the record label Sony BMG (jointly with Bertelsmann), Columbia Pictures and TriStar Pictures, as well as a complete archive of MGM films (jointly with Comcast)

[27].

Stock prices dynamics for the period from September 2017 to September 2020 (daily values) for selected companies can be seen in the figure 1.

The input data for the study are daily, weekly and monthly prices for the period from 09/11/2017 to 09/08/2020 obtained from the site Investing.com [20].

For the selected time series (TS), let's define the following notation:

$$Z^i = \langle z^i_j \rangle; i \in (\overline{1,6}), \tag{7}$$

where $j \in \{d, w, m\}$, index d denotes daily, w – weekly, m – monthly prices;

$i = 1$ – corresponds to the time series of Apple Inc., $i = 2$ – time series of Tesla Inc., $i = 3$ – time series of Alphabet Inc., $i = 4$ – time series of The Walt Disney Company, $i = 5$ – time series of Netflix Inc., $i = 6$ – time series of Sony Corporation.

2.3. Results

For a general understanding of the series dynamics based on the input data, historical volatility is calculated for each selected investment instrument (table 1).

Table 1
Historical volatility of the stock prices

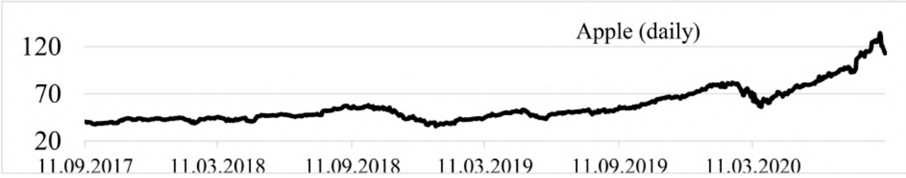
	Apple	Tesla	Alphabet	Disney	Netflix	Sony
Historical volatility (%)	40.6	107.4	39.7	40.3	43.9	81.5

Let's move on to the comparative analysis of the stock markets using the methodology of fractal analysis and calculation of memory depth characteristics. Note that February and March 2020 were the worst months for global stock markets since 2008. Stock indexes lost tens of percent, and experts said that the 11-year growth cycle since the last financial crisis has come to an end.

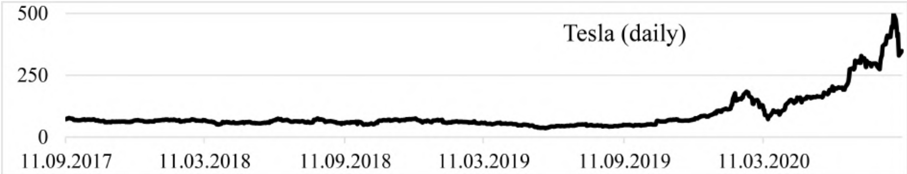
The cause of the fall – an outbreak of coronavirus, which grew into a pandemic. Against the background of the COVID-19 outbreak, investors have reconsidered their views on the future of the global economy [28]. The restrictive measures introduced in different countries have negatively affected almost all areas related to consumer activity: tourism, trade, catering, entertainment and others. Under quarantine, people spend less and move less. Bidders began to get rid of shares of airlines, oil companies, consumer electronics manufacturers and other companies, waiting for falling income and retvenue. Indices of the world's leading stock exchanges collapsed. For example, the Italian FTSE MIB index alone lost 29.8% from February 19 to March 11.

In connection with the consequences of the pandemic, the dynamics of stock prices for six corporations before and after the pandemic were analyzed. According to figure 1, we can see a fairly stable situation of all corporations until 2020, fluctuations in company share prices are quite small, almost imperceptible, but from February to March 2020, this situation is changing.

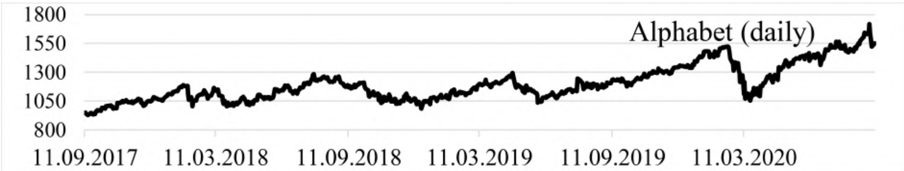
Consider the obtained values of the Hurst exponent (table 2).



(a)



(b)



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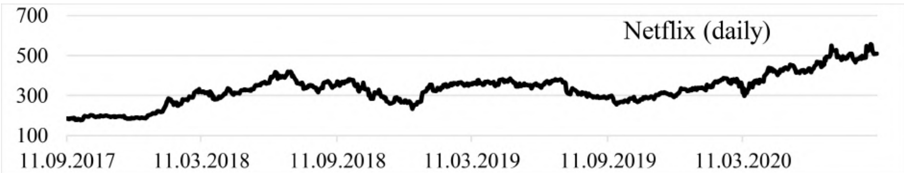

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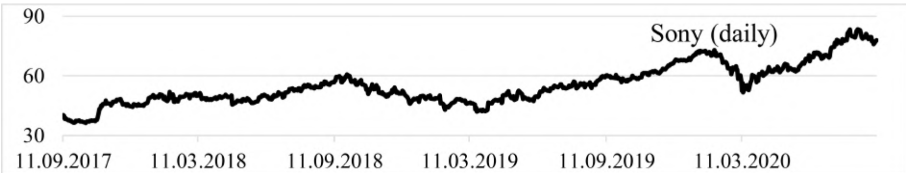


(f)

Figure 1: Stock prices dynamics for the period from September 2017 to September 2020 (daily values) for: a) Apple Inc.; b) Tesla Inc.; c) Alphabet Inc.; d) The Walt Disney Company; e) Netflix Inc.; f) Sony Corporation.

They indicate that all time series throughout the study period from September 2017 to

Table 2
The results of the application of The Hurst's rescaled range method for time series daily prices before the pandemic and during the pandemic

TS	H	H_{mixed}	TS	H	H_{mixed}
before the pandemic					
Z_d^1	0.92	0.64	Z_d^4	0.92	0.66
Z_d^2	0.89	0.57	Z_d^5	0.95	0.64
Z_d^3	0.91	0.63	Z_d^6	0.92	0.56
during the pandemic					
Z_d^1	0.92	0.65	Z_d^4	0.92	0.65
Z_d^2	0.90	0.64	Z_d^5	0.91	0.65
Z_d^3	0.92	0.71	Z_d^6	0.90	0.61

February 2020 are persistent, i.e. have a long-term memory. The value of the Hurst exponent for all time series is in the range $H \in [0.89; 0.95]$. For mixed values, the Hurst exponent is in the range $H \in [0.56; 0.66]$. The results of fractal analysis TS during a pandemic show that the dynamics of all financial instruments for the entire study period from February 2020 to September 2020 is persistent, i.e. have long-term memory. The value of the Hurst exponent for all time series is in the range $H \in [0.90; 0.92]$. For mixed values, the Hurst exponent is in the range $H \in [0.61; 0.71]$.

Table 3
The results of the application of The Hurst's rescaled range method for time series weekly and monthly prices before the pandemic and during the pandemic

TS	H before	H during	TS	H before	H during
Z_w^1	0.87	0.82	Z_w^4	0.87	0.86
Z_w^2	0.83	0.80	Z_w^5	0.91	0.85
Z_w^3	0.86	0.83	Z_w^6	0.87	0.84
Z_m^1	0.73	0.60	Z_m^4	0.80	0.56
Z_m^2	0.74	0.58	Z_m^5	0.80	0.66
Z_m^3	0.72	0.60	Z_m^6	0.74	0.60

The method of sequential R/S analysis [14] was used to study the dynamics and determine such a characteristic of fractal dynamics as long-term memory (i.e. its depth). As a result, a fuzzy set of memory depth for each TS is constructed. A visual representation of the fuzzy set of memory depth on the example of the time series of Apple and Tesla shares is shown in table 4.

Based on the fuzzy memory depth using formulas (3) - (6), the following characteristics of the time series dynamics were calculated: the greatest value of the memory depth l_{max}; the gravity center for the fuzzy set of memory depth l_{gc}; the most common memory depth l_{mc} and information entropy (H_{entrL}) [15]. The results of the calculations are shown in table 5.

When studying the dynamics of stock prices before the pandemic, it was found that for TS Z_d^1 the most often memory is stored for 5 and 6 days, where the gravity center to the fuzzy set

Table 4
The fuzzy set of memory depth for Apple Inc. and Tesla Inc. stock prices

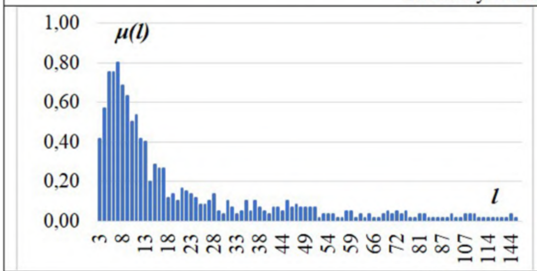
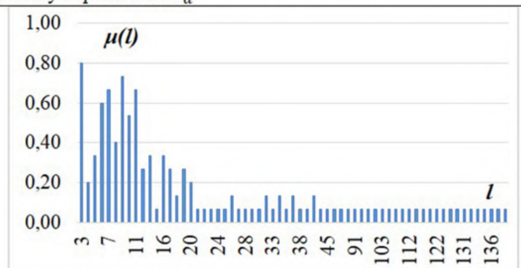
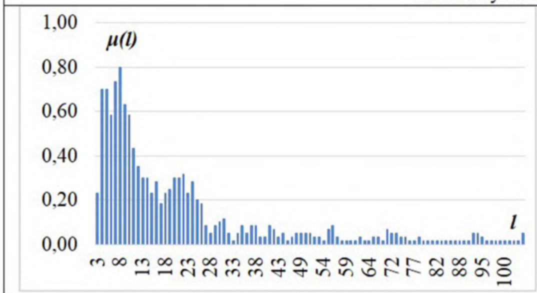
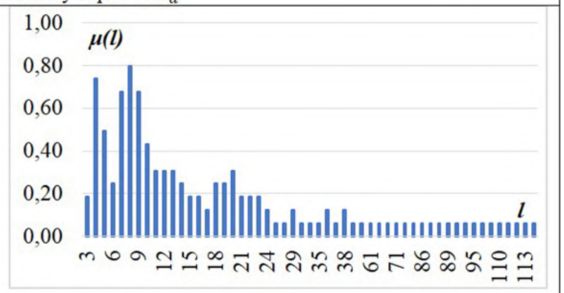
Before the pandemic	During the pandemic
The fuzzy set of memory depth for TS Z_d^1	
	
The fuzzy set of memory depth for Z_d^2	
	

Table 5
The main system characteristics of the fuzzy set of memory depth

TS	Z_d^1		Z_d^2		Z_d^3		Z_d^4		Z_d^5		Z_d^6	
Characteristics	Before	During	Before	During	Before	During	Before	During	Before	During	Before	During
l_{max}	145	154	104	114	153	153	152	138	122	119	154	135
l_{mc}	7	3	8	8	6	7	5	4,5,10	5	7	8	8
l_{gc}	21.8	30.8	21.7	23.8	18.8	28.1	17.6	21.4	19.7	18.8	17.9	28.3
H_{entrL}	22.1	20.1	23.1	18.0	19.5	17.6	18.1	20.3	20.2	16.1	18.8	18.8

of memory depth is 21.8, and the most common memory depth is 7, for TS Z_d^2 – during 4 and 5 days, where l_{mc} = 8 and l_{gc} = 21.7, for TS Z_d^3 – during 5, 8 days, where the center of gravity is 18.7, and the memory depth, which is most common from the fuzzy set of memory depths – 6, for TS Z_d^4 – during 6 and 7 days, where the center of gravity is 17.4, and the memory depth, which is most common from the fuzzy set of memory depth – 5, for TS Z_d^5 – during 6 and 8 days, where l_{mc} = 5 and l_{gc} = 19.7, and for TS Z_d^6 – during 5 and 9 days, where the center of gravity is 17.8 and the depth of memory is often 8, respectively (the number days is given in ascending order of the value of the membership function μ, $\mu \geq 0.60$).

Let's analyze the results of a consistent R/S analysis for the dynamics of stock prices during the pandemic. Long-term memory changed for all tested TS: for TS Z_d^1 the memory is stored for 7 and 11 days, for TS Z_d^2 – 7 and 9 days, for TS Z_d^3 – 5 and 8 days, for TS Z_d^4 – 4, 5 and 10 days, for TS Z_d^5 – 8 and 9 days, and for TS Z_d^6 – 5 and 7 days (the number of days is given in ascending order of the value of the membership function μ, $\mu \geq 0.60$). According to the results, during the pandemic period, long-term memory in some TS is shifted.

The following conclusions can be drawn from the analysis of the fuzzy set of memory depth.

The most stable and trend-resistant series were TS Z_d^2, Z_d^3 and Z_d^6. For these time series, the L_{mc} indicator is at a relatively high level and does not decrease during the pandemic, which indicates the trend stability of the time series. Shannon's entropy decreases (for TS Z_d^2, Z_d^3), which shows a decrease in uncertainty, or remains unchanged (for TS Z_d^6). It should be noted that for TS Z_d^2 the entropy was at the highest level, however, during the pandemic this indicator improved. That is, the crisis in the economy did not affect the increasing uncertainty of the time series.

For TS Z_d^1 negative is a significant decrease in L_{mc} from 7 to 3. TS Z_d^4 was marked by an increase in the uncertainty (entropy), and for TS Z_d^5, despite the decrease in entropy, l_{gc} and l_{max} decreased, which showed instability of these series to external risks.

It should be noted that the Hurst exponent of all series is at a high level, which indicates the persistence of the series.

In addition to the fractal analysis of corporate stock prices, we conduct a corresponding analysis of stock profitability.

In this regard, the TSs of stock profitability are built and studied:

$$P_s^i = \langle p_s^i \rangle \tag{8}$$

where $i = 1$ – corresponds to the time series of Apple Inc., $i = 2$ – time series of Tesla Inc., $i = 3$ – time series of Alphabet Inc., $i = 4$ – time series of The Walt Disney Company, $i = 5$ – time series of Netflix Inc., $i = 6$ – time series of Sony Corporation;

$p_s = \frac{z_d - z_{(d-s)}}{z_{(d-s)}} * 100\%$ – price profitability on day d relative to price on day $(d-s)$, i.e. profitability with lag $s = 1, 7, 10, 14, 21, 28, 30, 37, 42$.

The calculation results of the Hurst exponent for time series of profitability with the lag 1 ($s = 1$) indicate the randomness of TSs (table 6).

Table 6
The results of the application of The Hurst's rescaled range method for profitability time series before the pandemic and during the pandemic

TS	H before	H after	TS	H before	H after
Pd1	0.62	0.59	Pd4	0.56	0.63
Pd2	0.57	0.70	Pd5	0.64	0.61
Pd3	0.57	0.60	Pd6	0.59	0.61

As a result of the study of the time series of the delayed profitability, it was found that the nature of the dynamics of profitability varies from stochastic to persistent depending on the magnitude of the time lag and acquires the characteristics inherent in the "parent" TS of price. But the lag for each time series is different:

TS P^1 (Apple), P^6 (Sony) acquire persistence at a lag(s) value of 14 days, and the Hurst exponent is already 0.70;

TS P^3 (Alphabet), P^5 (Netflix) acquire persistence at a lag(s) value of 10 days, and the Hurst exponent is already 0.74;

TS P^4 (Disney) acquires persistence at a lag(s) value of 7 days, and the Hurst exponent is already equal to 0.71;

TS P^2 (Tesla) acquires persistence at a lag(s) value of 21 days, and the Hearst index is already 0.70.

A graphical representation of the values of the Hurst exponent for profitability TS with different lag is shown in figure 2.

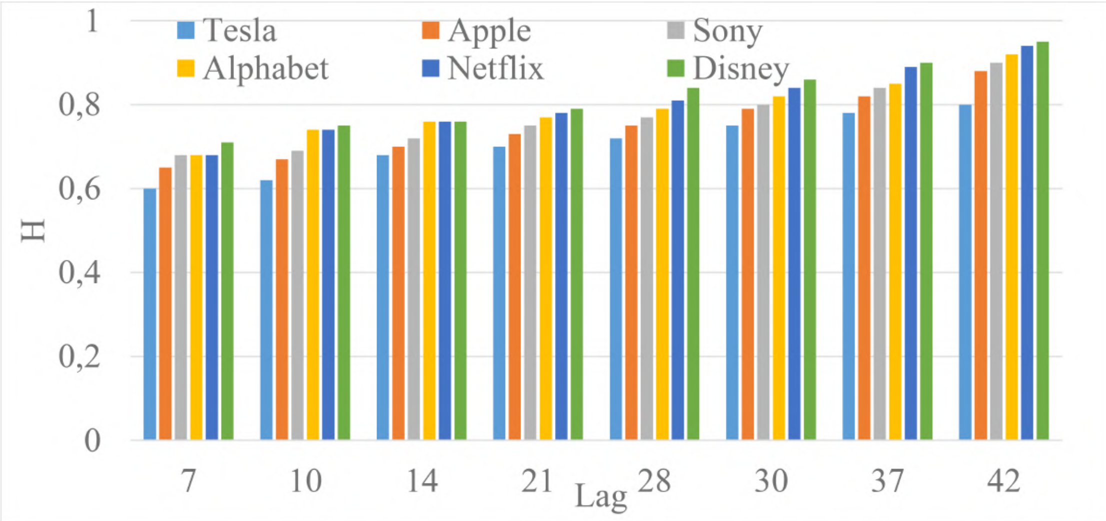


Figure 2: The value of the Hurst exponent for the profitability of shares of Apple, Tesla, Alphabet, Disney, Netflix and Sony with different lags.

Therefore, the time series P^4 (Disney) acquires persistence the fastest. We observe that with increasing lag, the Hurst exponent increases and becomes suitable for study by fractal analysis. This allows the investor to unify the analysis tools and forecast profitability in accordance with the characteristics of the price dynamics of a particular investment instrument.

3. Conclusions

Lack of investment strategy can cause loss of funds and complete frustration in the choice of a particular stock market by the investor. To reduce the level of risk, it is necessary to choose how long the wherewithal will be invested before making the investment. According to the term of investment, there are three strategies: short-term, medium-term and long-term strategy.

Short-term investment can last from a few days to 3 months. The medium term lasts from 3 months to 3 years. Long-term – from 3 years.

The results of the research allow us to offer the following (certain) recommendations for choosing an investment strategy for each selected security.

The first factor for choosing an appropriate strategy for investing in stocks is usually the price and volatility of the security and the sector as a whole. The media and entertainment sectors, automotive and technology have average volatility, ie price change, the rate of change is average. However, the selected financial instruments are characterized by a fairly high historical volatility (table 1). It follows that for all selected financial instruments, we can recommend a short-term investment strategy (table 7) with the possibility of investing for a small period.

Table 7
Choosing an investment strategy

	Short-term strategy	Medium-term strategy	Long-term strategy
Type of analysis	Volatility	Delayed profitability	Memory depth
Apple	+	++	
Tesla	+	++	++
Alphabet	++	+++	++++
Disney	+++		
Netflix	+++		
Sony			++

Netflix and Disney remain the best option for a short-term strategy, because due to the pandemic, which will cover the world for another one or two years, the stock prices of these companies will be constantly volatile. The price will fluctuate depending on the growth of demand for remote viewing of movies, series and shows, or the return of demand again for watching movies in regular cinemas. As a result, there is a constant risk of losing money if you invest them for more than 3 months.

Consider the second investment strategy – medium-term. To do this, we turn to the analysis of the results of complex fractal analysis. All series are persistent both before the pandemic and now, ie persistent (tables 2, 3). However, the profitability of instruments is not persistent, and the time series of delayed profitability with different time lag was used to study the level of trend stability (figure 2). Due to which it was determined that Tesla Inc., Apple Inc. and Sony gained the fastest profitability, ie you can invest in these shares for more than a week or two. Because in 7–14 days, the yield becomes persistent and it can be predicted for more than three months. These shares will generate passive income and can be reinvested again. In addition, these corporations are quite ambitious and have grand plans for the future, such as space exploration and shuttle construction, the creation of an electric car, etc., so the prices of their securities will only increase.

Alphabet Inc. has the most stable results, this corporation is often chosen for long-term investment, the results of volatility are the lowest, because the price does not fluctuate strongly enough, the dynamics of the time series is persistent, deferred profitability becomes persistent with a lag of 10 days. The results of the memory depth study show the stability of the corporation even under the influence of natural external factors such as the COVID-19. Stable memory depth characteristics during the pandemic were also demonstrated by shares of Tesla Inc. and Sony (table 7).

Note that these recommendations are formed only on the basis of this study and they may change depending on changes in the dynamics of financial instruments under the influence of various external and internal factors.

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Econophysics of cryptocurrency crashes: a systematic review

Andrii O. Bielinskyi[1], Oleksandr A. Serdyuk[2], Serhiy O. Semerikov[1,3,4,5] and Vladimir N. Soloviev[1]

[1]*Kryvyi Rih State Pedagogical University, 54 Gagarin Ave., Kryvyi Rih, 50086, Ukraine*
[2]*The Bohdan Khmelnytsky National University of Cherkasy, 81 Shevchenka Blvd., 18031, Cherkasy, Ukraine*
[3]*Kryvyi Rih National University, 11 Vitalii Matusevych Str., Kryvyi Rih, 50027, Ukraine*
[4]*Institute of Information Technologies and Learning Tools of the NAES of Ukraine, 9 M. Berlynskoho Str., Kyiv, 04060, Ukraine*
[5]*University of Educational Management, 52A Sichovykh Striltsiv Str., Kyiv, 04053, Ukraine*

Abstract
Cryptocurrencies refer to a type of digital asset that uses distributed ledger, or blockchain technology to enable a secure transaction. Like other financial assets, they show signs of complex systems built from a large number of nonlinearly interacting constituents, which exhibits collective behavior and, due to an exchange of energy or information with the environment, can easily modify its internal structure and patterns of activity. We review the econophysics analysis methods and models adopted in or invented for financial time series and their subtle properties, which are applicable to time series in other disciplines. Quantitative measures of complexity have been proposed, classified, and adapted to the cryptocurrency market. Their behavior in the face of critical events and known cryptocurrency market crashes has been analyzed. It has been shown that most of these measures behave characteristically in the periods preceding the critical event. Therefore, it is possible to build indicators-precursors of crisis phenomena in the cryptocurrency market.

Keywords
blockchain, cryptocurrency market, indicators-precursors of crisis phenomena, econophysics

1. Introduction

The instability of global financial systems concerning normal and natural disturbances of the modern market and the presence of poorly foreseeable financial crashes indicate, first of all, the crisis of the methodology of modeling, forecasting, and interpretation of modern socio-economic

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krivogame@gmail.com (A. O. Bielinskyi); serdyuk@ukr.net (O. A. Serdyuk); semerikov@gmail.com (S. O. Semerikov); vnsoloviev2016@gmail.com (V. N. Soloviev)
https://www.researchgate.net/profile/Andrii-Bielinskyi (A. O. Bielinskyi); http://iot.cdu.edu.ua/pidrozdily/kafedra-pm-ta-informatyky/personal-nij-sklad/serdiuk-oleksandr-anatoliiovych.html (O. A. Serdyuk); https://kdpu.edu.ua/semerikov (S. O. Semerikov); https://kdpu.edu.ua/personal/vmsoloviov.html (V. N. Soloviev)
0000-0002-2821-2895 (A. O. Bielinskyi); 0000-0002-1230-0305 (O. A. Serdyuk); 0000-0003-0789-0272 (S. O. Semerikov); 0000-0002-4945-202X (V. N. Soloviev)

realities. The doctrine of the unity of the scientific method states that for the study of events in socio-economic systems, the same methods and criteria as those used in the study of natural phenomena are applicable. Rapidly evolving coronavirus pandemic brings a devastating effect on the entire world and its economy as a whole [1, 2, 3, 4, 5, 6, 7]. Further instability related to COVID-19 will negatively affect not only on companies and financial markets, but also on traders and investors that have been interested in saving their investment, minimizing risks, and making decisions such as how to manage their resources, how much to consume and save, when to buy or sell stocks, etc., and these decisions depend on the expectation of when to expect next critical change [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. Despite the complexity of the problem, the results of recent studies indicate that significant success has been achieved within the framework of interdisciplinary approaches, and the theory of self-organization – synergetics [22, 23]. The modern paradigm of synergetics is a complex paradigm associated with the possibility of direct numerical simulation of the processes of complex systems evolution, most of which have a network structure, or one way or another can be reduced to the network. The theory of complex networks studies the characteristics of networks, taking into account not only their topology but also statistical properties, the distribution of weights of individual nodes and edges, the effects of dissemination of information, robustness, etc. [1, 2, 3, 4, 24, 25, 26].

Complex systems consist of a plurality of interacting agents possessing the ability to generate new qualities at the level of macroscopic collective behavior, the manifestation of which is the spontaneous formation of noticeable temporal, spatial, or functional structures [27, 28, 29, 30, 31, 32]. As simulation processes, the application of quantitative methods involves measurement procedures, where importance is given to complexity measures. Prigogine notes that the concepts of simplicity and complexity are relativized in the pluralism of the descriptions of languages, which also determines the plurality of approaches to the quantitative description of the complexity phenomenon [5].

Financial markets have been attracting the attention of many scientists like engineers, mathematicians, physicists, and others for the last two decades. Such vast interest transformed into a branch of statistical mechanics – econophysics [33, 34, 30, 31, 32, 35]. Physics, economics, finance, sociology, mathematics, engineering, and computer science are fields which, as a result of cross-fertilization, have created the multi-, cross-, and interdisciplinary areas of science and research such as econophysics and sociophysics, thriving in the last two and a half decades. These mixed research fields use knowledge, methodologies, methods, and tools of physics for modeling, explaining and forecasting economic, social phenomena, and processes. Accordingly, econophysics is an interdisciplinary research field, applying theories and methods originally developed by physicists to solve problems in economics, usually those including uncertainty or stochastic processes, nonlinear dynamics, and evolutionary games.

There are deep relationships (as well as crucial differences) between physics and finance [36] that have inspired generations of physicists as well as economists. In general, physicists apprehend financial markets as complex systems and, as such, they conducted numerous scientific investigations [37].

Though statistical physics cannot get along without quantum-mechanical ideas and notions in its fundamentals, the main sphere of its interest is the macroscopic description of systems with a large number of particles, the dynamic behavior of which can’t be brought to microscopic dynamical equations of quantum mechanics figured out for separate particles without the use of respective statistical postulates [38]. During last years an increasing flow of works was observed, in which detailed models of market process participants interactions and quantum-mechanical analogies, notions, and terminology based on methods of describing socio-economic systems are drawn to explain both particular peculiarities of modern market dynamics and economic functioning in whole [39, 40, 41]. Saptsin and Soloviev [42], Soloviev and Saptsin [43] have suggested a new paradigm of complex systems modeling based on the ideas of quantum as well as relativistic mechanics. It has been revealed that the use of quantum-mechanical analogies (such as the uncertainty principle, the notion of the operator, and quantum measurement interpretation) can be applied for describing socio-economic processes.

In this review, we will continue to study Prigogine's manifestations of the system complexity, using the current methods of quantitative analysis to determine the appropriate measures of complexity. The proposed measures of complexity, depending on the methodology and construction methods, can be divided into the following classes:

(1) informational,
(2) (multi-)fractal,
(3) chaos-dynamic,
(4) recurrent,
(5) irreversible,
(6) based on complex networks,
(7) quantum.

Econophysics, based on a rich arsenal of research on critical phenomena [44], very successfully copes with the description of similar events in economics and finance. These are crises and crashes that are constantly shaking the world economy. The introduced measures of complexity should, to one degree or another, respond to such phenomena.

The key idea here is the hypothesis that the complexity of the system before the crashes and the actual periods of crashes must change. This should signal the corresponding degree of complexity if they are able to quantify certain patterns of a complex system. A significant advantage of the introduced measures is their dynamism, that is, the ability to monitor the change in time of the chosen measure and compare it with the corresponding dynamics of the output time series. This allowed us to compare the critical changes in the dynamics of the system, which is described by the time series, with the characteristic changes of concrete measures of complexity. It turned out that quantitative measures of complexity respond to critical changes in the dynamics of a complex system, which allows them to be used in the diagnostic process and prediction of future changes.

The cryptocurrency market is a complex, self-organized system, which in most cases can be considered either as a complex network of market agents or as an integrated output signal of this network – a time series, for example, prices of individual cryptocurrency. The research on cryptocurrency price fluctuations being carried out internationally is complicated due to the interplay of many factors – including market supply and demand, the US dollar exchange rate, stock market state, the influence of crime, shadow market, and fiat money regulator pressure that introduces a high level of noise into the cryptocurrency data. Moreover, in the cryptocurrency market, to some extent, blockchain technology is tested in general. Hence, the cryptocurrency prices exhibit such complex volatility characteristics as nonlinearity and uncertainty, which are difficult to forecast, and any obtained results are uncertain. Therefore, cryptocurrency price prediction remains a huge challenge [45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59].

As can be seen, markets have seen significant numbers of investors selling off and rebalancing their portfolios with less risky assets. That has been leading to large losses and high volatility, typical of crisis periods. The economy key for preventing such activity may lie in cryptocurrency and constructing effective indicators of possible critical states that will help investors and traders fill in safety. Bitcoin, which is associated with the whole crypto market, has such properties as detachment and independence from the standard financial market and the proclaimed properties that should make it serve as the digital gold [60]. As was shown by Kristoufek [61], Bitcoin promises to be a safe-haven asset with its low correlation with gold, S&P 500, Dow Jones Industrial Average, and other authoritative stock indices even in the extreme events. But authors please not overestimate the cryptocurrency since according to their calculations and, obviously, the current structure of the system, gold remains more significant. But for ten years, this token has been discussed by many people, it has experienced a lot in such a short period, many people believe in it, and it has managed to form a fairly complex and self-organized system. The integrated actions from real-world merge in such dynamics and relevant information that is encoded in Bitcoin's time series can be extracted [62, 63, 64]. In the context of volatile financial markets, it is important to select such measures of complexity that will be able to notify us of upcoming abnormal events in the form of crises at an early stage.

In this review we:

- present such measures;
- study critical and crash phenomena that have taken place in the cryptocurrency market;
- try to understand whether crashes and critical events could be identified and predicted by such informative indicators or not.

This review is dedicated to the construction of such indicators based on the theory of complexity. According to our goals and actions, the paper is structured as follows. In Section 2, we present our classification of Bitcoin's crises for the period from July 16, 2010 to January 21, 2021. In Section 3, we describe the information measures of complexity. In Section 4, we describe the multifractal analysis methodology and its results for the crypto market. Section 5 defines what is chaos-based measures of complexity. In section 6, we deal with the recurrence quantification analysis of critical and crisis phenomena in the cryptocurrency market. Irreversible measure based on permutation patterns is defined in Section 7. Section 8 presents the theory and empirical results on network and multiplex measures of complexity and their robustness for digital currencies. Section 9 defines quantum complexity measures, the features of their manifestation on the crypto market are discussed. Section 10 contains conclusions and some recommendations for further research.

2. Data and classification

Bitcoin, being the most capitalized cryptocurrency, as a rule, sets and determines the main trends of the crypto market as a whole. Therefore, except for the part of the work where the study of collective processes in the market is carried out, we will use the time series of Bitcoin [65]. From figure 1 it can be seen that at the beginning of its existence, Bitcoin's dynamic was determined mainly by the processes of the formation of the market as a whole and characterized by high volatility, which, however, was not associated with critical phenomena. Bariviera et al. [66] find that the Hurt exponent changes significantly during the first years of existence of Bitcoin, and now it is less unstable than before. Moreover, they detect more evidence of information since 2014 [67].

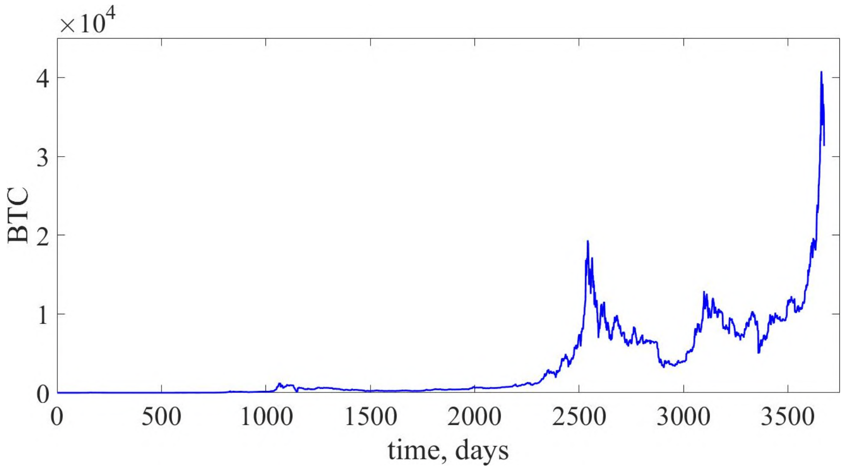


Figure 1: Bitcoin price development from July 16, 2010 to January 21, 2021.

Being historically proven, popular, and widely used cryptocurrency for the whole existence of cryptocurrencies in general, Bitcoin began to produce a lot of news and speculation, which began to determine its future life. Similar discussions began to lead to different kinds of crashes, critical events, and bubbles, which professional investors and inexperienced users began to fear. Thus, we advanced into action and set the tasks:

- classification of such critical events and crashes;
- construction of such indicators that will predict crashes, critical events in order to allow investors and ordinary users to work in this market.

Accordingly, during this period in the Bitcoin market, many crashes and critical events shook it. At the moment, there are various research works on what crashes are and how to classify such risk events in the market. The definition of these events still has been debatable. Nevertheless, the proposals of most authors have common elements that allow us to arrive at a consensus. Generally, the market crash is a sudden drastic decline in the prices of financial assets in a specific market [68]. Additionally, the applied model for a specific market takes an important place in the definition of "drastic decline". For instance, Wang et al. [68] take into account events with a minimum one-day decrease of 5% in the stock returns. These authors [26] identify financial crashes as a decrease of 25% or less of multi-year financial returns. Lleo and Ziemba [69] define a crash as a specific event of a day, which decreasing closing price exceeds a fall of 10% between the highest and the lowest value of the stock index in a year. Hong and Stein [70] postulate that the market crash is an unexpected event in which appearance was not accompanied by any financial news. Moreover, the price change during this event is rather negative. Also, it is worth mentioning the study of Shu and Zhu [71] where their classification of crashes included almost 50 crashes. It remains a little unclear which factors influence their choice of such an enormous amount of crashes in such a short period. Researchers emphasize these drops as such, with a fall of more than 15% and a duration of fewer than three weeks. Nevertheless, regarding this classification, we are going to emphasize the most relevant, where the complexity of the index started to decrease and whose initial deviation from regular behavior was noticeable in advance. Nowadays some people proclaim Bitcoin as a "digital gold". Gold as a material has served for jewelry and art as well as electronic or medical components. Limited supply and current acceptance of Bitcoin as a "digital gold" may erect it to the same level as gold. While some people back up Bitcoin's advantage, demonstrating its similarities with those of gold and silver [72], others argue that it is the new digital coin [73] due to its high volatility and unclear future development. However, researchers find its potential benefits during extreme market periods and provide a set of stylized factors that claim to be successful long-short strategies that generate sizable and statistically significant excess returns [74]. Despite volatile swings and many critics, Bitcoin has emerged and attracted much more confidence. Kristoufek [75], Li and Wang [76] consider that measures of financial and macroeconomic activity can be drivers of Bitcoin returns. Reviewing papers of the researches above, the experience of others and our own [77, 78, 79, 80, 81, 82, 83, 84, 85, 86], we have revised our classification of such leaps and falls, relying on Bitcoin time series during the entire period (01.01.2011–21.01.2021) of verifiable fixed daily values of the Bitcoin price (BTC) (https://finance.yahoo.com/cryptocurrencies). We emphasize that

- *crashes* are short and time-localized drops that last approximately two weeks, with the weighty losing of price each day. Their volatility is high. In percentage term, their decline exceeds 30 percent, and normalized returns proceed $\pm 3\sigma$ or near to it;
- *critical events* are those falls that, during their existence, have not had such massive changes in price as crashes.

Relying on these considerations, we emphasize 29 periods on Bitcoin time series, relying on normalized returns and volatility, where returns are calculated as

$$G(t) = \ln x(t + \Delta t) - \ln x(t) \cong [x(t + \Delta t) - x(t)]/x(t) \tag{1}$$

and normalized (standardized) returns as

$$g(t) \cong [G(t) - \langle G \rangle] / \sigma, \tag{2}$$

where $\sigma \equiv \sqrt{\langle G^2 \rangle - \langle G \rangle^2}$ is the standard deviation of G, Δt is time lag (in our case $\Delta t = 1$), and $\langle \ldots \rangle$ denotes the average over the time period under study and volatility as

$$V_T(t) = \frac{1}{n} \sum_{t'=t}^{t+n-1} |g(t')|$$

From the mentioned stylized facts on BTC dynamics, it was noticed how considerably it started to change near 2014. To gain a deeper understanding of its existence in the starting period, we divided the BTC time series into two periods: (01.01.2011-31.08.2016) and (01.09.2016-21.01.2021). More detailed information about crises, crashes, and their classification under these definitions is given in table 1 and table 2.

Table 1
List of Bitcoin major crashes and critical events since June 2011 till July 2016

№	Name	Days in correction	Bitcoin High Price, $	Bitcoin Low Price, $	Decline, %	Decline, $
1	07.06.2011-10.06.2011	3	29.60	14.65	50	14.95
2	11.10.2011-18.10.2011	7	4.15	2.27	45	1.88
3	15.01.2012-16.02.2012	32	7.00	4.27	39	2.73
4	15.08.2012-18.08.2012	3	13.50	8.00	40	5.50
5	08.04.2013-15.04.2013	7	230.00	68.36	70	161.64
6	28.04.2013-02.05.2013	4	144.00	98.09	32	45.91
7	19.06.2013-04.07.2013	15	111.29	68.50	38	42.79
8	04.12.2013-07.12.2013	3	1237.66	697.02	44	540.64
9	05.02.2014-21.02.2014	16	904.52	111.55	88	792.97
10	24.03.2014-09.04.2014	17	567.56	384.63	32	182.93
11	09.08.2014-17.08.2014	8	592.06	462.18	22	129.88
12	22.09.2014-04.10.2014	12	436.86	322.86	26	114.00
13	12.01.2015-14.01.2015	2	269.33	164.92	39	104.41
14	27.07.2015-23.08.2015	27	293.70	211.43	28	82.27
15	09.11.2015-11.11.2015	2	380.22	304.71	28	75.51
16	18.06.2016-21.06.2016	3	761.04	590.56	22	170.48
17	29.07.2016-01.08.2016	3	654.74	513.43	24	141.31

Table 2
List of Bitcoin major crashes and critical events since January 2017 till March 2020

№	Name	Days in correction	Bitcoin High Price, $	Bitcoin Low Price, $	Decline, %	Decline, $
18	04.01.2017-11.01.2017	7	1135.41	785.42	30	349.99
19	15.03.2017-18.03.2017	3	1253.43	971.38	23	282.05
20	10.06.2017-15.07.2017	35	2973.44	1914.09	36	1059.35
21	31.08.2017-13.09.2017	13	4921.85	3243.08	34	1678.77
22	08.11.2017-12.11.2017	4	7444.36	5878.13	21	1566.23
23	16.12.2017-30.12.2017	14	19345.49	12531.52	35	6813.97
24	06.01.2018-05.02.2018	30	17172.30	6937.08	60	10235.22
25	04.03.2018-05.04.2018	33	11504.42	6634.86	42	4869.56
26	04.05.2018-27.05.2018	23	9845.90	7118.88	28	2727.02
27	18.11.2018-15.12.2018	27	5615.26	3232.51	42	2382.75
28	12.07.2019-16.07.2019	4	11797.37	9423.44	20	2373.93
29	06.03.2020-16.03.2020	10	9122.55	5014.48	45	4108.07

Therefore, according to our classification crisis periods with numbers (1, 2, 4-6, 8, 9, 13, 18, 23-25, 27, 29) are *crashes*, all the rest – *critical events*.

Figure 2 confirms the importance of dividing the BTC time series in order to observe its dynamics in more detail. However, as it can be seen, we could separate time series in much deeper time scales.

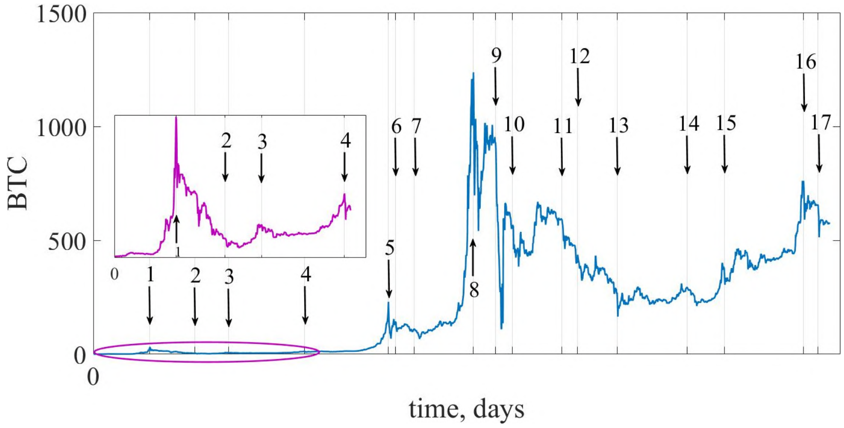


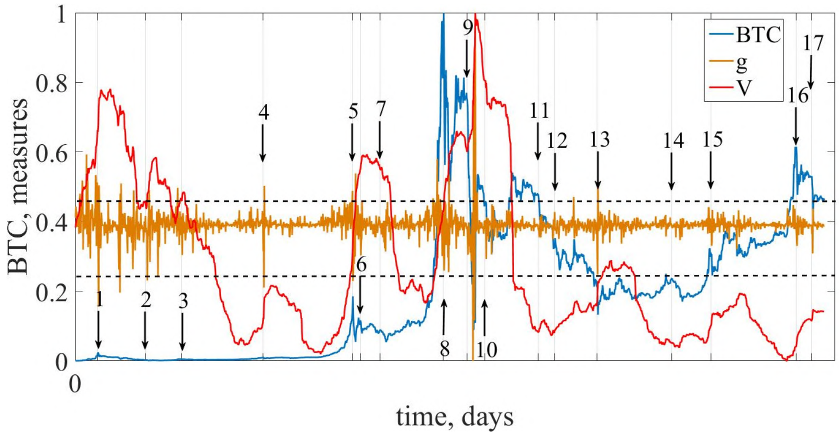
Figure 2: The dynamics of the daily values of the BTC price. The inset shows 1-4 crisis periods presented in table 1.

In figure 3 output Bitcoin time series for the first and the second periods, their normalized returns $g(t)$, and volatility $V_T(t)$ calculated for the window of size 100 are presented.

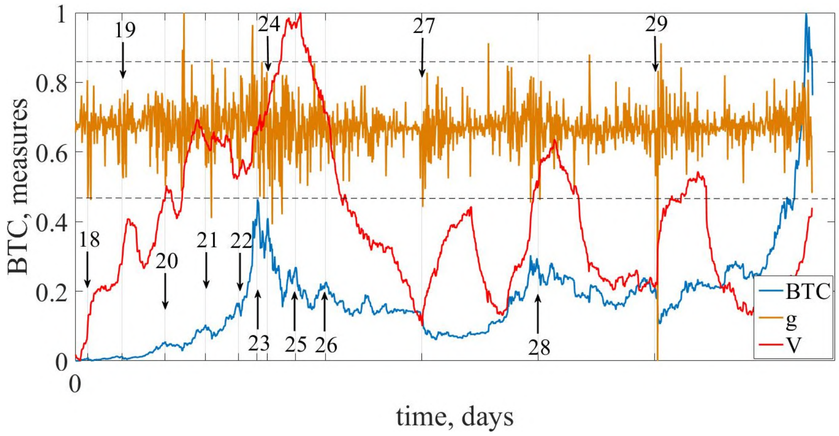
From figure 3 we can see that during periods of crashes and critical events normalized returns g increases considerably in some cases beyond the limits $\pm 3\sigma$. This indicates deviation from the normal law of distribution, the presence of the "heavy tails" in the distribution g, which are characteristics of abnormal phenomena in the market. At the same time volatility also grows. Such qualities are the foundation of our classification for crashes, as it has been mentioned already. All the rest events are critical. These characteristics serve as indicators of crashes and critical events as they react only at the moment of the above-mentioned phenomena and do not allow identifying the corresponding abnormal phenomena in advance. In contrast, most of the indicators described below will respond to critical changes and crashes in advance. It enables them to be used as indicators – precursors of such phenomena.

Calculations were carried out within the framework of the algorithm of a rolling (sliding, moving) window. For this purpose, the part of the time series (window), for which there were calculated measures of complexity, was selected, then the window was displaced along with the time series in a predefined value, and the procedure repeated until all the studied series had exhausted. Further, comparing the dynamics of the actual time series and the corresponding measures of complexity, we can judge the characteristic changes in the dynamics of the behavior of complexity with changes in the cryptocurrency. If this or that measure of complexity behaves in a definite way for all periods of crashes, for example, decreases or increases during the pre-crashes or pre-critical period, then it can serve as their indicator or precursor.

Calculations of measures of complexity were carried out both for the entire time series, and for a fragment of the time series localizing some of the emphasized crashes and critical events. In the latter case, fragments of time series of the same length with fixed points of the onset of crashes or critical events were selected and the results of calculations of complexity measures were compared to verify the universality of the indicators. Following some described below



(a)



(b)

Figure 3: The standardized dynamics, returns $g(t)$, and volatility $V_T(t)$ of BTC/USD daily values for first (a) and second (b) periods. Horizontal dotted lines indicate the $\pm 3\sigma$ borders. The arrows indicate the beginning of one of the crashes or the critical events.

procedures such time localization as, example, of length 100 or 200, either won‘t make any sense, or won‘t be possible as some of them are sensitive to time localization, or require a longer length of the time series as it is required by the procedure for better accuracy of further calculations

3. Informational measures of complexity

Complexity is a multifaceted concept, related to the degree of organization of systems. Patterns of complex organization and behavior are identified in all kinds of systems in nature and technology. Essential for the characterization of complexity is its quantification, the introduction of complexity measures, or descriptors [87].

We may speak of the complexity of a structure, meaning the amount of information (number of bits) of the structure; this is the minimum space we need to store enough information about the structure that allows us its reconstruction. We may also speak of the algorithmic complexity of a certain task: this is the minimum time (or other computational resources) needed to carry out this task on a computer. And we may also speak of the communication complexity of tasks involving more than one processor: this is the number of bits that have to be transmitted in solving this task [88, 89, 90].

Historically, the first attempt to quantify complexity was based on Shannon's information theory [91] and Kolmogorov complexity [92].

3.1. Lempel-Ziv complexity

Lempel-Ziv complexity (LZC) is a classical measure that, for ergodic sources, relates the concepts of complexity (in the Kolmogorov-Chaitin sense), and entropy rate [93, 94]. For an ergodic dynamical process, the amount of new information gained per unit of time (entropy rate) can be estimated by measuring the capacity of this source to generate new patterns (LZC). Because of the simplicity of the LZC method, the entropy rate can be estimated from a single discrete sequence of measurements with a low computational cost [95].

In this paper, we show that the LZC measure can be just such a measure of complexity, which is an early precursor of crisis phenomena in the cryptocurrency market [96, 97, 2, 80].

Historically, the first LZC measure system studies for financial time series were conducted by Da Silva [98], Da Silva et al. [99], Giglio and Da Silva [100], Giglio et al. [97]. They considered the deviation of LZC from that value for a random time series as a measure of actual market efficiency in absolute [96, 99, 100, 97] or relative [98] terms. Using this approach Da Silva et al. [99] were able to detect decreases in efficiency rates of the major stocks listed on the Sao Paulo Stock Exchange in the aftermath of the 2008 financial crisis. Lempel and Ziv [101] have surveyed the principal applications of algorithmic (Kolmogorov) complexity to the problem of financial price motions and showed the relevance of the algorithmic framework to structure tracking in finance. Some empirical results are also provided to illustrate the power of the proposed estimators to take into account patterns in stock returns. Brandouy et al. [102] proposed a generic methodology to estimate the Kolmogorov complexity of financial returns. Examples are given with simulated data that illustrate the advantages of our algorithmic method: among others, some regularities that cannot be detected with statistical methods can be revealed by compression tools. Applying compression algorithms to daily returns of the Dow Jones Industrial Average (DJIA), the authors concluded on an extremely high Kolmogorov complexity and by doing so, proposed another empirical observation supporting the impossibility to outperform the market. The structural complexity of time series describing returns on New York's and Warsaw's stock exchanges was studied using two estimates of the Shannon entropy rate based on the Lepel-Ziv and Context Tree Weighting algorithms [103]. Such structural complexity of the time series can be used as a measure of the internal (modelless) predictability of the main pricing processes and testing the hypothesis of an efficient market. Somewhat surprisingly, the results of Gao et al. [104], in which the authors computed the LZC from two composite stock indices, the Shanghai stock exchange composite index (SSE) and the DJIA, for both low-frequency (daily) and high-frequency (minute-to-minute) stock index data. The calculation results indicate that that the US market is basically fully random and consistent with the efficient market hypothesis (EMH), irrespective of whether low- or high-frequency stock index data are used. The Chinese market is also largely consistent with the EMH when low-frequency data are used. However, a completely different picture emerges when the high-frequency stock index data are used. Cao and Li [105] presents a novel method for measuring the complexity of a time series by unraveling a chaotic attractor modeled on complex networks. The complexity index, which can potentially be exploited for prediction, has a similar meaning to the LZC and is an appropriate measure of a series' complexity. The proposed method is used to research the complexity of the world's major capital markets. The almost absent sensitivity of the LZC to fluctuations in the time series indicates most likely errors in the calculation algorithm during the transformation of the time series. The complexity–entropy causality plane is employed in order to explore disorder and complexity in the space of cryptocurrencies [105]. They are found to exist in distinct planar locations in the representation space, ranging from structured to stochastic-like behavior.

A brief analysis of the problem indicates that so far, the Lempel-Ziv informational measure of the complexity has not been used to study the stability and behavior of the cryptocurrency market in a crisis. In this section, we use the Lempel-Ziv complexity measure to study the cryptocurrency market. Using the example of the most capitalized cryptocurrency – Bitcoin – we demonstrate the ability to identify the dynamics of varying complexity. Particularly relevant is the identification of the characteristic behavior of Bitcoin during the crisis phases of market behavior. By observing the dynamics of the Lempel-Ziv measure, precursors of crisis phenomena can be constructed [106].

3.1.1. The concept of Kolmogorov complexity

Let us begin with the well-known degree of complexity proposed by Kolmogorov [107]. The concept of Kolmogorov complexity (or, as they say, algorithmic entropy) emerged in the 1960s at the intersection of algorithm theory, information theory, and probability theory. A. Kolmogorov's idea was to measure the amount of information contained in individual finite objects (rather than random variables, as in the Shannon theory of information). It turned out to be possible (though only to a limited extent). A. Kolmogorov proposed to measure the amount of information in finite objects using algorithm theory, defining the complexity of an object as the minimum length of the program that generates that object. This definition is the basis of algorithmic information theory as well as algorithmic probability theory: an object is considered random if its complexity is close to maximum.

What is the Kolmogorov complexity and how to measure it? In practice, we often encounter programs that compress files (to save space in the archive). The most common are called zip, gzip, compress, rar, arj, and others. Applying such a program to some file (with text, data, program), we get its compressed version (which is usually shorter than the original file). After that, you can restore the original file using the paired program "decompressor". Therefore, approximately, the Kolmogorov complexity of a file can be described as the length of its compressed version. Thus, a file that has a regular structure and is well compressed has a small Kolmogorov complexity (compared to its length). On the contrary, a badly compressed file has a complexity close to its length.

Suppose we have a fixed method of description (decompressor) D. For this word x, we consider all its descriptions, i.e., all words y for which $D(y)$ it is defined and equal to x. The length of the shortest of them is called the Kolmogorov complexity of the word x in this way of description D:

$$KS_D(x) = \min\{l(y) \mid D(y) = x\}$$

where $l(y)$ denotes the length of the word. The index D emphasizes that the definition depends on the chosen method D. It can be shown that there are optimal methods of description. The better the description method, the shorter it is. Therefore, it is natural to make the following definition: the method D_1 is no worse than the method D_2 if

$$KS_{D_1} \leq KS_{D_2}(x) + c$$

for some c and all x.

Thus, according to Kolmogorov, the complexity of an object (for example, the text is a sequence of characters) is the length of the minimum program that outputs the text, and entropy is the complexity that is divided by the length of the text. Unfortunately, this definition is purely speculative. There is no reliable way of identifying this program uniquely, but there are algorithms that are actually just trying to calculate the Kolmogorov complexity of text and entropy.

A universal (in the sense of applicability to different language systems) measure of the complexity of the finite character sequence was suggested by Lempel and Ziv [101]. As part of their approach, the complexity of a sequence is estimated by the number of steps in the process that gives rise to it.

Acceptable (editorial) operations are: a) character generation (required at least for the synthesis of alphabet elements) and b) copying the "finished" fragment from the prehistory (i.e. from the already synthesized part of the text).

Let be Σ a complete alphabet, S – text (a sequence of characters) composed of elements Σ; $S[i]$ – i^{th} text symbol; $S[i : j]$ – a snippet of text from the i^{th} to j^{th} character inclusive ($i < j$); $N = |S|$ – length of text S. Then the sequence synthesis scheme can be represented as a concatenation

$$H(S) = S[1 : i_1]S[i_1 + 1 : i_2] \dots S[i_{k-1} + 1 : i_k] \dots S[i_{m-1} + 1 : N],$$

where $S[i_{k-1} + 1 : i_k]$ is the fragment S generated at the k^{th} step, and $m = m_H(S)$ is the number of process steps. Of all the schemes of generation is chosen the minimum number of steps. Thus, the Lempel-Ziv complexity of the sequence S is

$$c_{LZ}(S) = \min_H\{m_H(S)\}.$$

The minimum number of steps is provided by the choice to copy at each step the longest prototype from the prehistory. If you mark by the position number $j(k)$ from which the copying begins in step k the length of the copy fragment

$$l_{j(k)} = i_k - i_{k-1} - 1 = \max_{j \leq i_{k-1}}\{l_j : S[i_{k-1} + 1 : i_{k-1} + l_j] = S[j : j + l_j - 1]\}$$

and the k^{th} component of these complex decomposition can be written in the form

$$S[i_{k-1} + 1 : i_k] = \begin{cases} S[j(k) : j(k) + l_{j(k)} - 1] & \text{if } j(k) \neq 0, \\ S[i_{k-1} + 1] & \text{if } j(k) = 0. \end{cases}$$

The case $j(k) = 0$ corresponds to a situation where a symbol is in the position $i_{k-1} + 1$ that has not been encountered previously. In doing so, we use a character generation operation.

Complex text analysis can be performed in two regimes – segmentation and fragmentation. The first regime is discussed above. It gives an integrated view of the structure of the sequence as a whole and reduces it to disjoint but interconnected segments (without spaces). The other regime is to search for individual fragments characterized by an abnormally low complexity which means that they characterized by a sufficiently high degree of structure. Such fragments are detected by calculating local complexity within variable-length windows that slide along a sequence. Curves of change of local complexity along a sequence are called complex profiles. A set of profiles for different window sizes reveals the boundaries of anomalous fragments and their relationship.

We will find the LZC complexity for the time series, which is, for example, the daily values of the cryptocurrency price $x(t)$. To investigate the dynamics of LZC and compare it with cryptocurrency prices, we will find this measure of complexity for a fixed length (window) contract. To do this, we calculate the logarithmic returns accordingly to equation (1) and turn them into a sequence of bits.

You can specify the number of states that are differentiated (calculus system). Yes, for two different states we have 0, 1, for three – 0, 1, 2, etc. In the case of three states, unlike the binary coding system, a certain threshold σ is set and the states g are coded as follows [99, 100, 97]:

$$g = \begin{cases} 0 & \text{if } g < -\sigma, \\ 1 & \text{if } -b \leq g \leq b, \\ 2 & \text{if } g > b. \end{cases}$$

The algorithm performs two operations: (1) adds a new bit to an already existing sequence; (2) copies the already formed sequence. Algorithmic complexity is the number of such operations required to form a given sequence.

For a random sequence of lengths n, the algorithmic complexity is calculated by expression $LZC_r = n/\log n$. Then, the relative algorithmic complexity is the ratio of the obtained complexity to the complexity of the random sequence $LZC = LZC/LZC_r$.

Obviously, the classical indicators of algorithmic complexity are unacceptable and lead to erroneous conclusions. To overcome such difficulties, multiscale methods are used.

The idea of this group of methods includes two consecutive procedures: 1) coarse-graining ("granulation") of the initial time series – the averaging of data on non-intersecting segments, the size of which (the window of averaging) increased by one when switching to the next largest scale; 2) computing at each of the scales a definite (still mono scale) complexity indicator. The process of "rough splitting" consists in the averaging of series sequences in a series of non-intersecting windows, and the size of which – increases in the transition from scale to scale [108]. Each element of the "granular" time series follows the expression:

$$y_j^\tau = \frac{1}{\tau} \sum_{i=(j-1)\tau+1}^{j\tau} g(i), \quad \text{for } 1 \leq j \leq N/\tau, \tag{3}$$

with corresponding scale factor τ. The length of each "granular" row depends on the length of the window and is even N/τ. For a scale of 1, the "granular" series is exactly identical to the original one.

The coarse graining procedure for scales 2 and 3 is shown in figure 4.

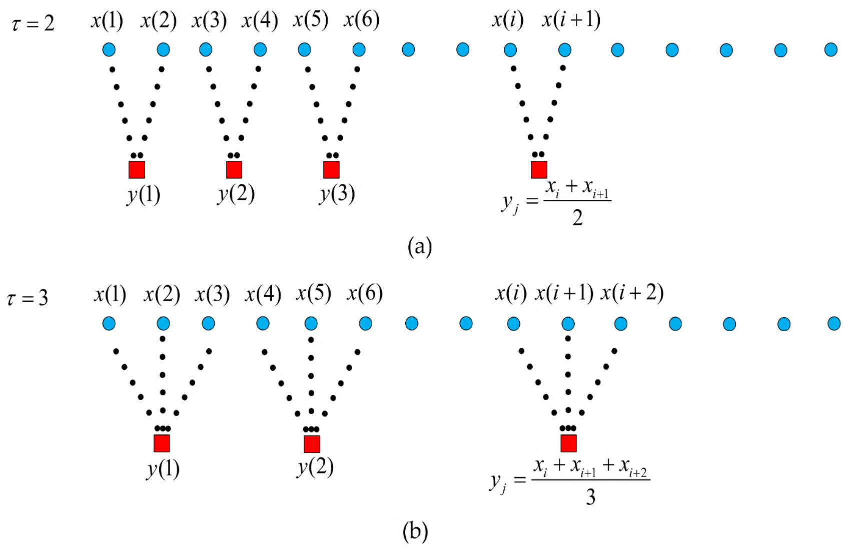


Figure 4: Coarse-graining procedure diagram: (a) scale factor $\tau = 2$; (b) scale factor $\tau = 3$.

To find the LZC measure of the time series, the rolling time windows were considered; the index for every window was calculated, and then the average was obtained.

Obviously, the crisis in the cryptocurrency market responds to noticeable fluctuations in standardized returns. Therefore, it is logical to choose σ as the value for the threshold value b.

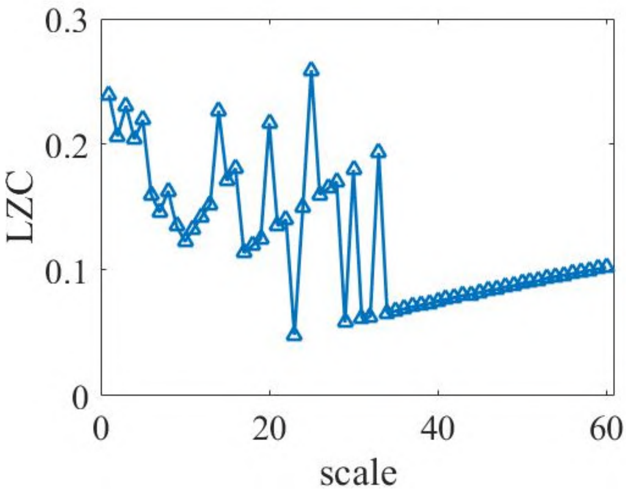
Figure 5 shows the dependence of the LZC on the scale. The absence of LZC fluctuations at scales exceeding 40 allows us to confine ourselves to this magnitude of the scale when calculating the multiscale measure.

Calculations of measures of complexity were carried for the two periods of BTC. Figure 6 presents the results of calculations of mono-(LZC_{m1}) and multi-(LZC_{m40}) scaling LZC measures. The calculations were performed for a rolling window of 100 days and an increment of 1 day.

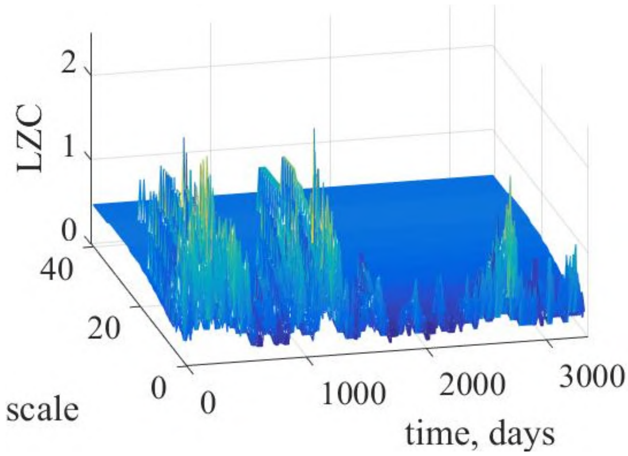
The data in figure 6 indicate that the LZC measure is noticeably reduced both in the case of mono-scale (m_1) and averaged over the scales from 1 to 40 (m_{40}) for all 29 crashes and critical events in the immediate vicinity of the crisis point.

As the results of calculations showed, the choice of the size of a rolling window is important: in the case of large windows, points of crises of different times can fall into the window, distorting the influence of each of the crises. When choosing small windows, the results fluctuate greatly, which makes it difficult to determine the actual point of the crisis. The used window length of 100 days turned out to be optimal for the separation of crises and fixing the LZC measure as an indicator.

Since the LZC measure begins to decrease even before the actual crisis point, it can be called an indicator-precursor of crisis phenomena in the cryptocurrency market.



(a)



(b)

Figure 5: Scale-dependent LZC (a) and its version with the rolling window approach (b).

3.2. Entropy as a measure of complexity

Nowadays, the most important quantity that allows us to parameterize complexity in deterministic or random processes is entropy. Originally, it was introduced by R. Clausius [109], in the context of classical thermodynamics, where according to his definition, entropy tends to increase within an isolated system, forming the generalized second law of thermodynamics. Then, the definition of entropy was extended by Boltzmann [110], Gibbs [111], linking it to molecular disorder and chaos to make it suitable for statistical mechanics, where they combined the notion of entropy and probability [112].

After the fundamental paper of Shannon [91] in the context of information theory, where entropy denoted the average amount of information contained in the message, its notion was significantly redefined. After this, it has been evolved along with different ways and successful

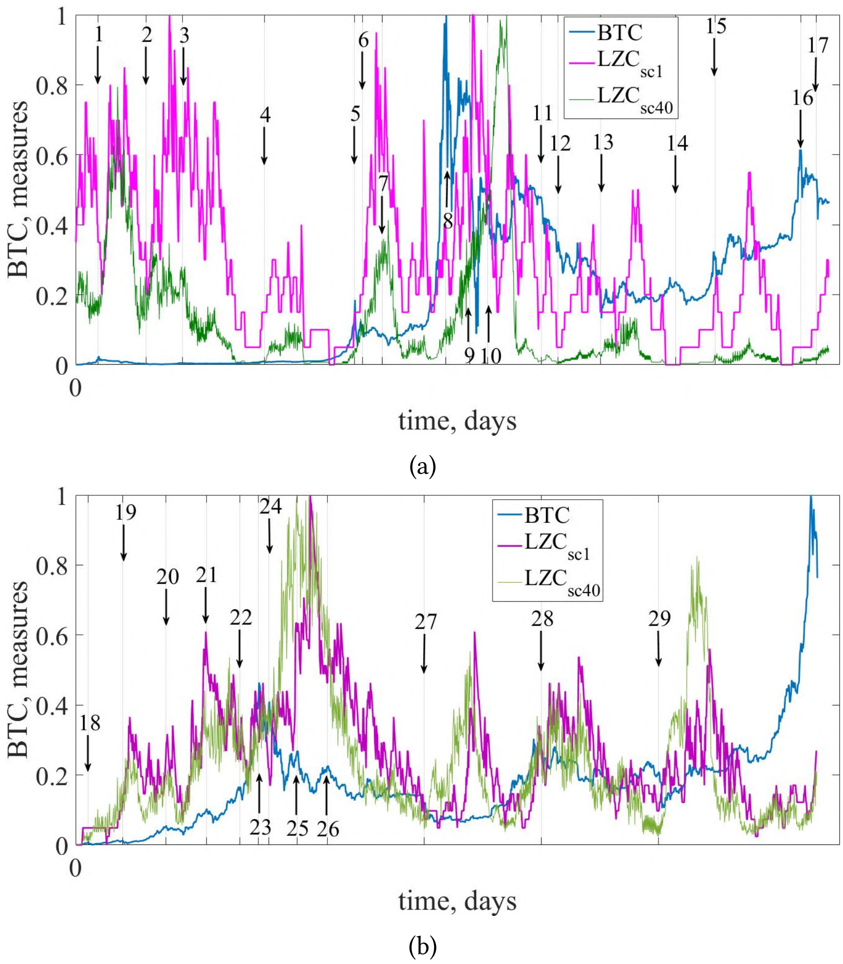


Figure 6: Comparative dynamics of BTC price fluctuations and mono- and multi-scaling LZC measures for first (a) and second (b) periods of the entire time series of Bitcoin.

enough used for the research of economic systems [113, 114, 115, 116].

A huge amount of different methods, as an example, from the theory of complexity, the purpose of which is to quantify the degree of complexity of systems obtained from various sources of nature, can be applied in our study. Such applications have been studied intensively for an economic behavior system.

The existence of patterns within the series is the core in the definition of randomness, so it is appropriate to establish such methods that will be based on the different patterns and their repetition [117]. In this regard, Pincus [118] described the methodology *Approximate entropy* (ApEn) to gain more detail analysis of relatively short and noisy time series, particularly, of clinical and psychological. Its development was motivated by the length constraints of biological data. Since then it has been used in different fields such as psychology [119], psychiatry [120], and finance [121, 122, 123, 124, 125]. Pincus and Kalman [125] considering both empirical data and models, including composite indices, individual stock prices, the random-walk hypothesis, Black-Sholes, and fractional Brownian motion models to demonstrate the benefits of ApEn applied to the classical econometric modeling apparatus. This research the usefulness of ApEn on the example of three major events of the stock market crash in the US, Japan, and India. During the major crashes, there is significant evidence of a decline of ApEn during and pre-crash periods. Based on the presented results, their research concludes that ApEn can serve as a base for a good trading system. Duan and Stanley [126] showed that it is possible to effectively distinguish the real-world financial time series from random-walk processes by examining changing patterns of volatility, approximate entropy, and the Hurst exponent. The empirical results prove that financial time series are predictable to some extent and ApEn is a good indicator to characterize the predictable degree of financial time series. Delgado-Bonal [127] gives evidence of the usefulness of ApEn. The researcher quantifies the existence of patterns in evolving data series. In general, his results present that degree of predictability increases in times of crisis.

Permutation entropy (PEn), according to the previous approach, is a complexity measure that is related to the original *Shannon entropy* (ShEn) that applied to the distribution of ordinal patterns in time series. Such a tool was proposed by Bandt and Pompe [128], which is characterized by its simplicity, computational speed that does not require some prior knowledge about the system, strongly describes nonlinear chaotic regimes. Also, it is characterized by its robustness to noise [129, 130] and invariance to nonlinear monotonous transformations [131]. The combination of entropy and symbolic dynamics turned out to be fruitful for analyzing the disorder for the time series of any nature without losing their temporal information.

As an example, Henry and Judge [132] applied PEn to the Dow Jones Industrial Average (DJIA) to extract information from this complex economic system. The result demonstrates the ability of the PEn method to detect the degree of disorder and uncertainty for the specific time that is explored. Sigaki et al. [133] applied PEn and statistical complexity over sliding time-window of daily closing price log-returns to quantify the dynamic efficiency of more than four hundred cryptocurrencies. Authors address to the efficient market hypothesis when the values of two statistical measures within a time-window cannot be distinguished from those obtained by chance. They found that 37% of the cryptocurrencies in their study stayed efficient over 80% of the time, whereas 20% were informationally inefficient in less than 20% of the time. Moreover, the market capitalization was not correlated with their efficiency. Performed analysis of information efficiency over time reveals that different currencies with similar temporal patterns formed four clusters, and it was seen that more young currencies tend to follow the trend of the most leading currencies. Sensoy [134] compared the time-varying weak-form efficiency of Bitcoin prices in terms of US dollars (BTC/USD) and euro (BTC/EUR) at a high-frequency level by PEn. He noticed that BTC/USD and BTCEUR have become more informationally useful since the beginning of 2016, namely Bitcoin in the same period. Researcher also found that with higher frequency in the Bitcoin market, we had lower price efficiency. Moreover, cryptocurrency liquidity (volatility) had a significant positive (negative) effect on the informational efficiency of its price.

Also, researh by Metin Karakaş [135] is dedicated both to Bitcoin and Ethereum. Here, the concept of entropy was applied for characterizing the nonlinear properties of the cryptocurrencies. For her goal, Shannon, Tsallis, Rényi, and Approximate entropies were estimated. From her empirical results, it was obtained that all entropies were positive. Of great interest was the results of ApEn which demonstrated larger value for Ethereum than for Bitcoin. In this case, it concluded that Ethereum had higher volatility.

Pele and Mazurencu-Marinescu-Pele [136] investigated the ability of several econometrical models to forecast value at risk for a sample of daily time series of cryptocurrency returns. Using high-frequency data for Bitcoin, they estimated the entropy of the intraday distribution of log-returns through the symbolic time series analysis STSA, producing low-resolution data from high-resolution data. Their results showed that entropy had strong explanatory power for the quantiles of the distribution of the daily returns. They confirmed the hypothesis that there was a strong correlation between the daily logarithmic price of Bitcoin and the entropy of intraday returns based on Christoffersen's tests for Value at Risk (VaR) backtesting, they concluded that the VaR forecast built upon the entropy of intraday returns was the best, compared to the forecasts provided by the classical GARCH models.

3.2.1. Time delay method

The state of the system can be described by the set of variables. Its observational state can be expressed through a d-dimensional vector or matrix, where each of its components refers to a single variable that represents a property of the system. After a while, the variables change, resulting in different system states.

Usually, not all relevant variables can be captured from our observations. Often, only a single variable may be observed. *Thakens' theorem* [137] that was mentioned in previous sections ensures that it‘s possible to reconstruct the topological structure of the trajectory formed by the state vectors as the data collected for this single variable contains information about the dynamics of the whole system.

For an approximate reconstruction of the original dynamics of the observed system, we project the time series onto a Reconstructed Phase Space [138, 131, 139] with the commonly used time delay method [131] which relied on the *embedding dimension* and *time delay*.

The embedding dimension is being the dimensionality of the reconstructed system (corresponds to the number of relevant variables that may differ from one system to another. The time delay parameter specifies the temporal components of the vector components. As an example, in recurrence analysis that will be described in section 6, Webber, Jr. and Zbilut [140] recommend setting the embedding dimension between 10 and 20. Regarding the analysis of financial systems, values between 1 and 20 for the embedding dimension are considered to be reasonable as well as the time delay.

3.2.2. Shannon entropy

The general approach can be described as follows. Formally, we represent the underlying dynamic state of the system in probability distribution form P and then the Shannon entropy S with an arbitrary base (i.e. 2, e, 10) is defined as:

$$S[P] = -\sum_{i=1}^{N} p_i \log p_i. \tag{4}$$

Here, in equation 4, p_i represents the probability that price i occurs in the sample's distribution of the Bitcoin time series, and N is the total amount of data in our system. Dealing with continuous probability distributions with a density function $f(x)$, we can define the entropy as:

$$H(f) = -\int_{-\infty}^{+\infty} f(x)\log f(x)dx. \tag{5}$$

According to the approach, the negative log increases with rarer events due to the information that is encoded in them (i.e., they surprise when they occur). Thus, when all p_i have the same value, i.e. where all values are equally probable, and $S[P]$ reaches its minimum for more structured time series (events that are more certain). Equation 5 is obeyed to the same rules as 4. In figure 7 are the empirical results for Shannon entropy and Bitcoin time series.

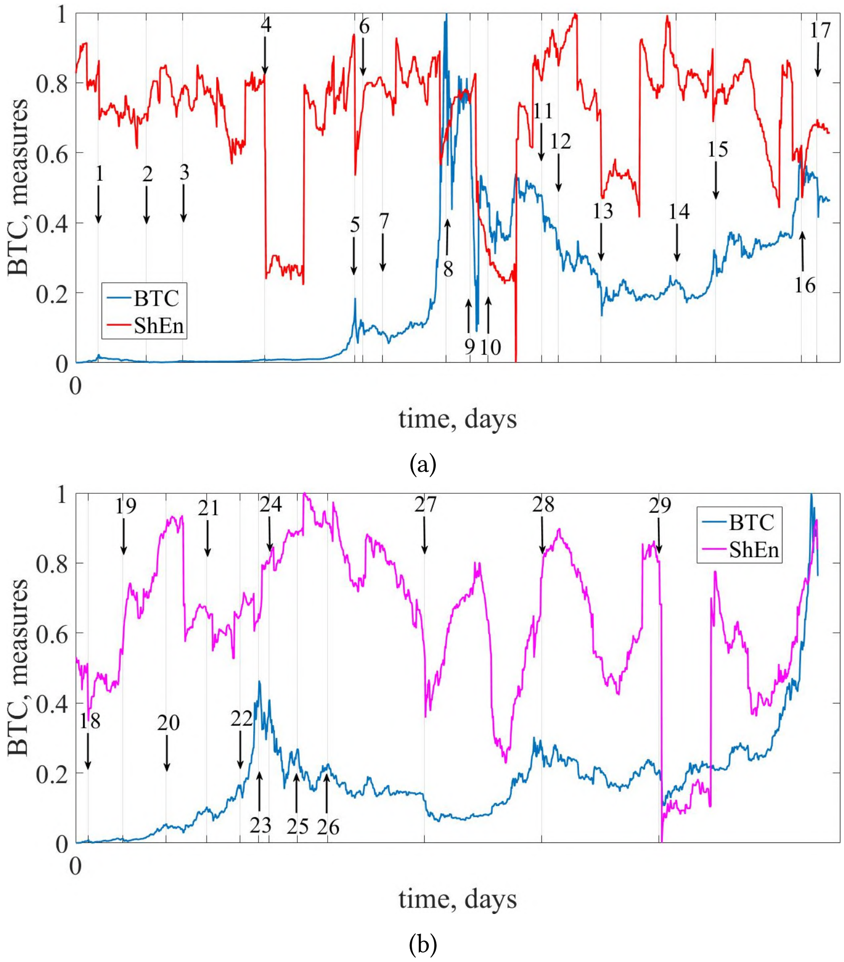


Figure 7: ShEn dynamics along with the first (a) and the second (b) periods of the entire time series of Bitcoin.

It can be seen from the figure that Shannon's entropy is rapidly increasing at the very moment of the crisis itself and is an excellent indicator of crisis phenomena.

3.2.3. Approximate entropy

To gain more detail analysis of the complex financial systems, it is known other entropy methods have become known, particularly, ApEn developed by Pincus [118] for measuring regularity in a time series.

When calculating it, given N data points $\{x(i)\,|\,i = 1,\dots,N\}$ are transformed into subvectors $\vec{X}(i) \in \Re^{d_E}$, where each of those subvectors has $[x(i), x(i+1),\dots,x(i+d_E-1)]$ for each i, $1 \leq i \leq N-m+1$. Correspondingly, for further estnimations, we would like to calculate a probability of finding such patterns whose Chebyshev distance $d[\vec{X}(i), \vec{X}(j)]$ does not exceed a positive real number r:

$$C_i^{d_E}(r) = (N-d_E+1)^{-1} \sum_{j=1}^{N-d_E+1} \mathscr{H}(r - d[\vec{X}(i), \vec{X}(j)])$$

where $\mathscr{H}(\cdot)$ is the Heviside function which count the number of instances $d[\vec{X}(i), \vec{X}(j)] \leq r$.

Next, we estimate

$$F^{d_E}(r) = (N-d_E+1)^{-1} \sum_{i=1}^{N-d_E+1} \ln(C_i^{d_E}(r)),$$

and ApEn of a corresponding time series (for fixed d_E and r) measures the logarithmic likelihood that patterns that are close for d_E adjacent observations remain close on the next comparison:

$$ApEn(d_E, r, N) = F^{d_E}(r) - F^{d_E+1}(r), \tag{6}$$

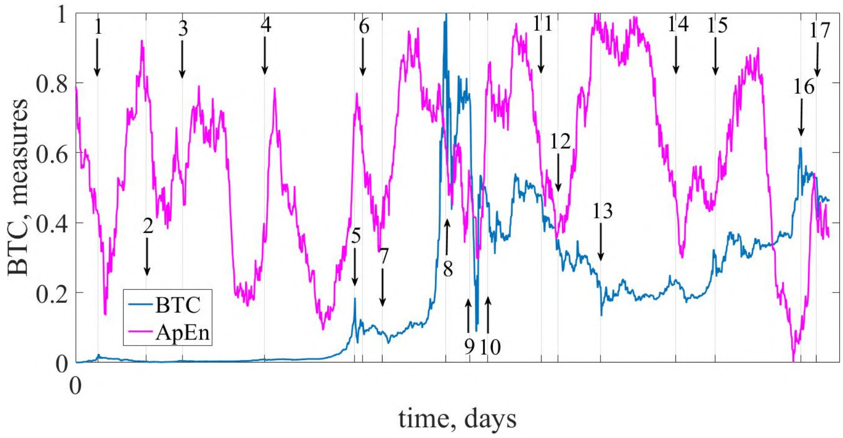
i.e., equation (6) measures the logarithmic likelihood that sequences of patterns that are close for d_E observations will remain close after further comparisons. Therefore, if the patterns in the sequence remain close to each other (high regularity), the ApEn becomes small, and hence, the time series data has a lower degree of randomness. High values of ApEn indicate randomness and unpredictability. But it should be considered that ApEn results are not always consistent, thus it depends on the value of r and the length of the data series. However, it remains insensitive to noise of magnitude if the values of r and d_E are sufficiently good, and it is robust to artifacts and outliers. Although ApEn remains usable without any models, it also fits naturally into a classical probability and statistics frameworks, and, generally, despite its shortcomings, it is still the applicable indicator of system stability, which significantly increased values may prognosticate the upcoming changes in the dynamics of the data.

The empirical results for the corresponding measure of entropy along with two periods of BTC are presented in figure 8.

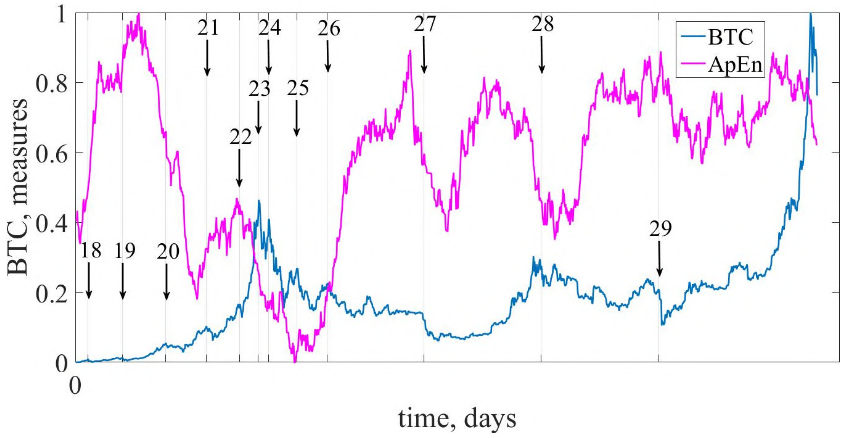
Long before the crisis, the value of this type of entropy begins to decrease, the complexity of the system decreases. This measure, in our opinion, is one of the earliest precursors of the crisis.

3.2.4. Permutation entropy

PEn, according to the previous approach, is a complexity measure that is related to the fundamental Information theory and entropy proposed by Shannon. Such a tool was proposed by Bandt and Pompe [128], which is characterized by its simplicity, computational speed that does not require some prior knowledge about the system, strongly describes nonlinear chaotic



(a)



(b)

Figure 8: ApEn dynamics along with the first (a) and the second (b) periods of the entire time series of Bitcoin calculated with rolling window of 100 days and the step size of 1 day.

regimes. Also, it is characterized by its robustness to noise [129, 130] and invariance to nonlinear monotonous transformations [131]. The combination of entropy and symbolic dynamics turned out to be fruitful for analyzing the disorder for the time series of any nature without losing their temporal information. According to this method, we need to consider "ordinal patterns" that consider the order among time series and relative amplitude of values instead of individual values. For evaluating PEn, at first, we need to consider a time series $\{x(i)\,|\,i = 1, \dots, N\}$ which relevant details can be "revealed" in d_E-dimensional vector

$$\vec{\mathrm{X}}\,(i) = [x(i), x(i+\tau), \dots, x(i+(d_E-1)\tau)]\,,$$

where $i = 1, 2, \dots, N - (d_E - 1)\tau$, and τ is an embedding delay of our time delayed vector. After it, we consider $d_E!$ permutation patterns $\pi = (k_0, k_1, \dots, k_{d_E-1})$ of symbols $(0, 1, \dots, d_E - 1)$ if the following condition for each $\vec{\mathrm{X}}(i)$ is satisfied:

$$x(i+k_0) \le x(i+k_1) \le \dots \le x(i+k_{d_E-1}).$$

We will use ordinal pattern probability distribution as a basis for entropy estimation. Further, let us denote $f(\pi_l)$ as the frequency of occurrence of the pattern π_l. Then, the relative frequencies of permutations in the time series are defined as

$$p(\pi_l) = \frac{f(\pi_l)}{N - (d_E - 1)\tau},$$

where the ordinal pattern probability distribution is given by $P = \{p(\pi_l) \,|\, l = 1, \ldots, d_E!\}$. Finally, permutation entropy (denoted by $S[P]$) of the corresponding time series presented in the following form:

$$S[P] = -\sum_{l=1}^{d_E!} p(\pi_l) \log p(\pi_l).$$

Then, to get more convenient values, we calculate *Normalized permutation entropy* as:

$$E_s[P] = \frac{S[P]}{S_{max}},$$

whose $S_{max} = \ln d_E!$ represents the maximum value of $E_S[P]$ (a normalization constant), and normalized entropy has a range $0 \leq PEn \leq 1$. Here, the maximal entropy possible value is realized when all $d_E!$ possible permutations have an equal probability of occurrence (more noise and random data). With the much lower entropy value, we get a more predictable and regular sequence of the data. Therefore, the *PEn* gives a measure of the departure of the time series from a complete noise and stochastic time series.

There must be predefined appropriate parameters on which *PEn* relying, namely, the embedding dimension d_E is paramount of importance because it determines $d_E!$ possible states for the appropriate probability distribution. With small values such as 1 or 2, parameter d_E will not work because there are only few distinct states. Furthermore, for obtaining reliable statistics and better detecting the dynamic structure of data, d_E should be relevant to the length of the time series or less [141]. For our experiments, $d_E \in \{3, 4\}$ and $\tau \in \{2, 3\}$ indicate the best results. Hence, in figure 9 we can observe the empirical results for permutation entropy, where it serves as indicator-precursor of the possible crashes and critical events.

Information measures of complexity due to their initial validity and transparency, ease of implementation and interpretation of the results occupy a prominent place among the tools for the quantitative analysis of complex systems.

4. Fractal and multifractal measures of complexity

The economic phenomena that cannot be explained by the traditional efficient market hypothesis can be explained by the fractal theory proposed by Mandelbrot and Wheeler [142]. Before, fractal studies focus on the Rescaled Range (R/S) analysis were proposed by Hurst [143, 144] in the field of hydrology. However, Lo [145] discovered that the R/S method is sensitive to short-term autocorrelation, which may lead to a bias error of nonstationary time series. To solve this problem, Peng et al. [146] proposed a widely used detrended fluctuation analysis (DFA) that uses a long-range power law to avoid significant long-term autocorrelation false detection [147].

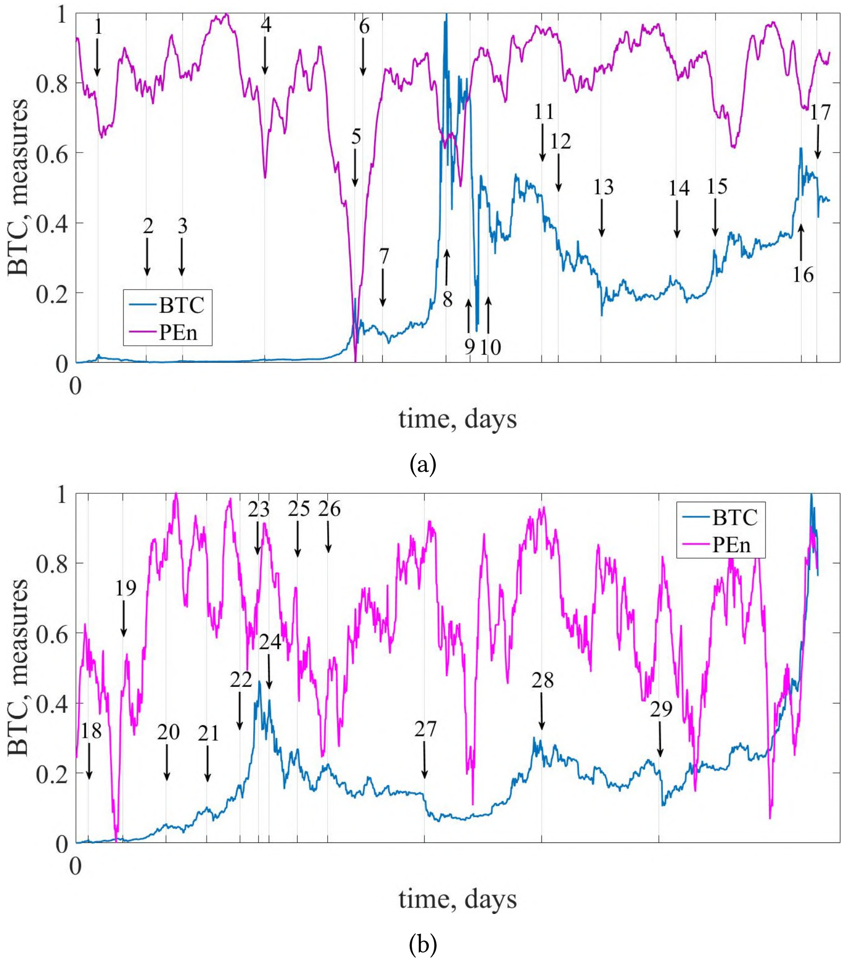


Figure 9: PEn dynamics along with the first (a) and the second (b) periods of the entire time series of Bitcoin.

As a multifractal extension (MF) of the DFA approach, Kantelhardt et al. [148] introduced the MF-DFA method that for a long time has been successfully applied for a variety of financial markets, such as stock [149, 150, 151, 152, 153, 154, 155, 156, 157], commodity [158, 159, 160, 161, 155], tanker [162], derivative [163], foreign exchange rates [164, 165, 166, 167, 168], and electricity markets [169]. An especially interesting application of multifractal analysis is measuring the degree of multifractality of time series, which can be related to the degree of efficiency of financial markets [170, 171, 172, 173].

Podobnik and Stanley [174] extended DFA by introducing a detrended cross-correlation analysis (DCCA) approach that can be used to study long-range cross-correlation between two non-stationary time series. Guided by ready-made approaches, Zhou [175] proposed a multifractal detrended cross-correlation analysis (MF-DCCA) [176], which is a combination of MF-DFA and DCCA. Then the number of interesting methods has been proposed, such as the method of MF-PX-DFA and MF-PX-DMA [177], MF-X-DMA [178], MF-HXA [179], MF-X-PF

[180], etc. These increase the efficiency of some applications of the MF-DCCA method. The MF-DCCA method has been widely applied to describe the multifractal characteristics of two cross-correlated nonstationary time series in the financial field such as the foreign exchange market [181, 182], the stock market [183, 184, 185], the crude oil market [186, 187, 188], carbon market [189, 190], and the commodity market [191, 155]. Zhang et al. [192] also employ MF-DCCA to examine the relationship between mass media and new media [192] and to quantify cross-correlation between investor sentiment proxies [193], i.e., fears [194] and Twitter happiness sentiment [195].

Along with common multifractal methods, Gronwald and Sattarhof [196] applied an intermittency coefficient for the evaluation of financial market efficiency. While the random walk corresponds to the most genuine form of market efficiency, the larger the value of the intermittency coefficient is, the more inefficient a market would be. In an empirical application using data from the largest current market for tradable pollution permits, the European Union Emissions Trading Scheme, they show that this market becomes more efficient over time. Besides, the degree of market efficiency is overall similar to that for the US stock market; for one sub-period, the market efficiency is found to be higher. While the first finding is anticipated, the second finding is noteworthy, as various observers expressed regarding the information efficiency of this newly established artificial market.

Since Bitcoin was born, it has attracted the considerable attention of researchers from different fields of science that apply modern methods and models of analysis of the peculiarities of the dynamics of the popular digital currency, namely, the methods of multifractal analysis to gain a deeper understanding of its inherent nonlinear statistical properties.

Using 1-min returns of Bitcoin price, Takaishi investigated statistical properties and MF of Bitcoin time series [197]. His results present that 1-min returns distribution is fat-tailed, and kurtosis largely deviates from the Gaussian expectation. Although with large time scales, kurtosis is anticipated to approach the Gaussian expectation, he found that convergence to that is very slow. Skewness is found to be negative at short time scales and becomes consistent with zero at large time scales. Also, he analyzed daily volatility-asymmetry by using GARCH, GJR, and RGARCH models and found no evidence of volatility asymmetry. On exploring MF using MF-DFA, it was confirmed that the Bitcoin time series exhibits MF. The sources of MF are also investigated, and it is confirmed that both temporal correlation and the fat-tailed distribution contribute to the MF, and the degree of MF for the temporal correlation is stronger than that for the fat-tailed distribution.

Generally, Bariviera [67], Bariviera et al. [66] investigated the long memory of the Bitcoin market using the Hurst exponent. Their research proves the advantages of the DFA methods, basically, because it is more robust and less sensitive to departures from conditions of stationarity. They find that daily returns suffered a regime switch. From 2011 until 2014 the Hurst exponent was showing persistence behavior, whereas after 2014, it equals to white noise, while daily volatility exhibits persistent behavior during the period under study. Also, daily volatility presents stronger fluctuations than in daily returns. In particular, that volatility characteristic is the main peculiarity of the Bitcoin market.

Kirichenko et al. [198] conducted a comparative correlation and fractal analysis to time series of the Bitcoin cryptocurrency rate and community activities in social networks associated with famous cryptocurrency. The results of their study show a significant correlation and similar multifractal structure between the Bitcoin rate and the community activities. Time series fractal analysis indicated the presence of self-similar and multifractal properties.

As an example, Jiang et al. [199] attempt to investigate the time-varying long-term memory in the Bitcoin market through a sliding window approach and by employing a new efficiency index [200]. The daily dataset for the period from 2010 to 2017 is utilized, and some interesting findings emerge that:

- generalized Hurst exponents in the Bitcoin market are above 0.5;
- long-term memory exists in the Bitcoin market;
- high degree of inefficiency ratio;
- Bitcoin market does not become more efficient over time;
- rolling window approach can help to obtain more reliable results. Some conclusions for those who deal with the cryptocurrency market were made.

Al-Yahyaee et al. [201] provides the results on the efficiency of the Bitcoin market compared to gold, stock, and foreign exchange markets. By applying the MF-DFA approach, researchers found that the long-memory feature and MF of the Bitcoin market were more robust and, therefore, more inefficient than the gold, stock, and currency markets.

Gajardo et al. [202] applied MF-ADCCA to analyze the presence and asymmetry of the cross-correlations between the major currency rates, Bitcoin, the DJIA, gold price, and the oil crude market. They found that multifractality existed in every cross-correlation studied, and there was an asymmetry in the cross-correlation exponents under the different trends of the WTI, Gold, and the DJIA. Bitcoin showed greater multifractal spectra than the other currencies on its cross-correlation with the WTI, the Gold, and the DJIA. Bitcoin presented a different relationship between commodities and stock market indices, which had to be taken into consideration when investing. The reason is that over the years the currency was traded and over time, it has earned the trust of the community.

The nonlinear patterns of the volatility of the seven Bitcoin markets were investigated by Lahmiri et al. [203]. Using four diverse distribution inferences: Normal, Student-t, Generalized Error, and t-Skewed distribution, they explored the fractional long-range dependence in conjunction with the potential inherent stochasticity of volatility time series. Their results testify to the existence of long-range memory in Bitcoin market volatility, irrespectively of distributional inference. The entropy measurement, which indicates a high degree of randomness in the estimated series, shows the same. As Bitcoin markets are highly disordered and risky, they cannot be considered suitable for hedging purposes. Their exploration provides strong evidence against the efficient market hypothesis.

Zhang et al. [204] investigated the cross-correlations of the return-volume relationship of the Bitcoin market. In particular, they selected eight exchange rates whose trading volume accounts for more than 98% market shared to synthesize Bitcoin indexes. The empirical results based on MF-DCCA revealed that: (1) return-volume relationship exhibited the nonlinear dependencies and power-law cross-correlations; (2) all cross-correlations were multifractal, and there were anti-persistent behaviors of cross-correlation for $q = 2$; (3) the price of small fluctuations was more persistent than that of the volume, while the volume of larger fluctuations was more anti-persistent; (4) the sliding window approach showed that the cross-correlations of return-volume were anti-persistent in the entire sample period.

Similarly to our article [205] where we applied the MF-DFA method to Ukrainian and Russian stock markets, we use it here to explore the multifractal property of Bitcoin and construct reliable indicator for it.

4.1. Multifractal detrended fluctuation analysis (MF-DFA)

As an extension to the original DFA [146, 206, 207], the multifractal approach [148, 208] estimates the Hurst exponent of a time series at different scales. Based on a given time series $\{x(i) \,|\, i = 1, \dots, N\}$, the MF-DFA is described as follows:

(i) The profile $Y(i)$ (accumulation) is defined as:

$$Y(i) = \sum_{j=1}^{i} (g(j) - \langle g \rangle), \tag{7}$$

where $\langle g \rangle$ stands for the average of returns.

(ii) The profile $Y(i)$ is then divided into $N_s \equiv \text{int}(N/s)$ non-overlapping time segments of equal length s and the local trend Y_v^{fit} for each segment is calculated by the least-square fit. Since the length of the time series is not always a multiple of s, a short period at the end of the profile, which is less than the window size, may be removed. For taking into account the rejected part and, therefore, to use all the elements of the sequence, the above procedure is repeated starting from the end of the profile. Therefore, the total $2N_s$ segments are obtained together, and the variance is computed as

$$F^2(v, s) = \frac{1}{s} \sum_{i=1}^{s} \left[Y((v-1)s + i) - Y_v^{fit}(i) \right]^2, \text{for } v = 1, \dots, N_s \tag{8}$$

and

$$F^2(v, s) = \frac{1}{s} \sum_{i=1}^{s} \left[Y(N - (v - N_s)s + i) - Y_v^{fit}(i) \right]^2, \text{for } v = N_s + 1, \dots, 2N_s \tag{9}$$

Various types of MF-DFA such as linear, quadratic, or higher order polynomials can be used for eliminating local trend in segment v; we use a cubic order polynomial.

(iii) Considering the variability of time series and the possible multiple scaling properties, we obtain the q^{th} order fluctuation function by averaging over all segments:

$$F_q(s) = \left[\frac{1}{2N_s} \sum_{v=1}^{2N_s} \left[F^2(v, s) \right]^{q/2} \right]^{1/q}. \tag{10}$$

The index q can take any non-zero value. For $q = 0$, $F_q(s)$ is divergent and can be replaced by an exponential of a logarithmic sum

$$F_0(s) = \exp \left[\frac{1}{4N_s} \sum_{v=1}^{2N_s} \ln F^2(v, s) \right].$$

(iv) At least, we determine the scaling behavior of the fluctuation function by analyzing $\log F_q(s)$ vs $\log s$ graphs for each value of q. Here, $F(s)$ is expected to reveal power-law scaling

$$F_q(s) \sim s^{h(q)} \tag{11}$$

for large n. The scaling exponent $h(q)$ can be considered as generalized Hurst exponent. With $q = 2$ MF-DFA transforms into standard DFA, and $h(2) = H$, where H is the well-known Hurst exponent.

- $h(2) \in [0.0, 0.5) \rightarrow$ anti-persistency. The process under study tends to decrease (increase) after a previous increasing (decreasing);
- $h(2) = 0.5 \rightarrow$ uncorrelated process. The fluctuations that depend on q tend to a random walk behavior [131];
- $h(2) \in (0.5, 1.0] \rightarrow$ persistency. If a process tends to increase (decrease) for a some period T, then it expected to continue to increase (decrease) for a similar period of time;
- $h(2) > 1.0 \rightarrow$ nonstationary process, stronger long-range correlations are present.

Those intervals with time intervals ν will dominate which variance $F^2(\nu, s)$ is large and q values are positive. Therefore, for positive values of q , $h(q)$ describes the scaling behavior of time intervals with large fluctuations. Large fluctuations are usually characterized by smaller scaling coefficients of $h(q)$ for multifractal series. On the contrary, for negative values of q , time intervals with a small variance $F^2(\nu, s)$ will dominate. Thus, $h(q)$ will describe the scaling behavior of time intervals with small fluctuations.

(v) Another way of characterizing multifractality of a time series is in terms of the multifractal scaling exponent $\tau(q)$ which is related to the generalized Hurst exponent $h(q)$ from the standard multifractal formalism and given by [146]:

$$\tau(q) = qh(q) - 1. \tag{12}$$

Equation (12) reflects temporal structure of the time series as a function of moments q, i.e., it represents the scaling dependence of small fluctuations for negative values of and large fluctuations for positives values. If (12) represents linear dependence of q, the time series is said to be monofractal. Otherwise, if (12) has a nonlinear dependence on q, then the series is multifractal.

(vi) The different scalings are better described by the singularity spectrum $f(\alpha)$ which can be defined as:

$$\alpha = \frac{d\tau(q)}{dq} = h(q) + q\frac{dh(q)}{dq},$$
$$f(\alpha) = q\alpha(q) - \tau(q) = q^2\frac{dh(q)}{dq} + 1,$$

with α – the Hölder exponent or singularity strength. Following the methods described above, we present results that reflect multifractal behavior of the Bitcoin time series.

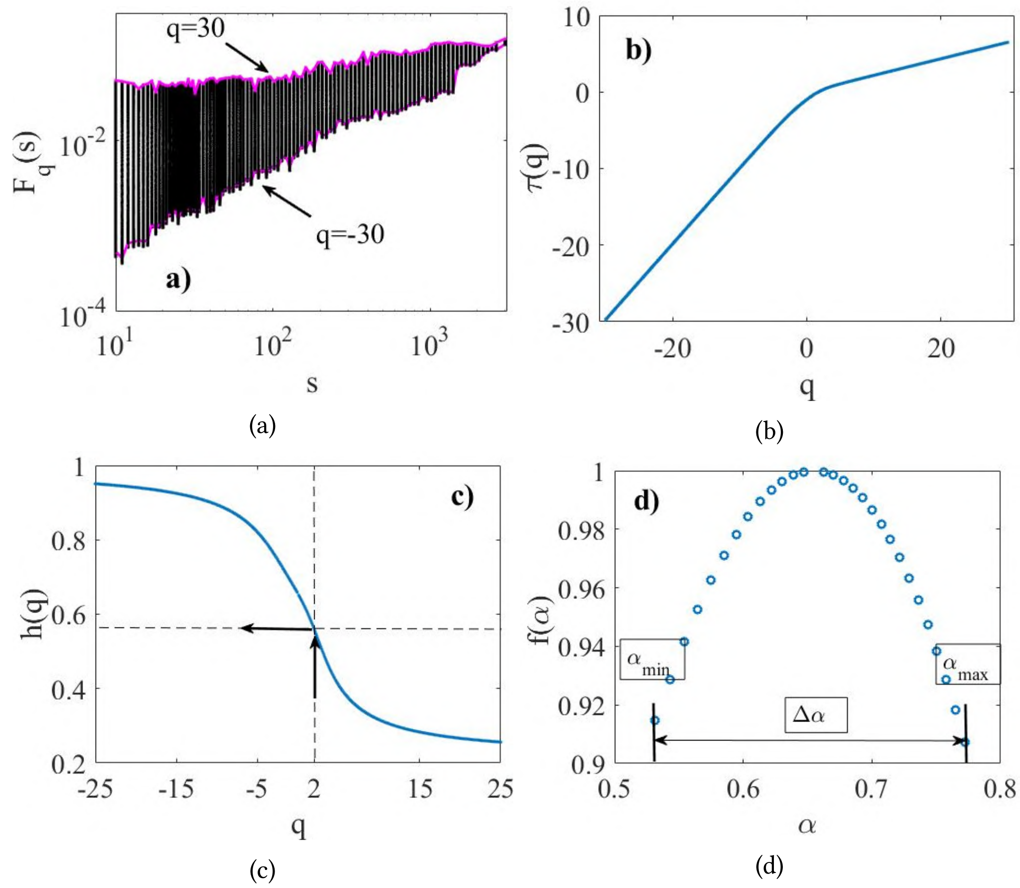


Figure 10: The fluctuation function $F_q(s)$ (a), multifractal scaling exponent $\tau(q)$ (b), $h(q)$ versus q of the BTC return series (c), and singularity spectrum $f(\alpha)$ (d) obtained from MF-DFA for BTC time series.

Figure 10a presents $F_q(s)$ in the log-log plot. The slope changes dependently on q , which indicates the multifractal property of a time series. As it was pointed out, multifractality emerges not only because of temporal correlation, but also because the Bitcoin returns distribution turns out to be broad (fat-tailed) [148], and this distribution could contribute to the multifractality of the time series. The same dependence can be observed in the remaining plots. The scaling exponent $\tau(q)$ remains nonlinear, as well as generalized Hurst exponents that can serve as evidence that Bitcoin exhibit multifractal property.

In the case of multifractals, the shape of the singularity spectrum typically resembles an inverted parabola (see figure 10d); furthermore, the degree of complexity is straightforwardly quantified by the width of $f(\alpha)$, simply defined as $\Delta\alpha = \alpha_{max} - \alpha_{min}$, where α_{max} and α_{min} correspond to the opposite ends of the α values as projected out by different q-moments (equation (10)).

In the figure below we present the width of the spectrum of multifractality that changes over time accordingly to the sliding window approach. The whole figure consists of both a three-dimensional plot (singularity spectrum) and two-dimensional representation of its surface.

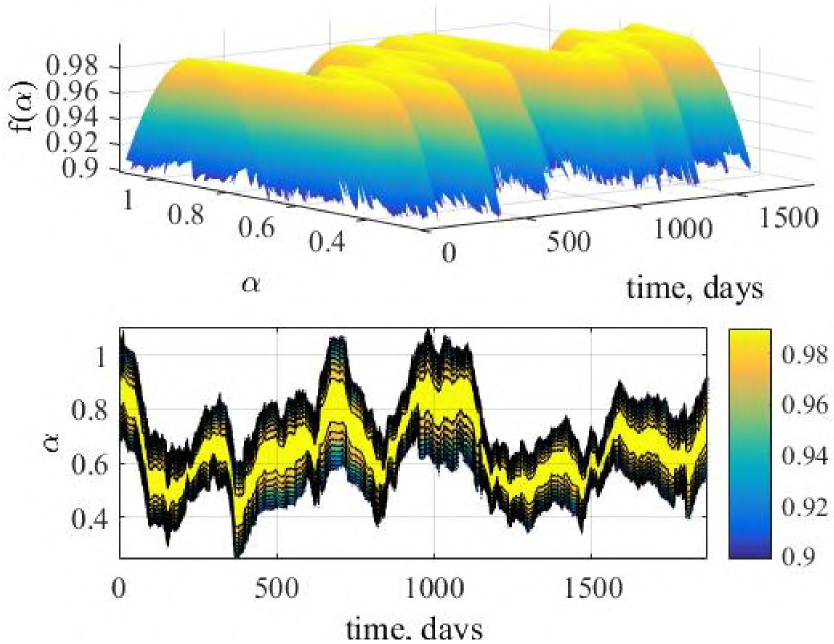


Figure 11: Changes in the spectrum of multifractality in time.

If the series exhibited a simple monofractal scaling behavior, the value of singularity spectrum $f(\alpha)$ would be a constant. As can be observed, here our series exhibits a simple multifractal scaling behavior, as the value of singularity spectrum $f(\alpha)$ changes dependently on α , i.e., it exhibits different scalings at different scales. Moreover, with the sliding window of the corresponding length, we understand that at different time periods Bitcoin becomes more or less complex. The value of $\Delta\alpha$ gives a shred of additional evidence on it.

As we can see from the presented results, the width of the singularity spectrum after the crisis starts to increase, which tells us that more violent price fluctuations are usually expected. With the decreasing width of the singularity spectrum, the series is expected to hold the trend. As the rule, it reaches its minimum before the collapse of the BTC price.

5. Chaos-dynamical measures

Apparently random fluctuations in financial systems often tend to exhibit varying levels of complexity and chaos. Regarding limited data, it becomes hard to define the boundaries of predictability of them. The analysis of such systems, the processes driving their dynamics chaos theory has been considered in various fields such as economics, finance, physics, and others [209, 210, 211, 212, 213, 214]. Regarding the analysis of Bitcoin dynamics, the knowledge about its completely random and, at the same time, deterministic processes can potentially explain fluctuations in time series of different nature. Considering the financial sector, evidence on deterministic chaos, the knowledge of such moments when two initially close trajectories

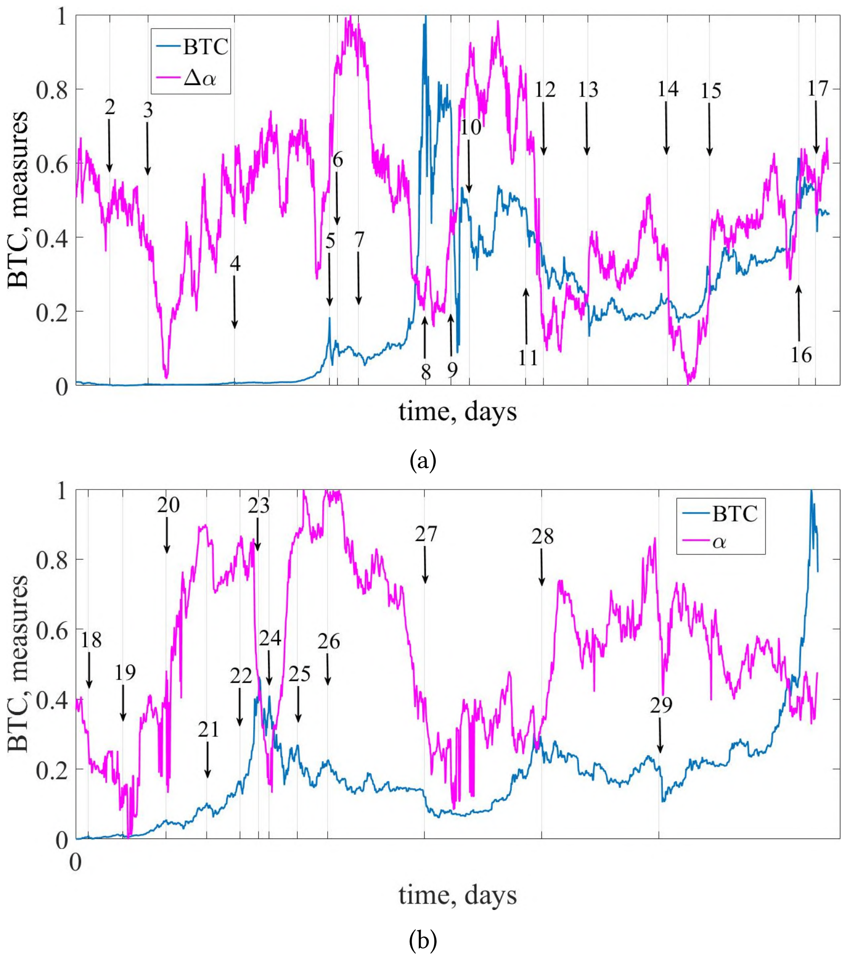


Figure 12: The comparison of the corresponding first (a) and second (b) Bitcoin time intervals with the width of the multifractality spectrum measure.

start to diverge, and of the periods for which they will stay close to each other would have important implications for regulators and traders, who will develop effective short-term trading strategies. During the years, chaos theory has been providing approaches to study some interesting properties of time series. The most widespread are: correlation dimension, the BDS test, Kolmogorov entropy, Lyapunov exponent, close returns test, etc. [215, 216, 217, 218].

Endowing Bitcoin time series with the sliding window approach and efficient methods of Lyapunov exponents, and Levy alpha-stable distribution, we are going to reflect its transition between chaotic and non-chaotic behavior. Also, as it has been observed, such unstable events as market crashes correspond to fat tails. Thus, the analysis of such extreme events can be understood throughout Levy alpha-stable distribution.

5.1. Lyapunov exponents

The evolution of the system exhibits *sensitive dependence* on *initial conditions*. It means that initially close trajectories that evolve may rapidly diverge from each other and have totally different outcomes. Accordingly, with small uncertainties that amplify enormously quickly, long-term predictions turn out to be impossible. On the other hand, in a system with *attraction points* or *stable points*, the distance between them decreases asymptotically in time or with the number of points, which tend to converge [219].

To present the idea more precisely, let's consider two consecutive points presented as $x(t)$ and its initially close neighbor as $x(t) + \delta(t)$, where $\delta(t)$ represents a tiny deviation in time t, as presented in figure 13.

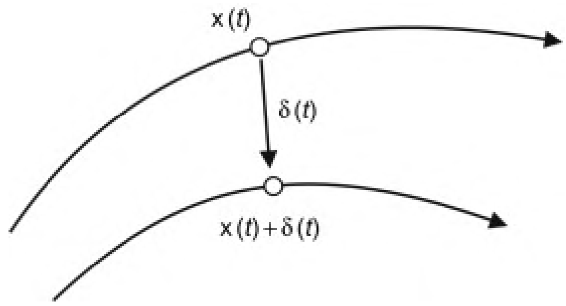


Figure 13: Divergence of two initially close trajectories in a dynamical system [220].

As two initially close points disturbed by some event, they start to diverge, and the distance between them grows following exponential law:

$$\|\delta(t)\| \approx \|\delta(0)\| \exp(\lambda t) \tag{13}$$

with λ that denotes the *Lyapunov exponent* (LE); $\delta(t)$ is the distance between the reference point and its nearest neighbor after t iterations; $\delta(0)$ is the initial distance between the reference point and its nearest neighbor perturbed with a small error at $t = 0$.

LE is a measure of the exponential rate of nearby trajectories in the phase-space of a dynamical system. In other words, it quantifies how fast converge or diverge trajectories that start close to each other, quantifying the strength of chaos in the system.

In such cases when our system n-dimensional, we have as many LEs as the dimensions in it. To define them, we consider the evolution of an infinitesimal sphere of perturbed initial conditions. During its evolution, the sphere will become distorted into an infinitesimal ellipsoid. Defining the length of the i^{th} principal axis as $\delta_i(t)$, there are n-Lyapunov exponents given by:

$$\|\delta_i(t)\| \approx \|\delta_i(0)\| \exp(\lambda_i t), \quad \text{for } i = 1, \ldots, n. \tag{14}$$

To identify whether the motion is periodic or chaotic, especially, for large t it is recommended to contribute to the *largest Lyapunov exponent* (LLE) among the others of the n-dimensional dynamical system [220], as the diameter of ellipsoid starts to be controlled by it. Exactly the LLE is used to quantify the predictability of the systems, since exponential divergence means that in the system where the initial difference was infinitesimally small, start to rapidly lose its predictability behaving differently. However, it should be noted that other exponents also contain important information about the stability of the system, including the directions of convergence and divergence of the trajectories [221].

The existence of at least one positive LE is generally seen as a strong indicator of chaos. Positive LE means that initially similar, phase space trajectories that are sensitive to initial conditions and diverge exponentially fast, characterize chaotic behavior of the system. Negative LE responds to the cases when trajectories remain close to each other, but it is not necessarily implied stability, and we have to examine our system in more detail. Zero or very close to zero exponents indicate that perturbations made along the trajectory neither diverge nor converge.

With the great interest in LE, more and more methods and proposals for their calculating have appeared. Unfortunately, there has not been obtained accepted and universal method for estimating the whole spectrum of Lyapunov exponents from a time series data. One of the most common and popular algorithms have been applied by Wolf et al. [222], Sano and Sawada [223], and later improved by Eckmann et al. [224], Rosenstein et al. [225], Parlitz [226], Balcerzak et al. [227]. Here, we followed the methods proposed by Gao et al. [228, 229], Soloviev et al. [230, 82], Soloviev and Stratiychuk [231] to compute the spectrum of Lyapunov exponents. With Rosenstein's algorithm, we compute only the LLE from an experimental time series. As again suggested by Eckmann et al. [232] one of the measures from recurrence quantification analysis can be considered for estimation of the LLE since it detects in a similar way highly non-monotonic behavior.

With the high growth in computer science, computer simulations of complex and chaotic systems become increasingly appreciated. For at least two decades, with development in numerical computations and quantitative analysis, no doubt left that chaos theory suggests the same unstable fluctuation that may be as common as the extreme events and critical transitions in financial markets. For instance, Scheinkman and Lebaron [233] explored several indications of nonlinear dynamic structure in stock market returns. In their opinion, the weaknesses of such studies are based on time series that are not long enough to reveal the strange (fractal) attractors. On the other hand, the reason may be chaos that comprises a class of signals intermediate between regular periodic or quasiperiodic motions and unpredictable, truly stochastic behavior [234]. Kulkarni [235] denotes that, probably, random financial fluctuations often exhibit varying levels of fluctuations, chaos. Kulkarni's paper represents the efficiency of LE for the complexity analysis of shortly limited data. The analysis constitutes weakly chaotic behavior which alternates with non-chaotic over the entire period of analysis.

Lyapunov exponents are a natural first choice in exploring and indicating such chaotic behaviors that occur in it. They do not only classify the system but also tell us the limits of predictability of the chaotic system [234]. During the last few decades, there was plenty of scientific research that was related to chaos systems, chaos behavior and, namely, to the LE. The earliest papers, in which Bajo-Rubio et al. [236], Dechert and Gencay [237] try to use LE to detect chaos dynamics in financial time series, it is determined that linear, deterministic processes are characterized with negative LE from nonlinear, chaotic processes with the largest exponent (where it is positive). Besides, there is an article in which Gençay [238] presents a methodology to compute the empirical distributions of LE's using a blockwise bootstrap technique. This method provides a formal test of the hypothesis that the LLE equals some hypothesizes value, and can be used to test the system for the presence of chaotic dynamics. Such methodology is particularly useful in those cases where the largest exponent is positive but very close to zero.

Shreemoyee and Vikhyat [239] in their paper investigated the local fractal and chaotic properties of financial time series by calculating two exponents, the Local Hurst Exponent and LE. As was seen in their research, all calculations were made with the algorithm of the sliding window where they had considered two major financial indices of the US: the DJIA and the S&P 500. Regarding the considered measures, they attempted to predict the major crashes that took place in these markets.

Srinivasan et al. [240] have provided an explanation and motivation for reconstructed phase spaces using the methods of time delay and SVD embedding. They explained the meaning of LE and an algorithm for its estimation for the corresponding chaotic, deterministic, and periodic time series. From their presented results it is seen that estimated positive and zero exponents converge to the expected, documented values. Mastroeni and Vellucci [241] obtained empirical results with the help of the LLE and a determinism test that shows that commodity and futures prices are representatives of a nonlinear deterministic, rather than stochastic systems. Similarly to Shreemoyee and Vikhyat [239], Plakandaras et al. [242] measured the Hurst exponent and LE in the sliding window to focus on persistence and chaotic behavior — two prime characteristics of uncertainty indices. For such purpose, they analyzed 72 popular indices constructed by forecasting models, text mining from news articles, and data mining from monetary variables. More specifically, researchers found that almost all uncertainty indices are persistent, while the chaotic dynamics are detected only sporadically and for certain indices during recessions of economic turbulence. Chakrabarti and Sen [243] in one of their chapters explored whether the global markets are intrinsically unstable where unpredictability, disorder, and discontinuities are inherent and not aberrations. They investigated a huge amount of literature and examine the possible non-linear, particularly chaotic nature of the global stock markets. Their study explores the possible presence of chaos in two phases: over the period for 1998-2005 and from 2006 to 2011. Over 30 indices had been investigated. Empirical results showed that for the first phase, 29 indices are deterministic. But 10 of them are found to be non-chaotic. Estimated determinism factors for all the indices are quite high, but Lyapunov exponent is presented to be non-positive for at least 6 of them, where others are chaotic.

As it is seen, chaos theory and its tools remain a huge challenge for researchers of different fields of science and, namely, in the financial industry, and, as it was suggested by Plakandaras et al. [242], the examination of persistent and chaos should be a prerequisite step before using financial indices in economic policy model. The world of Lyapunov exponents remains a growing interest in their definition, numerical methods, and application to various complex systems. In summary, LLE allows us to establish [220]:

- transition region between stable and unstable;
- stability region;
- unstable region;
- chaotic region, including a possible transition between unstable and chaotic.

5.1.1. Eckmann et al. method

Firstly, according to the approach of Eckmann et al. [224], we need to reconstruct attractor dynamics from a single time series $\{x(i) \,|\, i = 1, \ldots, N\}$ with the embedding dimension d_E , and after this, we construct d_E-dimensional orbit representing the time evolution

$$\vec{\mathrm{X}}\,(i) = [x(i), x(i+1), \ldots, x(i + (d_E - 1))]\,, \qquad \text{for } i = 1, \ldots, N - d_E + 1.$$

Then, we have to determine the most neighboring trajectories with $\vec{\mathrm{X}}(i)$:

$$\left\|\vec{\mathrm{X}}(i) - \vec{\mathrm{X}}(j)\right\| = \max_{0 \le \alpha \le d_E - 1} \{|x(i+\alpha) - x(j+\alpha)|\}\,. \tag{15}$$

We sort the $x(i)$ so that $x(\Pi(1)) \le x(\Pi(2)) \le \cdots \le x(\Pi(N))$ and store the permutation Π, and its inverse Π^{-1}. Then, we try to find the neighbors of $x(i)$ in dimension 1 by looking at $k = \Pi^{-1}(i)$ and scan the $x(\Pi(s))$ for $s = k+1, k+2, \ldots$ and $k-1, k-2, \ldots$ until $x(\Pi(s)) - x(i) > r$. For chosen embedding dimension $d_E > 1$, we select the value of s for which further condition is true

$$|x(\Pi(s) + \alpha) - x(j + \alpha)| \le r, \quad \text{for } \alpha = 0, 1, \ldots, d_E - 1.$$

After we embedded our system in d_E dimensions, we need to determine the $d_E \times d_E$ matrix M_i that will describe time evolution of the vectors that surround trajectory $\vec{\mathrm{X}}(i)$, and how they map onto $\vec{\mathrm{X}}(i+1)$ state. The matrix M_i is obtained by looking for neighbors

$$M_i(\vec{\mathrm{X}}(i) - \vec{\mathrm{X}}(j)) \approx \vec{\mathrm{X}}(i+1) - \vec{\mathrm{X}}(j+1). \tag{16}$$

Nevertheless, the vectors $\vec{\mathrm{X}}(i) - \vec{\mathrm{X}}(j)$ may not span $\mathfrak{R}^{d_E}$. In this case, such indeterminacy may lead to spurious exponents which confuse the analysis. To overcome such obstacles, the projection of the trajectories is determined on a subspace of dimension $d_M \le d_E$. Thus, the manifold on which the dynamics takes place corresponds to the local dimension d_M, where d_E should be somewhere larger than d_M to avoid the presence of false neighbors [234, 131]. Hence, the trajectory $\vec{\mathrm{X}}(i)$ is associated with a d_M-dimensional vector

$$\vec{\mathrm{X}}(i) = [x(i), x(i+\tau), \ldots, x(i + (d_M - 1)\tau)] = [x(i), x(i+\tau), \ldots, x(i + d_E - 1)]\,, \tag{17}$$

where $\tau = (d_E - 1)/(d_M - 1)$. When $\tau > 1$, the condition (16) is replaced by

$$M_i(\vec{\mathrm{X}}(i) - \vec{\mathrm{X}}(j)) \approx \vec{\mathrm{X}}(i+\tau) - \vec{\mathrm{X}}(j+\tau). \tag{18}$$

The M_i is then defined by the linear least-square method [244]. The last step of the algorithm is the classical QR matrix decomposition to find orthogonal matrices Q_i and upper-triangular matrices R_i with non-negative diagonal elements such that

$$M_{1+i\tau} Q_i = Q_{i+1} R_{i+1}, \quad \text{for } i = 0, 1, 2, \ldots\,.$$

As it was proposed by Touzé and Chaigne [245], Eckmann et al. [224], Eckmann and Ruelle [138], in order to calculate d_M Lyapunov exponents, the equation for the k^{th} Lyapunov exponent with K number of points on the attractor, for which the Jacobian has been estimated, the diagonal eigenvalues of the matrix R_i and the sampling step Δt is given by:

$$\lambda_k = \frac{1}{\Delta t}\frac{1}{\tau}\frac{1}{K}\sum_{i=0}^{K-1}\ln(R_i)_{kk}.$$

Thus, with linearizations by using the diagonal elements from the QR decomposition, we can calculate Lyapunov exponents.

The calculation results for the LLE on the example of BTC are presented in figure 14.

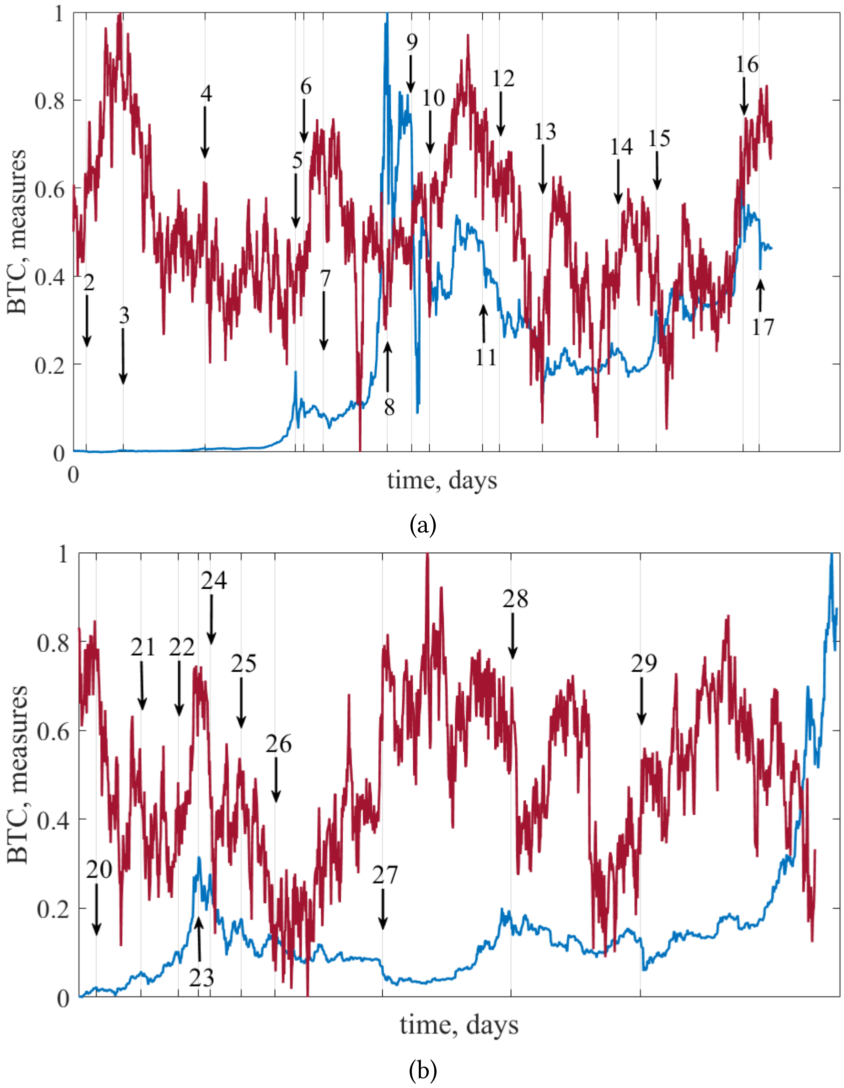


Figure 14: The dynamics of the LLE (λ_{max}) with Eckmann et al. method within the time window of the length 250 days and the time step of 1 day for the first (a) and second (b) periods. Exponents are calculated with $d_E = 3$ and $d_M = 2$.

Let us pay attention to the behavior of λ_{max} at moments of the known failures noted in the list of crashes and critical events. Definitely, we can see that in the pre-crisis period, the value of LLE decreases markedly, then increases in the post-crisis period.

5.1.2. Rosenstein's et al. method

Rosenstein's algorithm [225] uses the delay embedding method that reconstructs the most important features of a multi-dimensional attractor into a single one-dimensional time series of some finite size N. For the time series $\{x(i) \,|\, i = 1, \dots, N\}$, each delay embedded vector $\vec{\mathrm{X}}(i)$ will be presented similarly to the vector (17) with embedding dimension d_E and time delay τ. Then, in the reconstructed trajectory, we initialize searching for in the state space for the nearest neighbor $\vec{\mathrm{X}}(j)$ of the trajectory $\vec{\mathrm{X}}(i)$:

$$\delta_i(0) = \min_{\vec{\mathrm{X}}(i)} \left\| \vec{\mathrm{X}}(i) - \vec{\mathrm{X}}(j) \right\|, \quad \text{for } |i - j| > \text{mean period},$$

where $\| \, \|$ is the Euclidian norm, $\vec{\mathrm{X}}(j)$ is the nearest neighbor, and $\vec{\mathrm{X}}(i)$ is the reference point.

From equation (13) we have already known that the distance between states $\vec{\mathrm{X}}(i)$ and $\vec{\mathrm{X}}(j)$ will grow in time accordingly to a power law, where λ is a good approximation of the LLE. For further estimations, we look at the logarithm of the distance trajectory $\ln \delta_i(k) \approx \lambda(k \cdot \Delta t) + \ln c_i$, where $\delta_i(k)$ is the distance between i^{th} pair of the nearest neighbors defined in equation (18) after k time steps, c_i is the initial separation of them, and Δt is the time interval between measurements (sampling period of the time series).

Further result of this algorithm is not a numerical value, but a function of time

$$y(k, \Delta t) = \frac{1}{\Delta t} \frac{1}{M} \sum_{i=1}^{M} \ln \delta_i(k)$$

with the size of the reconstructed time series $M = N - (d_E - 1)\tau$, and a set of approximately parallel lines $\delta_i(k)$ whose slope roughly proportional to the LLE. Then, it is proposed to be calculated as the angle of inclination of its most linear section. Finding such a section turns out to be a non-trivial task, and sometimes it is impossible to specify such a section at all. Despite this problem, Rosenstein's method is easy for implementing and computing.

The LLE behavior for a window procedure with the length of 250 days and the step size of 1 day is shown in figure 15.

It can be seen that, as before, the LLE is also sensitive to the crisis conditions of BTC.

5.1.3. Scale-dependent Lyapunov exponent (SDLE)

We briefly describe the idea and the formal foundations of the method SDLE, introduce new measures of complexity, and illustrate their effectiveness with the example of the BTC index. Let us have a single observation conducted at a discrete time interval Δt in the form of a time series $\{x_i \,|\, i = 1, \dots, N\}$ where $t = i \cdot \Delta t$. After reconstructing the phase space, let us consider the ensemble of trajectories.

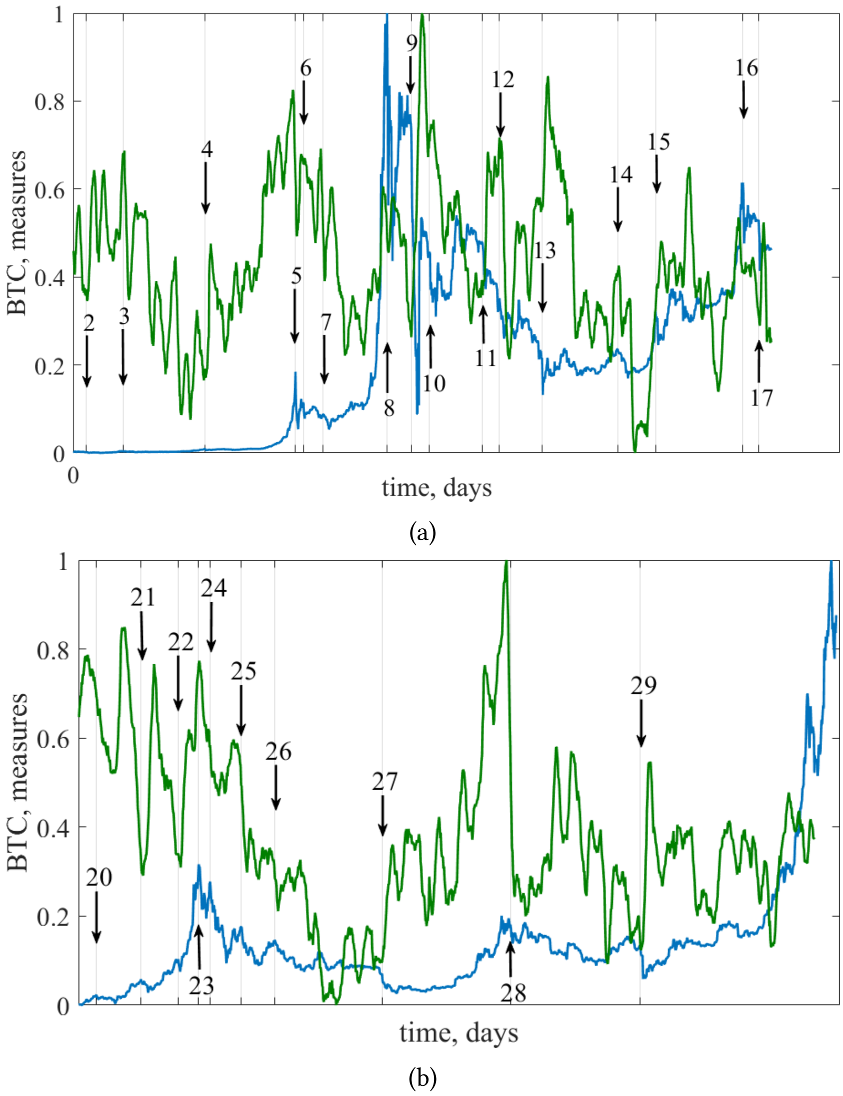


Figure 15: The dynamics of the LLE calculated with Rosenstein et. al. method within the time window length of 250 days and the time step of 1 day for first (a) and second (b) periods. Exponents are calculated with $d_E = 3$ and $\tau = 2$.

Let us denote the initial distance between two close trajectories $\delta(0)$, and their average distance at time t and $t + \Delta t$ as $\delta(t)$ and $\delta(t + \Delta t)$ respectively. Note that the classical algorithm for calculating the LLE (λ_{max}) is based on the assumption (13) and its estimation as $(\ln(\delta(t) - \delta(0)))/t$. Depending on $\delta(0)$, this property may not be true even for truly chaotic systems. To calculate the SDLE, we check whether the following inequality holds for a pair of vectors $\left(\vec{X}(i), \vec{X}(j)\right)$:

$$\delta(k) \leq \left\|\vec{X}(i) - \vec{X}(j)\right\| \leq \delta(k) + \Delta\delta(k), \quad \text{for } k = 1, 2, 3, \dots,$$

where $\delta(k)$ and $\Delta\delta(k)$ are arbitrarily chosen small values of distances, and

$$\left\|\vec{X}(i) - \vec{X}(j)\right\| = \sqrt{\sum_{w=1}^{d_E} (x(i+(w-1)\tau) - x(j+(w-1)\tau))^2}.$$

Geometrically, the last inequality defines a shell in high-dimensional space. Next, we investigate the dynamics of the same pairs of vectors $\left(\vec{X}(i), \vec{X}(j)\right)$ in the middle of the shell and perform averaging over the ensemble by indices i, j. Since the exponential or power functions are of the greatest interest, we assume that logging and averaging can be reversed. Finally, the following equation looks like:

$$\lambda(\delta(t)) = \left\langle \ln\left\|\vec{X}(i+t+\Delta t) - \vec{X}(j+t+\Delta t)\right\| - \ln\left\|\vec{X}(i+t) - \vec{X}(j+t)\right\|\right\rangle / \Delta t,$$

with the sampling intervals t and Δt; the angle brackets correspond to the averaging over the indices i, j inside the shell and

$$\delta(t) = \left\|\vec{X}(i+t) - \vec{X}(j+t)\right\| = \sqrt{\sum_{w=1}^{d_E} (x(i+(w-1)\tau+t) - x(j+(w-1)\tau+t))^2}.$$

Finally, note that

$$I = \ln\delta(t) = \ln\delta(0) + \int_0^t \lambda(\delta(t))dt.$$

Time series, characterizing economic systems of varying degrees of complexity, differ in magnitudes $\Delta\lambda = \lambda_{max} - \lambda_{min}$, $\Delta\delta = \delta_{max} - \delta_{min}$, and I. As an example, the integral measure I is calculated for a sliding window of 400 days and step size of 1 day for the daily values of the BTC index (see figure 16).

Like the previous LLE indicators, SLDE is also a leading indicator. However, its disadvantage is the impossibility of accurate calculations for small window sizes which limits the possibilities of its usage.

5.2. Lévy alpha-stable distribution

Financial crises that regularly shake the world economy are characterized by noticeable fluctuations in stock indices, thereby causing noticeable changes in the statistical distributions of empirical data [112, 246].

In 1900, Bachelier proposed the first model for the stochastic process of returns – an uncorrelated random walk with independent, identically Gaussian distributed (i.i.d) random variables [247]. This model is natural if one considers the return over a time scale Δt to be the result of many independent "shocks", which then lead by the central limit theorem to a Gaussian distribution of returns [247]. Some stylized facts of daily returns [248, 249, 250] reveal that distributions are leptokurtic and, therefore, Gaussian distribution does not fit well to the data. Lux and Sornette [251] pronounce that the distribution of such data may be not only leptokurtic, but it can also be characterized by fat tails [252, 253, 254, 255]. Thus, it should belong to the class

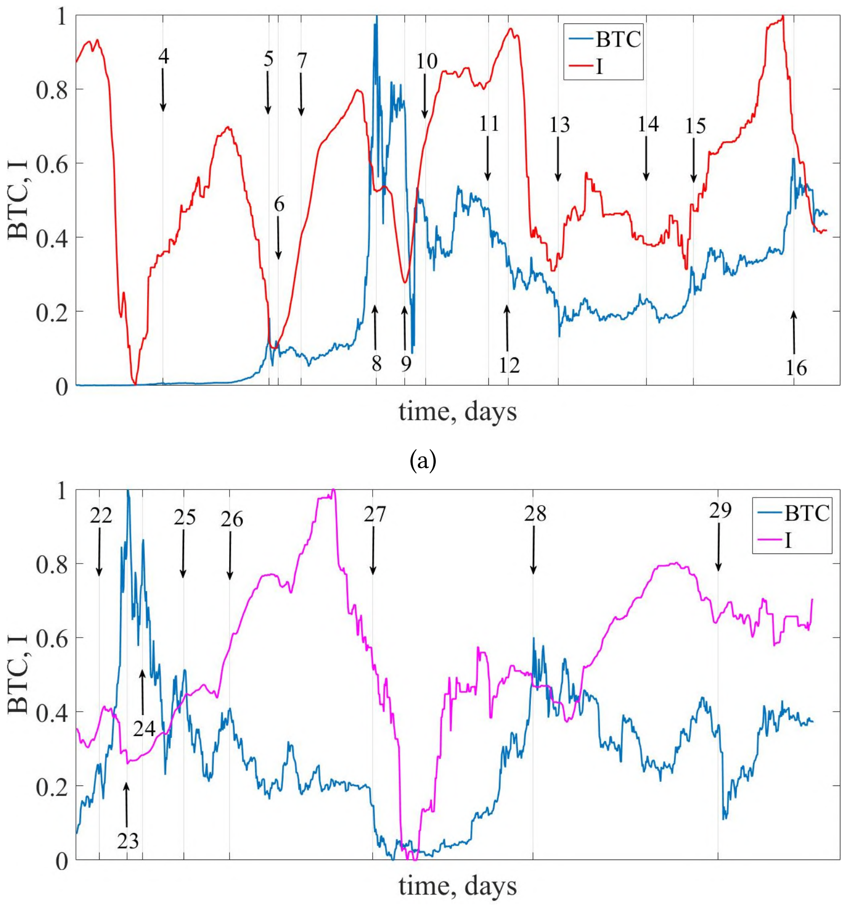


Figure 16: The dynamics of the SDLE exponents within the time window of length 400 days and the time step of 1 day for first (a) and second (b) periods.

of fat-tailed distributions. Formally, it is said that they follow a *power law*. The emergence of power-law behavior in price fluctuations is argued to be a consequence of underlying complex mechanisms, such as feedback effects and correlations in financial markets [256, 257, 258, 259]. Some theories associate this phenomenon with market impact and the distribution of large investors [260, 248], while other studies model the power-law behavior as a consequence of limited information and the true value of companies [261]. Such property is a symptom of self-organization and complexity which are prominent for economic systems. In Chakraborty et al. [262] paper it was established that currencies of several frontiers that are outside of *inverse cubic law* (with an exponent of $\alpha \simeq 3$) belong to the Levy-stable regime and are expected to be yet emerging and having sudden large changes such as crashes and critical events, while those of most developed exhibited *inverse cubic law*.

Recently, it has been reported that Bitcoin is becoming more mature, following *inverse cubic law* [263, 264, 265]. Besides, Begušić et al. [266], motivated by the rise of novel assets based on blockchain technology, presented a detailed analysis of trade-level data of the BTC/USD pairs from five large Bitcoin exchanges: Mt. Gox, BTC-e, Bitstamp, Bitfinex, and Kraken. They applied two estimation methods and a resampling-based technique to statistically validate if the main cryptocurrency follows power-law behavior or not. Their study presented that the exponent α lie within the range $2 < \alpha < 2.5$ that gives the evidence that the cryptocurrencies market is much more volatile than the stock market, and that Bitcoin returns exhibit much heavier tails. Moreover, they find that such a phenomenon is universal as such behavior holds across multiple exchanges and tiny intervals. Their results imply that Bitcoin lies outside of the Levy-stable region which provides the existence of a finite second moment, and a basis for the usage of standard financial theories for portfolio optimization and risk management.

In figure 17 the daily returns of Bitcoin and contrast it with a sequence of i.i.d. Gaussian random variables are presented.

It is obvious that the distribution of returns has heavy tails and in general case can be described as

$$P(g > x) \sim x^{-(1+\alpha)}, \quad \alpha \in (0, 2]. \tag{19}$$

Figure 17 b it can be seen that Bitcoin exhibits the inverse cubic law.

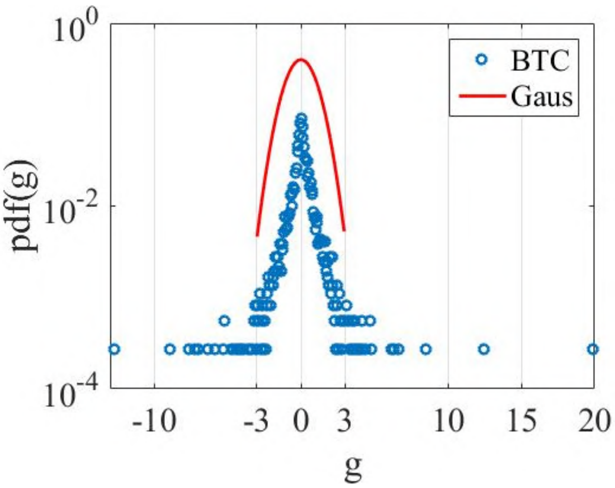
In the analysis of cotton prices, Mandelbrot observed that in addition to being non-Gaussian, the process of returns shows another interesting property: "time scaling" – that is, the distributions of returns for various choices of Δt, ranging from 1 day up to 1 month have similar functional forms [267]. Motivated by (i) pronounced tails, and (ii) a stable functional form for different time scales, Mandelbrot proposed that the distribution of returns is consistent with a Levy stable distribution [268, 269, 267] – that is, the returns can be modeled as a Levy stable process. Levy stable distributions arise from the generalization of the Central Limit Theorem (CLT) to random variables that do not have a finite second moment.

The CLT [270], which offers the fundamental justification for approximate normality, points to the importance of alpha-stable distribution: they are the only limiting laws of normalized sums of independent, identically distributed random variables. Gaussian distributions, the best-known member of the stable family, have long been well understood and widely used in all sorts of problems. However, they do not allow for large fluctuations and are thus inadequate for modeling high variability. Non-Gaussian stable models, on the other hand, do not share such limitations. In general, the upper and lower tails of their distributions decrease like a power function. In literature, this is often characterized as heavy or long tails. In the last two or three decades, data that seem to fit the stable model has been collected in fields as diverse as economics, telecommunications, hydrology, and physics [249].

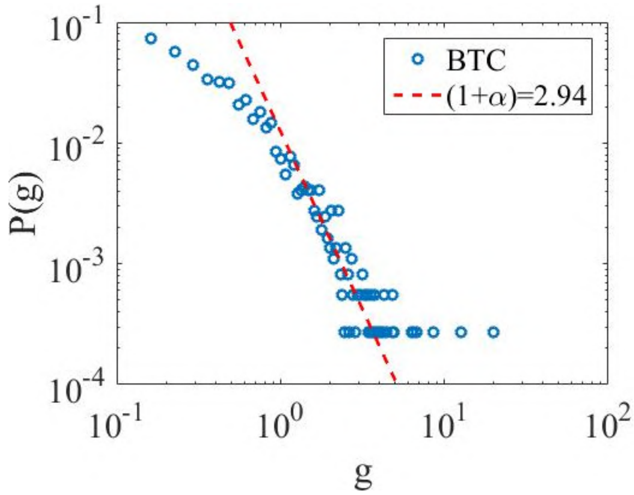
Consequently, a probability model with a power tail can be suitable for identifying processes with extreme events. It was discovered that alpha-stable distributions fit better than the Gaussian distribution to financial and spot markets. It is still debatable whether Lévy stable distribution is applicable since there is not enough theoretical material, and there is not a universal analyzing method for estimating parameters of Lévy stable distribution.

5.2.1. Lévy's stable distribution properties

Lévy stable distribution being the generalization of the CLT, became an addition to a wide class of distributions. The term *stable* is such characteristic of distribution where the shape



(a)



(b)

Figure 17: Probability density function of BTC daily normalized returns for the whole period (a). Cumulative distribution of the normalized BTC daily returns (b). Fits yield values $\alpha = 1.94 \pm 0.02$.

(up to scale and shift) retains under addition: if $X, X_1, X_2, \dots, X_n$ are independent, identically distributed random variables, then for every n

$$X_1 + X_2 + \cdots + X_n \stackrel{d}{=} c_n X + d_n \tag{20}$$

for some constants $c_n > 0$ and $d_n \in \mathbb{R}$, where $X_1, \dots, X_n$ are independent, identical copies of X.

The class of all laws that satisfy condition (20) is presented by 4 parameters: $\alpha \in (0, 2]$ is the *index of stability* or *characteristic exponent* where a smaller value of α corresponds to more severe tails of the distribution. The parameter $\beta \in [-1, 1]$ is called the *skewness* parameter of the law. If $\beta = 0$, the distribution is symmetric. In the case when $\beta > 0$, it is skewed toward the right, otherwise to the left. The last two parameters stand for the *scale* $\gamma \in [0, \infty)$ and $\delta \in (-\infty, \infty)$

the *location* parameters of the distribution. Since random variables X is characterized by four parameters, we will denote α-stable distribution by $S(\alpha, \beta, \gamma, \delta)$ and write

$$X \sim S(\alpha, \beta, \gamma, \delta). \tag{21}$$

Lévy stable distributions cannot be defined in closed form expression except few cases: the case of $(\alpha, \beta) = (2, 0)$ corresponds to the Gaussian distribution, $(\alpha, \beta) = (1, 0)$ to the Cauchy distribution. Instead, it is expressed in terms of their Fourier transforms or characteristic functions (CF). If the density $f(x)$ exists, CF of that density can be expressed as

$$\lambda(k) = E\exp(ikX) = \int_{-\infty}^{\infty} \exp(ikx) f(x) dx,$$

where k denotes the Fourier transformed variable. Thus, the inverse Fourier transform

$$f(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-ikx)\lambda(k) dk$$

allows us to reconstruct probability density function with a known characteristic function.

As we do not have any analytical expression for the probability density of a random variable X_i, for Lévy stable distribution, if the variable x_i follows $S(\alpha, \beta, \gamma, \delta)$, the CF can be expressed as [271]:

$$\lambda(k) = \begin{cases} \exp i\delta k - \gamma^{\alpha}|k|^{\alpha}\left[1 - i\beta \mathrm{sgn}(k)\tan(\frac{\pi\alpha}{2})\right], & (\alpha \neq 1), \\ \exp i\delta k - \gamma|k|\left[1 + i\beta \mathrm{sgn}(k)\frac{2}{\pi}\ln|k|\right], & (\alpha = 1). \end{cases} \tag{22}$$

5.2.2. Methods for estimation of stable law parameters

There are numerous approaches that can estimate stable distribution parameters. Since the probability density functions are not always expressed in a closed form, there are some challenges to overcome the analytic difficulties. Thus, there have been constructed a variety of methods: the approximate maximum likelihood (ML) estimation [272, 273], quantiles method [274, 275], fractional lower order moment method [276, 277], method of log-cumulant [278], the logarithmic moment method [279] and more. Unfortunately, some of those methods cannot be applied due to computational problems associated with a limited range of estimation, restricted range of parameters, high computational costs, or requiring a large number of data. However, several of them should be mentioned.

5.2.3. Maximum likelihood method

DuMouchel [280] was the first to obtain approximate ML estimates of α and γ (assuming $\delta = 0$). A multinomial approximation to the likelihood function is used in his approach. Under some additional assumptions on $\hat{\alpha}$ and the likelihood function, DuMouchel has shown the obtained estimates to be consistent and asymptotically normal. However, the computational effort involved seems considerable.

A direct method can be formulated, after Brorsen and Yang [272], as follows. The standard symmetric probability density functions defined by Zolotarev [281] is presented as:

$$f_\alpha(x) = \frac{\alpha}{\pi|1-\alpha|} x^{1/(\alpha-1)} \times \int_0^{\pi/2} U_\alpha(\eta, 0) e^{-x^{\alpha/(\alpha-1)} U(\eta,0)} d\eta, \tag{23}$$

for $\alpha \neq 1, x > 0$, where U_α is defined by:

$$U_\alpha(\eta, \eta_0) = \left(\frac{\sin\alpha(\eta-\eta_0)}{\cos\eta}\right)^{\alpha/(1-\alpha)} \left(\frac{\cos\eta - \alpha(\eta-\eta_0)}{\cos\eta}\right) \tag{24}$$

and η_0 is explained here [282]. Therefore, the parameters α, γ, and δ can be estimated from the observations $\{x_i \,|\, i = 1, 2, \dots, N\}$ by maximizing the log likelihood function:

$$\sum_{i=1}^{N} \log f_\alpha(z_i) = n \log\frac{\alpha}{\alpha-1} + \sum_{i=1}^{N} \frac{\log z_i}{\alpha-1} + \sum_{i=1}^{N} \log \int_0^{\pi/2} U_\alpha(\eta, 0) e^{-z_i^{\alpha/(\alpha-1)} U_\alpha(\eta,0)} d\eta, \tag{25}$$

where $z_i = |x_i - \delta|/\gamma$.

To avoid the discontinuity and non-differentiability of the symmetric α-stable density function at $\alpha = 1$, $alpha$ is restricted to be greater than one. Caution must be used when evaluating the integrals in equations (23) and (25), since the integrals are singular at $\eta = 0$.

An obvious disadvantage of this method is that it is a highly nonlinear optimization problem and no initialization and convergence analysis is available.

5.2.4. Quantiles methods

This method is focused on empirical quantiles, which has been introduced by Fama and Roll, with the assumptions that $\alpha > 1$, $\beta = 0$, and $\delta = 0$ [274]. However, it was much more appreciated through McCulloch [275] after its extension to include asymmetric distribution for $\alpha \in [0.6, 2]$.

In order to quantify the four parameters of the stable distribution, we consider N independent variables x_i that follow alpha-stable distribution (21) [283, 275]. Then, we need to define five empirical quantiles of probability levels 5%, 25%, 50%, 75%, and 95%. Then we have to obtain two intermediate quantities:

$$\begin{cases} \hat{v}_\alpha = \frac{\hat{Q}_{0.95} - \hat{Q}_{0.05}}{\hat{Q}_{0.75} - \hat{Q}_{0.25}}, & \hat{v}_\beta = \frac{\hat{Q}_{0.95} + \hat{Q}_{0.05} - 2\hat{Q}_{0.05}}{\hat{Q}_{0.95} - \hat{Q}_{0.05}}, \\ \hat{\alpha} = \Psi_1(\hat{v}_\alpha, \hat{v}_\beta), & \hat{\beta} = \Psi_2(\hat{v}_\alpha, \hat{v}_\beta), \end{cases}$$

where $\hat{Q}_p (p = 0.05, 0.25, 0.5, 0.75, 0.95)$ is the corresponding sample data with which quantile is calculated, and Ψ_1 with Ψ are the interpolating functions the values of which can be found in Table I-IV by McCulloch [275]. Further, the scale parameter is given by:

$$\hat{\gamma} = \frac{\hat{Q}_{0.75} - \hat{Q}_{0.25}}{\Psi_3(\hat{\alpha}, \hat{\beta})},$$

where $Psi_3(\hat{\alpha}, \hat{\beta})$ is given in Table V [275]. For simplicity of the location parameter, we can define the variable which is predefined in the following form:

$$\xi = \begin{cases} \delta + \beta\gamma \tan \frac{\pi\alpha}{2}, & \text{if } \alpha \neq 1, \\ \delta, & \text{if } \alpha = 1. \end{cases} \tag{26}$$

In consistence with the corresponding parameter ξ which can be estimated by $\hat{\xi} = \hat{Q}_{0.5} + \hat{\gamma}\Psi_5(\hat{\alpha, \beta})$ (Ψ_5 can be obtained through linear interpolation according to Table VII [275]), the location parameter δ is given by:

$$\hat{\delta} = \hat{\xi} + \hat{\beta}\hat{\gamma} tan \frac{\pi\hat{\alpha}}{2}.$$

5.2.5. Empirical characteristic function method

Analyzing data, we often assume that they are ergodic [284]. In general, if random variables are ergodic with the integrable function $f(x)$, and the measure $\rho(x)dx$ in the space M, then the following equation holds [285]:

$$\lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \exp(ikx_i) = \int_{-\infty}^{\infty} \exp(ikx)\rho(x)dx. \tag{27}$$

Then, to consider characteristic functions, equation (27) comes out to be the following ergodic equality [285]:

$$\lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \exp(ikx_i) = \int_{-\infty}^{\infty} \exp(ikx) f(x)dx \tag{28}$$

for which we have

$$\hat{\lambda}(k) = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \exp(ikx_i). \tag{29}$$

This assumption allows us to empirically obtain the probability distribution. Hence, the empirical characteristic function $\lambda_N(k)$ can be calculated as

$$\hat{\lambda}_N(k) = \frac{1}{N} \sum_{i=1}^{N} \exp(ikx_i). \tag{30}$$

Then, according to Koutrouvelis [271, 286] regression type from (22) it can be derived that

$$\log(-\log(|\lambda(k)|^2)) = \log(2\gamma^\alpha) + \alpha \log(k). \tag{31}$$

The imaginary and real parts of $\lambda(k)$ are given by

$$\begin{cases} \lambda_{\mathrm{I}}(k) = \exp(-|\gamma k|^\alpha) \cdot i \sin[\delta k - |\gamma k|^\alpha \beta \mathrm{sgn}(k)\omega(k, \alpha)], \\ \lambda_{\mathrm{R}}(k) = \exp(-|\gamma k|^\alpha) \cdot \cos[\delta k - |\gamma k|^\alpha \beta \mathrm{sgn}(k)\omega(k, \alpha)], \end{cases} \tag{32}$$

where

$$\omega(k,\alpha) = \begin{cases} \tan\frac{\pi\alpha}{2}, & \alpha \neq 1, \\ \frac{2}{\pi}\ln|k|, & \alpha = 1. \end{cases}$$

Suppose $\Upsilon := \arctan(\lambda_{\mathrm{I}}(k)/\lambda_{\mathrm{R}}(k))$. Then, in the condition $\alpha \neq 1$, the last two equations lead, apart from considerations of principal values, to

$$\Upsilon(k) = \delta k - \beta\gamma^{\alpha}\tan(\frac{\pi\alpha}{2})\mathrm{sgn}(k)|k|^{\alpha}. \tag{33}$$

Equation (31) depends only on α and γ, and it suggests that we estimate these parameters by regressing

$$y = \log(-\log|\lambda_N(k)|^2)$$

on $\omega = \log(k)$ in the model

$$y_l = m + \alpha\theta_l + \epsilon_l, \quad \text{for } l = 1,\dots,L, \tag{34}$$

where $y_l = \log(-\log(\hat{\lambda}(k_l))^2)$, $m = \log(2\gamma^{\alpha})$, $\theta_l = \log(k_l)$, and ϵ_l responds for an error term. The proposed real data set for L (see Koutrouvelis [271], Table I) is given by $k_l = \pi l/25$ ($l = 1,\dots,L$).

With estimated and fixed parameters α and γ, the symmetric parameter β and location parameter δ can be obtained by linear regression estimation

$$z_q = \delta k_q - \beta\gamma^{\alpha}\tan(\frac{\pi\alpha}{2})\mathrm{sgn}(k_q)|k_q|^{\alpha} + v_q, \text{for } q = 1,\dots,Q, \tag{35}$$

where $z_q = \Upsilon_N(k_q) + \pi l_N(k_q)$, v_l denotes an error term, and the proposed real data set for Q (see Koutrouvelis [271], Table II) is $k_q = \pi q/50$ ($q = 1,\dots,Q$).

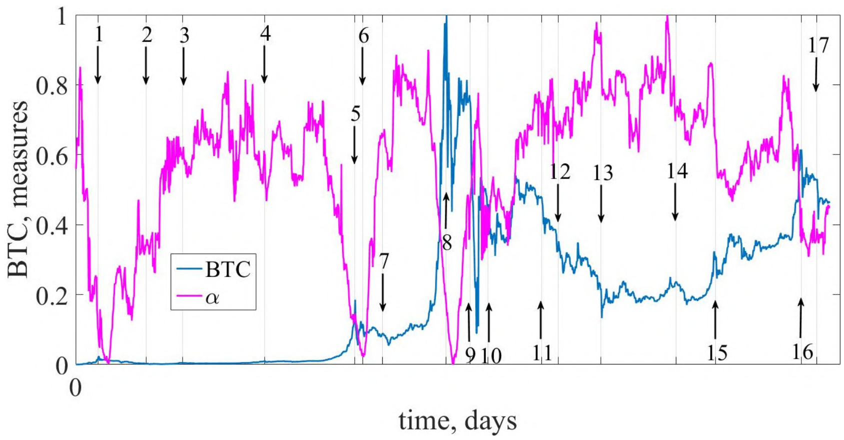
5.2.6. Related studies and corresponding results

Recently, the use of dynamic indicators, precursors of crashes in stock markets using the parameters of a α-stable distribution was proposed by us in the papers [77, 78, 287] and later repeated in a recent paper [30]. From the data above, we estimate the parameters α and β of the stable distribution that the best describes the empirical returns. Figure 18 shows the dynamics of the parameter α as a more informative indicator.

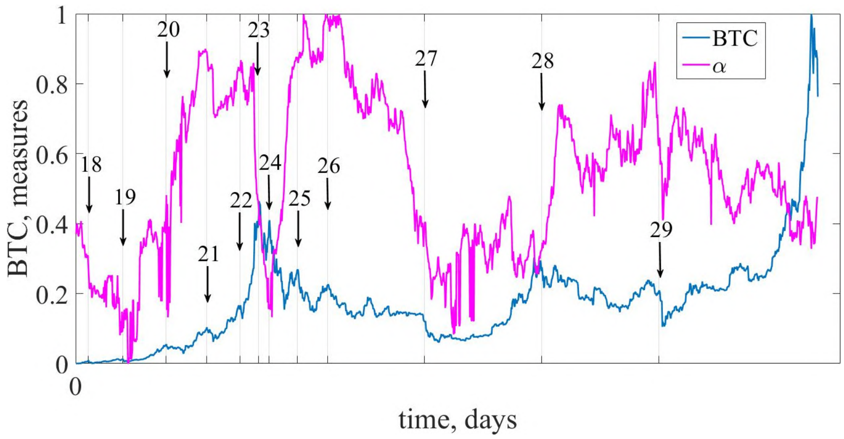
From the figure 18 we can see that our parameters start to decrease in crisis states. Such abnormal behavior can serve as an indicator or precursor of crashes and critical states.

6. Recurrence analysis

In 1890 the mathematical foundations of recurrence were introduced by Henri Poincaré, resulting in the *Poincaré recurrence theorem* [288]. This theorem states that certain systems will return to their arbitrarily close, or exactly the same initial states after a sufficiently long but finite time. Such property in the case of deterministic behavior of the system allows us to make conclusions regarding its future development.



(a)



(b)

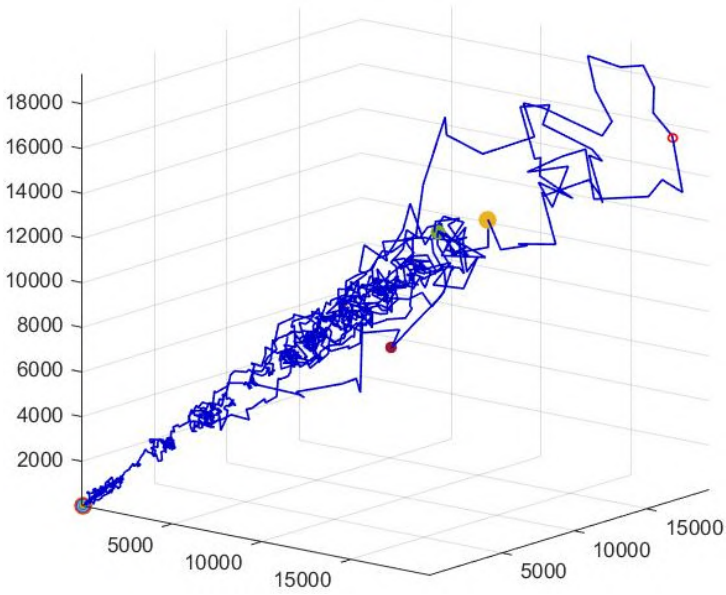
Figure 18: The Bitcoin time series and estimated for them parameter α. Vertical arrows indicate crashes and critical events.

6.1. Recurrence plot

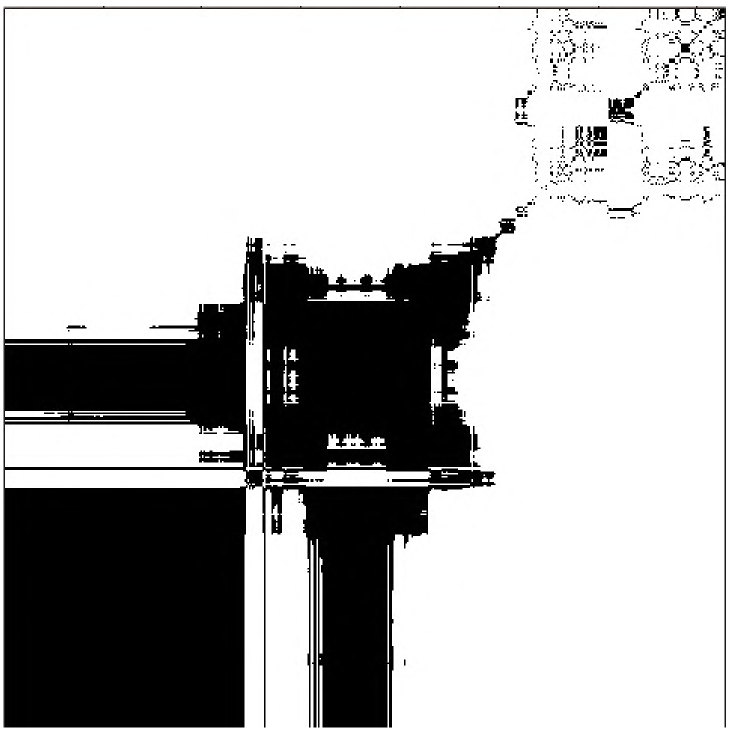
Recurrence plot (RP) have been introduced to study dynamics and recurrence states of complex systems. When we create RP, at first, from recorded time series we reconstruct phase-space trajectory. Then, according to Eckmann et al. [232], we consider a trajectory $\vec{X}(i)$ on the reconstructed trajectory. The recurrence plot is an array of dots in a $N \times N$ matrix, where dot is placed at (i, j) whenever $\vec{X}(j)$ is sufficiently close to $\vec{X}(i)$, and both axes are time axes which mathematically can be expressed as

$$R_{ij} = \mathscr{H}(\epsilon - \left\|\vec{X}(i) - \vec{X}(j)\right\|), \quad \text{for } i, j = 1, \ldots, N, \tag{36}$$

where $\| \ \|$ is a norm (representing the spatial distance between the states at times i and j); ϵ is a predefined recurrence threshold, and $\mathscr{H}$ is the Heaviside function. As a result, the matrix captures a total of N^2 binary similarity values. A synthetic example is presented in figure 19.



(a)



(b)

Figure 19: Phase portrait (a) and corresponding RP (b) for BTC.

Typically an L_p-norm is applied to determine the pairwise similarity between two vectors.

Accordingly to Webber, Jr. and Zbilut [140], there are such candidates that can serve as a distance measure:

- the L_1-norm (Manhattan distance);
- the L_2-norm (Manhattan distance);
- the L_∞-norm (Manhattan distance);

In accordance with our results, Maximum distance seems to be a suitable choice. It is often used as it is independent of the phase space dimension, easy to calculate, and allows some analytical expression [289, 290, 291].

Also, as it can be seen from equation (36), the similarity between vectors is determined by a threshold ϵ. The choice of $\epsilon > 0$ ensures that all vectors that lie within this radius are similar to each other, and that dissimilarity up to a certain error is permitted [288].

The fixed radius for recurrent states is the commonly used condition, which leads to equally sized ϵ-neighborhoods. The shape in which neighborhoods lie is determined by the distance metric. Applying the fixed threshold with the distance metric, we define recurrence matrices that are symmetric along the middle diagonal. The self-similarity of the multi-dimensional vectors reflects in the middle diagonal which is commonly referred to as *line of identity* (LOI). In contrast, it is not guaranteed that a recurrence matrix is symmetric, if the condition of fixed number of nearest neighbors is applied. For specific purposes (e.g., quantification of recurrences), it can be useful to exclude the LOI from the RP, as the trivial recurrence of a state with itself might not be of interest [292].

6.1.1. Recurrence plots and their structures

The visualization of trajectories and hidden patterns of the systems is the "destiny" of RP [293, 292].

The dots within RP, representing the time evolution of the trajectories, exhibit characteristic large-scale and small-scale patterns. Large-scale patterns of RP can be classified as *homogeneous*, *periodic*, *drift*, and *disrupted* [294, 295, 290]:

- *Homogeneous* typify behavior of autonomous and stationary systems, which consist of a large number of recurrence points that are homogeneously distributed (relaxation times are short).
- *Periodic* represents long, uninterrupted, and diagonally oriented structures that represent which indicate periodic behavior. These lines are usually distributed regularly.
- Paling or darkening from the LOI to the outer corners in RP is characteristic of *drift*.
- *Disrupted* distribution of recurrence points may serve as an indicator of drastic changes as well as extreme events in the system dynamics. In these cases, RP can be used to find and assess extreme and rare events by scoring the frequency of their repeats.

6.1.2. Recurrence quantification analysis

For a qualitative description of the system, the graphic representation of the system suits perfectly. However, the main disadvantage of graphical representation is that it forces users to subjectively intuit and interpret patterns and structures presented within the recurrence plot. Also, with plots' increasing size, they can be hardly depicted on graphical display as a whole. As a result, we need to work with separated parts of the original plot. Analysis in such a way may create new defects that which should distort objectivity of the observed patterns and lead to incorrect interpretations. To overcome such limitation and spread an objective assessment among observers, in the early 1990s by Webber and Zbilut [296, 297] were introduced definitions and procedures to quantify RP's complexity, and later, it has been extended by Marwan et al. [295].

The small-scale clusters can represent a combination of *isolated dots* (chance recurrences). Similar evolution at different periods in time or in reverse temporal order will present *diagonal lines* (deterministic structures) as well as *vertical/horizontal lines* to inscribe laminar states (intermittency) or systems that paused at singularities. For the quantitative description of the system, such small-scale clusters serve the base of the recurrence quantification analysis (RQA).

Usually, first acquaintance with classical RQA starts with recurrence point density, or, as it is known, recurrence rate (RR):

$$RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j}.$$

It enumerates the probability that any state of the system will recur. It is the simplest measure that is computed by taking the number of nearest points forming short, spanning row and columns of the recurrent plot. It summarizes them and divides by the number of possible points in the recurrence matrix of size N^2.

The remaining measure relies on the frequency distribution of line structures in the RP. First, we consider the histogram of the length of the diagonal structures in the RP

$$P(l) = \sum_{i,j=1}^{N} \left\{ (1 - R_{i-1,j-1}) \cdot (1 - R_{i+l,j+l}) \cdot \prod_{k=0}^{l-1} R_{i+k,j+k} \right\}.$$

The fraction recurrence points on the recurrence plots that form line segments of minimal length μ parallel to the matrix diagonal is the measure of determinism (DET):

$$DET^{(\mu)} = \frac{\sum_{l=\mu}^{N} l \cdot P(l)}{\sum_{i,j=1}^{N} R_{i,j}} = \frac{\sum_{l=\mu}^{N} l \cdot P(l)}{\sum_{l=1}^{N} l \cdot P(l)}.$$

Systems that exhibit deterministic dynamics are mainly characterized by diagonal lines. Long diagonal lines indicate periodic signals, but short diagonal lines stand for chaotic behavior. Regarding the quantitative analysis, typically, only the lines with minimal length $\mu = 2$ are considered. If $\mu = 1$, then DET and RR are identical. For some systems, DET becomes more reliable if $\mu > 2$. Here, μ serves as a filter, excluding the shorter lines. However, it should be noted that too large μ may spoil the histogram $P(l)$ and thus the reliability of DET.

The results of calculations of window dynamics of the considered recurrence measures are presented in Figure 20. RR and DET are calculated for local time series of 50 days and a step of 1 day. In this case, the beginning of a crash or critical event is at point 100.

It is evident that the two recurrent measures during abnormal periods decrease long before the actual anomaly. The complex system becomes less recurrent and less deterministic which is

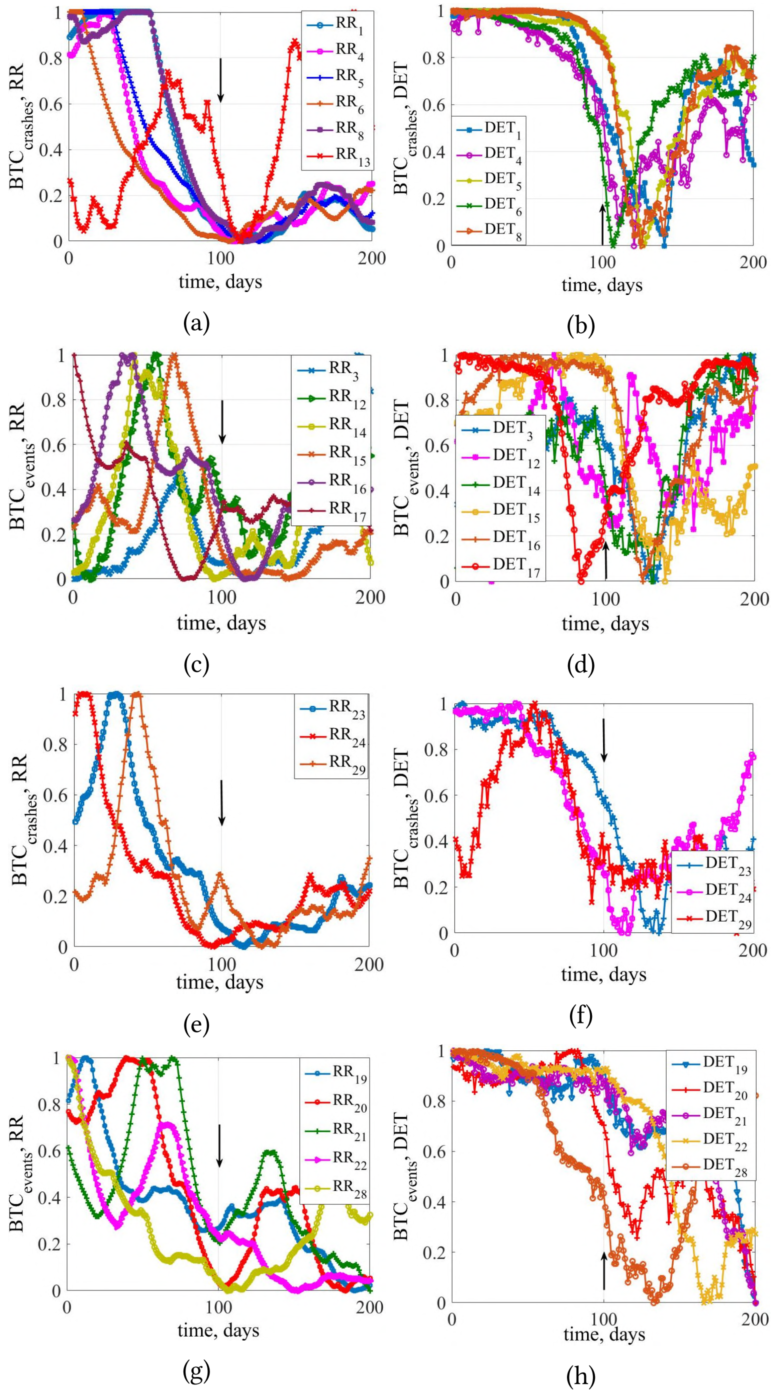


Figure 20: Dynamics of RR and DET for crashes (a, b, e, f) and critical events (c, d, g, h).

logical in the periods approaching critical phenomena. And, consequently, RR and DET can be used as precursors of critical and crash phenomena.

6.1.3. Chaos recurrent measures

The corresponding measure of entropy is related to the recurrence properties that may be peculiar for the nonlinear complex system and important class of recurrence quantifiers are those that try to capture the level of complexity of a signal [298, 79, 85]. In accordance with this study, the entropy diagonal line histogram (DLEn) is of the greatest interest which uses the Shannon entropy of the distribution of diagonal lines $P(l)$ to determine the complexity of the diagonal structures within the recurrence plot. One of the most know quantitative indicators of the recurrence analysis can be defined as:

$$DLEn = -\sum_{l=l_{min}}^{l=l_{max}} p(l) \ln p(l)$$

and

$$p(l) = \frac{P(l)}{\sum_{l=l_{min}}^{N} P(l)},$$

where $p(l)$ captures the probability that a diagonal line has exactly length l, and DLEn reflects the complexity of deterministic structure in the system. Further calculations were provided and presented in figure 21 for both Bitcoin time series.

However, as follows from the analysis of the entropy indicators, the results may differ for different data preparation. Further, we take into account two types of Shannon entropy-based approaches: recurrence period density entropy (RPDEn) and recurrence entropy (RecEn).

The RPDEn is the quantitative measure of the recurrence analysis that is useful for characterizing the periodicity or absolutely random processes in the time series. It is useful for quantifying the degree of repetitiveness [299, 300]. Considering embedded data point $\vec{X}(i)$ from the phase space and suitable threshold ϵ in d_E-dimensional space. Then the trajectory is followed forward in time until it has left the corresponding threshold ϵ. Subsequently, the time j at which the trajectory first returns to this ball and the period T of previous and current states is recorded. The procedure is repeated for all states of the RPs, forming a histogram of recurrence times $R(T)$. The histogram is then normalized to give the recurrence time probability density:

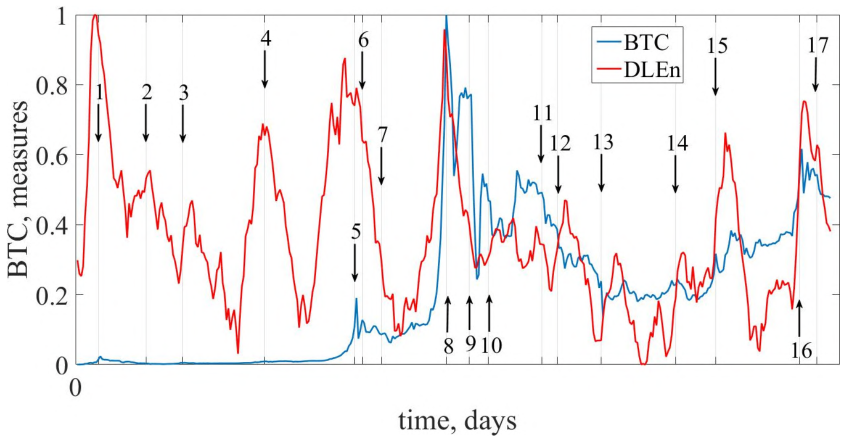
$$P(T_i) = \frac{R(T_i)}{\sum_{i=1}^{T_{max}} R(T_i)},$$

where $T_{max} = \max\{T_i\}$. The normalized entropy of the obtained density can be defined as:

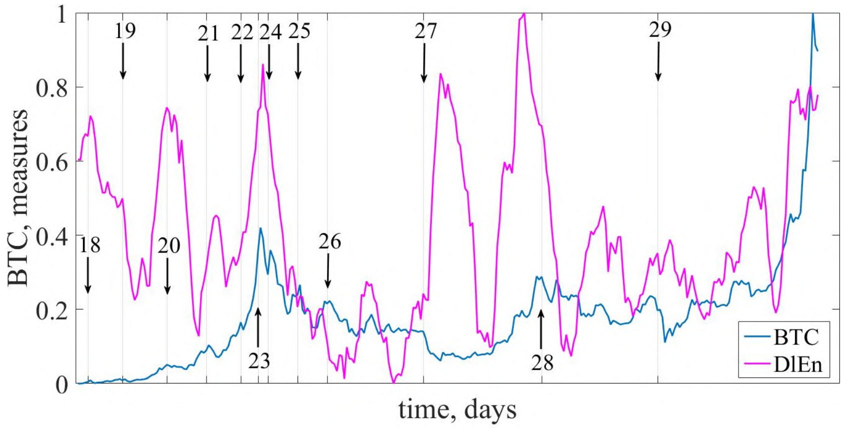
$$RPDEn = \frac{-\sum_{i=1}^{T_{max}} P(T_i) \ln P(T_i)}{\ln T_{max}}. \tag{37}$$

In fact, based on the length of the sequences of neighboring points in the phase space: the more points are neighborhoods, the lower the value of the entropy according to equation (37). The comparing of RPDEn and Bitcoin's critical states can be seen in figure 22.

However, recent articles [298, 301] present a slightly different technique for calculating recurrent entropy using a novel way to extract information from the recurrence matrix. To properly define it, we need to define the microstates $F(\epsilon)$ for the RP that are associated with



(a)



(b)

Figure 21: *DLEn* dynamics along with first (a) and second (b) periods of the entire time series of Bitcoin.

features of the dynamics of the time series. Selecting the appropriate metric and using the Heaviside function, we evaluate the matrices of dimensions $N \times N$ that are sampled from the RP. The total number of microstates for a given N is $N_{ms} = 2^{N^2}$. The microstates are populated by $\overline{N}$ random samples obtained from the recurrence matrix such that:

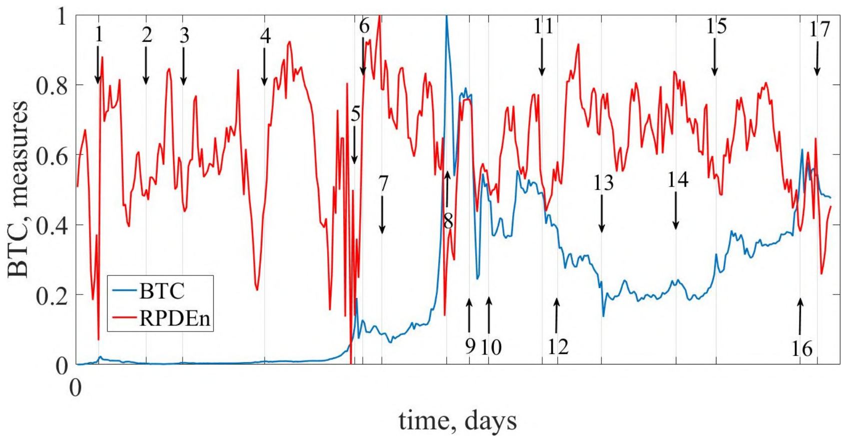
$$\overline{N} = \sum\nolimits_{i=1}^{N_{ms}} n_i$$

with n_i representing the number of times that a microstate i is observed. The probability of occurrence of the related microstate i can be obtained as:

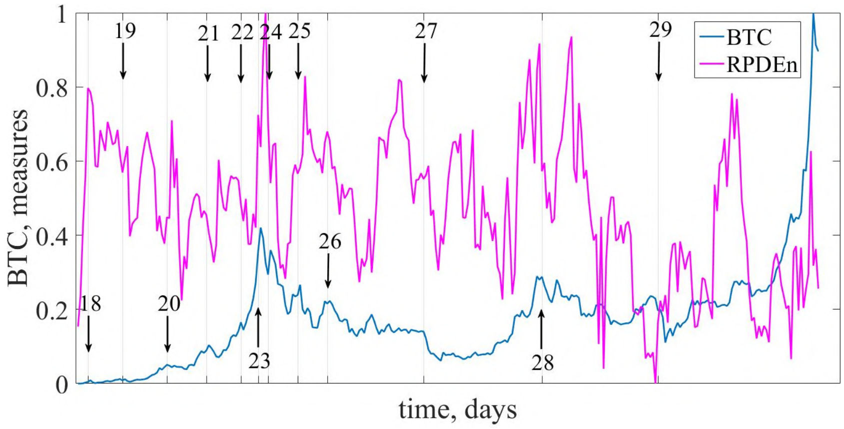
$$p_i = n_i \cdot \left(\overline{N}\right)^{-1}.$$

The RecEn of the RP associated with the probability distribution of the corresponding microstates is given by the following equation:

$$RecEn = \sum_{i=1}^{N_{ms}} p_i \ln p_i. \tag{38}$$



(a)



(b)

Figure 22: *RPDEn* dynamics along with first (a) and second (b) periods of the entire time series of Bitcoin.

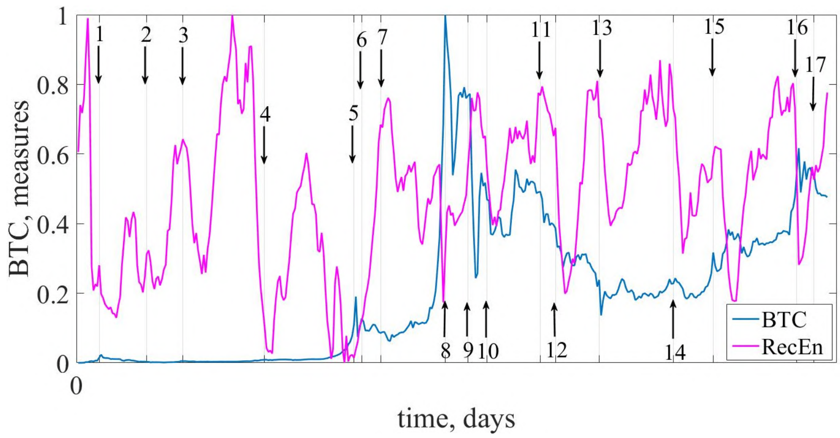
In figure 23 we can see the performance of *RecEn* accordingly to the described above method.

A vertical line of length l starting from a dot (i, j) means that the trajectory starting from $\vec{\mathrm{X}}(j)$ remains close to $\vec{\mathrm{X}}(i)$ during $l-1$ time steps. A diagonal black line of length l starting from a dot i, j means that trajectories starting from $\vec{\mathrm{X}}(i)$ and $\vec{\mathrm{X}}(j)$ remain close during $l-1$ time steps, thus these lines are related to the divergence of the trajectory segments. The average diagonal line length

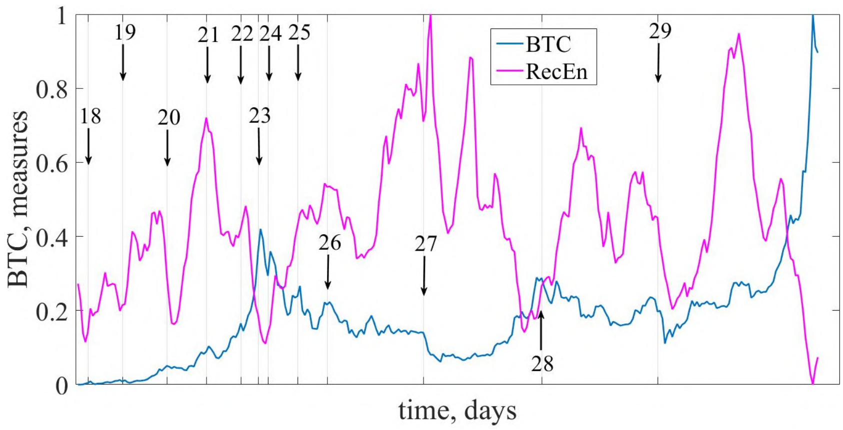
$$L_{mean} = \frac{\sum_{l=l_{min}}^{N} l \cdot P(l)}{\sum_{l=l_{min}}^{N} P(l)}$$

is the average time that two segments of the trajectory are close to each other and can be interpreted as the mean prediction time. Here, $P(l)$ is a histogram of diagonal lines of length l.

Another measure (L_{max}) considers the length of the longest diagonal line found in the RP. In other words, it means the maximum time that two segments of the trajectory are close to each



(a)



(b)

Figure 23: *RecEn* dynamics along with first (a) and second (b) periods of the entire time series of Bitcoin.

other, and the following equation can be defined as

$$L_{max} = \max\left(\{l_i \,|\, i = 1, \dots, N_l\}\right),$$

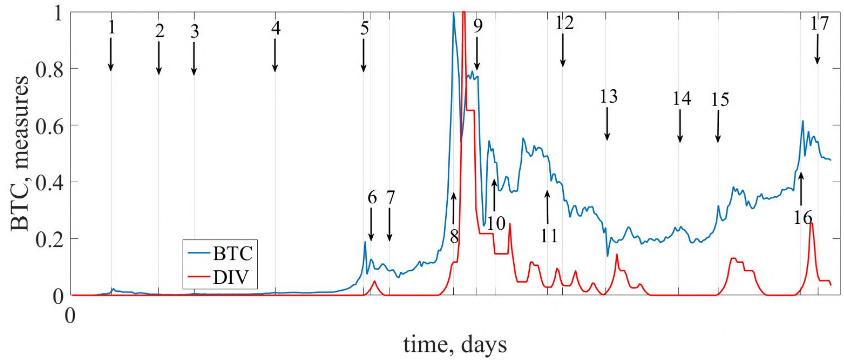
where $N_l = \sum_{l \geqslant l_{min}} P(l)$ is the total number of diagonal lines.

Respectively, the inverse of L_{max} characterizes the exponential divergence of the phase space trajectory [302, 303]. Faster the trajectory segments diverge, shorter are the diagonal lines and higher is the measure of *divergence* (DIV). It is given by the following equation:

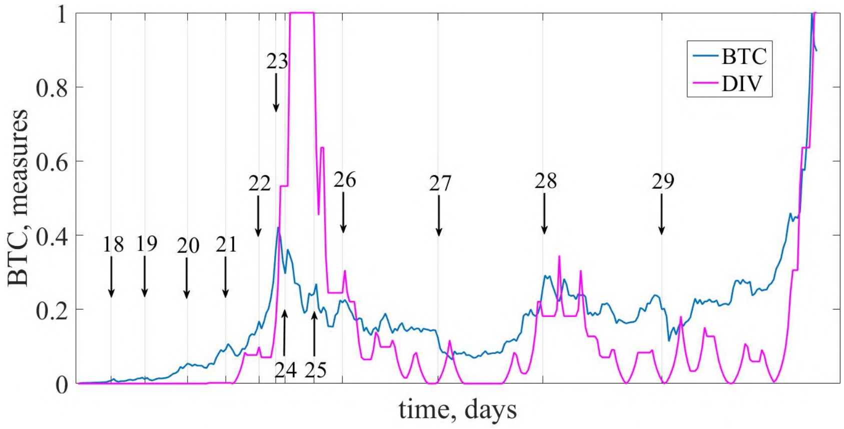
$$DIV = 1/L_{max}.$$

Therefore, the measure of *DIV*, according to Eckmann et al. [232], can be used to estimate the largest positive Lyapunov exponent. The comparative dynamics of the measure of divergence and BTC time series are presented in figure 24.

A comparative analysis of the measures under consideration revealed an obvious advantage of the recursive measure. In addition to the smoothness of the measure itself, it can be calculated for windows of small sizes, which leads to inaccurate or incorrect results for other methods.



(a)



(b)

Figure 24: The measure of divergence along with first (a) and second (b) periods for BTC time series.

7. Irreversibility

Complex systems are open systems that exchange energy, matter, and information with the environment. Investigating complex systems in the natural sciences, Prigogine made a fundamental generalization, indicating the need for consideration of the phenomena of irreversibility and non-equilibrium as principles of selection of space-time structures that are implemented in practice [304]. Later it became clear that this generalization extends to complex systems of another nature: social, economic, biomedical, etc. [305]. Prigogine believed that the most important changes in the modern scientific revolution are related to the removal of previous restrictions in the scientific understanding of time. The nonlinear world is characterized by features of temporality, i.e., irreversibility and transience of processes and phenomena. Self-organization is considered as a spontaneous process of formation of integrating complex systems. It is due to the ambiguity of choice at bifurcation points that time in theories of self-organization becomes truly irreversible. In contrast to linear dynamic theories – classical, relativistic, quantum (where time is reversed), in the thermodynamics of dissipative structures created by Prigogine, time ceases to be a simple parameter and becomes a concept that expresses the pace and direction of events.

Thus, the irreversibility of time is a fundamental property of non-equilibrium dissipative systems, and its loss may indicate the development of destructive processes [108, 305].

A stationary process X is called statistically inverse in time, if for any N, the series $\{x(i) \mid i = 1, \dots, N\}$ and $\{x(i) \mid i = N, \dots, 1\}$ will have the same compatible probability distributions [306]. The irreversibility of time series indicates the presence of nonlinearities in the dynamics of a system far from equilibrium, including non-Gaussian random processes and dissipative chaos. Since the definition of the irreversibility of the time series is formal, there is no a priori optimal algorithm for its quantification. Several methods for measuring the irreversibility of time have been proposed [108, 305, 307, 308, 309, 310, 311, 312, 313].

In the first group of methods, the symbolization of time series is performed, and then the analysis is performed by statistical comparison of the appearance of a string of symbols in the forward and reverse directions [308].

Sometimes additional compression algorithms are used [307]. An important step for this group is the symbolization – the conversion of the time series into a character series requires additional special information (e.g., division of the range or size of the alphabet) and, therefore, contains the problem of the algorithm's dependence on these additional parameters. The second problem arises when considering the large-scale invariance of complex signals. Since the procedures of typical symbolizations are local, taking into account different scales can cause some difficulties [108].

Another group of methods in formalizing the index of irreversibility does not use the symbolization procedure but is based on the use of real values of the time series or returns.

One of such approaches is based on the asymmetry of the distribution of points in the Poincare map, built on the basis of the values of the analyzed time series [310, 313].

Recently, a fundamentally new approach to measuring the irreversibility of time series has been proposed, which uses the methods of complex network theory [309, 312] and which combines two tools: the algorithm for visibility of time series recovery into a complex network and the Kullbak-Leibler divergence algorithm [312]. The first forms a directional network according to the geometric criterion. The degree of irreversibility of the series is then estimated by the Kullbak-Leibler divergence (i.e., the resolution) between the distribution of the input and output stages of the associated count. This method is computationally efficient, does not require any special symbolization of the process, and, according to the authors, naturally takes into account multiscale.

In this study, we consider the irreversibility of time as a measure of the complexity of the system.

Let us consider non-reversible measures of complexity based on the construction and analysis of ordinal permutation patterns.

7.1. Time series irreversibility measure based on permutation patterns

The concept of permutation patterns (PP) was introduced by Bandt and Pompe [128]. PP is based on the idea of finding the order patterns that result in sorted (ascending) sub-sequences, and of then studying the probability distribution of these patterns. Zanin et al. [314] introduce a new method, based on permutation entropy [128, 315, 316], to evaluate irreversibility of time series at various temporal scales. The proposed one presents various advantages: (1) it has no free parameters other than the embedding dimension of the permutation entropy; (2) similar to visibility graph methods [312, 317], it is temporally local, and therefore allows assessing fluctuations; (3) assessing significance is straightforward, and does not rely on scaling arguments as in visibility graph methods; and (4) it has a convergence speed advantage over visibility graph methods graph methods. They also demonstrated how the proposed approach can help elucidate the complex irreversibility dynamics of financial time series, representing 30 major European stocks and 12 world indices [314] and gait analysis on late stages of neurodegenerative dementias [318]. This technique was also used to analyze the irreversibility of time series in ecology (the time series of lynx abundance), epidemiology (dengue prevalence), economy (the S&P price-index series), neuroscience (electroencephalographic data from an epileptic patient) [319, 320], and heart rate [321].

The idea of irreversibility analysis implies that regarding each permutation pattern that can be obtained following procedure from section 3.2.4, there will be reversed one under the operation of time reversal. Example, if for an embedded vector $\vec{\mathrm{X}}$, a pattern π_i is found, reversing time series will necessary imply π_i^r. As an example, let us consider a fragment of BTC time series for the period 21.01.2021-31.01.2021:

$$\mathrm{X}^d = [30825.70, 33005.76, 32067.64, 32289.38, 32366.39]$$

and

$$\mathrm{X}^r = [32366.39, 32289.38, 32067.64, 33005.76, 30825.70] \, .$$

According to mentioned steps, we will construct embedded matrix of overlapping column vectors with $d_E = 3$ and $\tau = 1$. Our sampled data is partitioned as follows:

$$\mathrm{X}_t^d(d_E, \tau) = \begin{bmatrix} 30825.70 & 33005.76 & 32067.64 \\ 33005.76 & 32067.64 & 32289.38 \\ 32067.64 & 32289.38 & 32366.39 \end{bmatrix} \tag{39}$$

and for $\mathrm{X}_t^r(d_E, \tau)$:

$$\begin{bmatrix} 32366.39 & 32289.38 & 32067.64 \\ 32289.38 & 32067.64 & 33005.76 \\ 32067.64 & 33005.76 & 30825.70 \end{bmatrix}. \tag{40}$$

After it, our time-delayed vectors are mapped to *permutations* or *ordinal patterns* of the same size. Our example consists $3! = 6$ different ordinal patterns in total. They can be paired together, such that each pattern composing a pair is the time reversal of the other. For instance:

$$\{0, 1, 2\} \leftrightarrow \{2, 1, 0\}$$
$$\{1, 0, 2\} \leftrightarrow \{2, 0, 1\}$$
$$\{1, 2, 0\} \leftrightarrow \{0, 2, 1\}$$

with $\leftrightarrow$ representing a time reversal transformation.

As an example, the corresponding permutation of the first column from (39) would be $\phi(30825.70, 33005.76, 32067.64]) = 021$ since we arrange values in ascending order and replace them by their ordinal ranking from original placement. Therefore, after mapping from the time-series data into a series of permutations ($\phi : \mathbb{R}^{d_E} \to S_{d_E}$), we obtain the ordinal matrices first of all for initial time series:

$$\begin{bmatrix} 0 & 1 & 0 \\ 2 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}. \tag{41}$$

and its reversed version:

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}. \tag{42}$$

Finally, the probability of each pattern in initial and reversed time series is calculated as

$$p(\pi) = \frac{\#\{t \leq N - (d_E - 1)\tau, \phi(X_t^{d_E,\tau}) = \pi\}}{N - (d_E - 1)\tau}, \tag{43}$$

forming the probability distributions $P^d = \left[p_{0,1,2}, p_{2,1,0}, p_{1,0,2}, p_{2,0,1}, p_{1,2,0}, p_{0,2,1}\right]$ and $P^r = \left[p_{2,1,0}, p_{0,1,2}, p_{2,0,1}, p_{1,0,2}, p_{0,2,1}, p_{1,2,0}\right]$. Using the Kullback-Leibler divergence, we can define the degree of irreversibility in a time series:

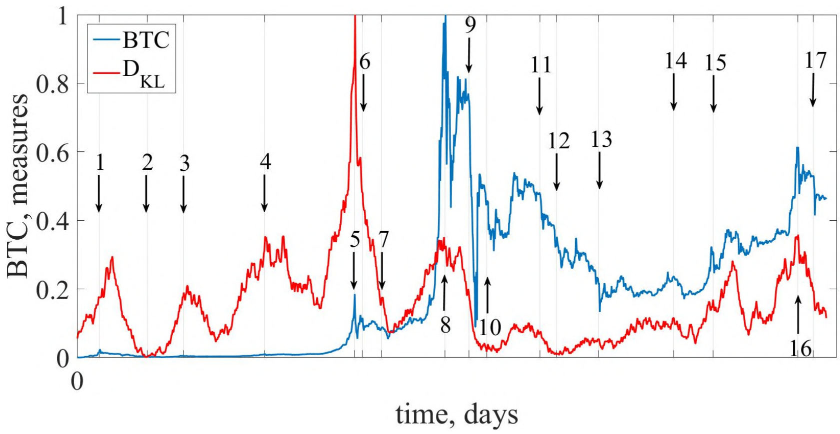
$$D_{KL} = \sum_{i=1}^{d_E!} P^d(i) \log \frac{P^d(i)}{P^r(i)}. \tag{44}$$

If $P^d \approxeq P^r$, the time series is presented to be reversible, thus yielding a $D_{KL} \approxeq 0$ and vise versa. Estimating varying bitcoin's irreversibility according to (44) with time window of 100 days and time step of 1 day, we obtain following results.

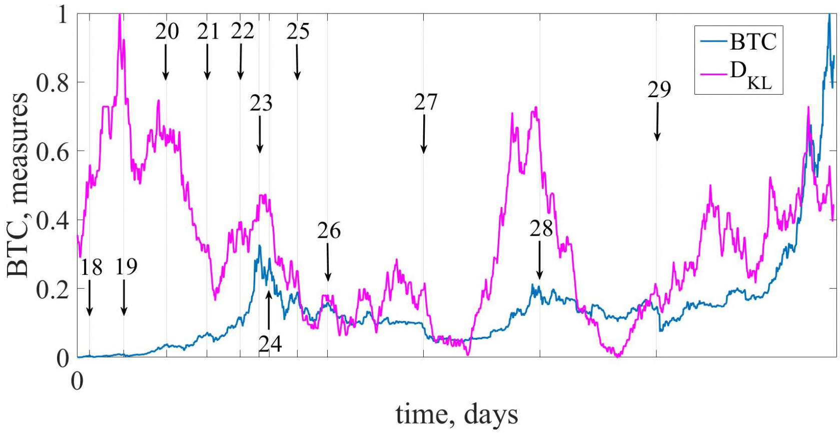
The figures show that time series are significantly irreversible. When moving the original rows of their irreversible disappears. Draws attention and noticeable unevenness introduced measures, which correlate with the fluctuations of the input time series. Identifying significant changes in the time series and comparing them with the corresponding changes of non-reversible measures of complexity, it is possible to construct the corresponding indicators.

8. Single and multiplex networks

The new interdisciplinary study of complex systems, known as the complex networks theory, laid the foundation for a new network paradigm of synergetics [322]. In the framework of the complexity paradigm, it became apparent that we should move from well-studied systems and processes, taking into account the minimal number of new entities that are characteristic of the social sciences or the humanities. Apparently, one of these entities is the bonds, that is, what characterizes the interaction of the elements that are part of the system, that makes parts of the whole. The set of these links is called the network. Investigating networks, we take into account their topology, statistical properties, the distribution of weights of individual nodes and edges, the effects of information dissemination, robustness, etc. [1, 323, 324, 4]. Complex networks include electrical, transport, information, social, economic, biological, neural, and



(a)



(b)

Figure 25: Dynamics of permutation-based time irreversibility measure for the first (a) and second (b) periods.

other networks [325, 326, 3]. The network paradigm has become dominant in the study of complex systems since it allows you to enter new quantitative measures of complexity not existing for the time series [327]. Moreover, the network paradigm provides adequate support for the core concepts of Industry 4.0 [322].

Previously, we introduced various quantitative measures of complexity for individual time series [58, 328, 329, 330]. However, except a graph for an individual time series, it is necessary to take into account the interconnection interaction, which can be realized within the framework of different models [331]. We will consider it by simulating so-called multiplex networks, the features of which are reduced to a fixed number of nodes in each layer, but they are linked by different bonds [331].

Recently, the first papers using the spectral and topological characteristics of dynamic systems presented as networks have appeared. Thus, in [332], it has been investigated universal and non-universal allometric scaling behaviors in the visibility graphs of 30 world stock market indices. It has been established that the nature of such behavior is due to the returns distribution that is characterized by fat-tails, the nonlinear long-term correlation, and a coupling effect between the set of influential factors.

Birch [333] compared the mean degree value and clustering coefficient for a group of companies included in the DAX 30 index basket. He observed the companies from the DAX 30 index for two time periods: the first from the beginning of 2008 through the end of 2009 and the second from the beginning of 2010 up to the end of 2011 as these include the dates – a period of crisis (7th October 2008 – 31st December 2008) and a period of recovery (7th May 2010 – 3rd August 2010). Contrary to expectations, the results differed little from the relatively low accuracy of the horizontal visibility graph procedure compared to visibility graph.

Liu et al. [334] collected from the Chinese stock market the data of 2571 stock companies in 2012 and the data of 2578 stock companies in 2013. Every year, data of these stock companies are randomly arranged. These data are then converted into some complex networks based on the visibility graph method. For these complex networks, degree distribution and clustering coefficient are considered. These results show that complex networks have power-law distribution and small-world properties.

Yan and van Tuyll van Serooskerken [335] construct an indicator to measure the magnitude of the super-exponential growth of stock prices, by measuring the degree of the price network, generated from the price time series. Twelve major international stock indices have been investigated. The work results show that this new indicator has strong predictive power for financial extremes, both peaks, and troughs. By varying the model parameters, the authors show the predictive power is very robust. The new indicator has a better performance than the indicator based on a well-known model of log-periodic oscillations of Sornette [336].

Vamvakaris et al. [337] analyzed high-frequency data from the S&P 500 via the horizontal visibility graph method and find that all major crises that have taken place worldwide for the last twenty years, affected significantly the behavior of the price-index. Nevertheless, they observe that each of those crises impacted the index in a different way and magnitude. These results suggest that the predictability of the price-index series increases during periods of crisis.

Serafino et al. [338] studied the visibility graphs built from the time series of several stock market indices. They proposed a validation procedure for each link of these graphs against a null hypothesis derived from ARCH-type modeling of such series. Building on this framework made it possible to devise a market indicator that turned out to be highly correlated and even predictive of financial instability periods.

Grau et al. [25] examined the characteristics of the daily price series of 16 different cryptocurrencies between July 2017 and February 2018. Using Minimum Spanning Tree (MST) and hierarchical analysis by dendrograms that were obtained from Pearson correlation between daily returns, they visualized and identified a high correlation between price movements of all the currencies. From the obtained results, it is seen that the most interconnected with all cryptocurrencies is Ethereum, while Bitcoin standed for one of its branches. They concluded that Ethereum might be a benchmark currency in the cryptocurrency market, rather than Bitcoin.

Coquidé et al. [339] constructed the Google matrices of bitcoin transaction for all year quarters from the very start of it till April 10, 2013. From PageRank and CheiRank probabilities that serve as analogues to trade import and export, they determined the dimensionless trade balance of each user and analyzed the direct and hidden (indirect) links between top PageRank users of BCN using the recently developed reduced Google matrix algorithm. They modeled the contagion propagation of the transactions assuming that a user goes bankrupt if its dimensional balance exceeds a certain bankruptcy threshold k. Their results present that the phase transition neighboring with the critical threshold $k = k_c \approx 0.1$ below which almost all users remain safe. For $k > 0.55$ almost all users remain safe, and for $0.1 < k > 0.55$ more than a half of users go bankrupt. Moreover, their result present that even being not very close to the critical threshold $k_c \approx 0.1$, almost all top PageRank and CheiRank users rapidly become bankrupts that give the evidence about their strong interconnectivity. With the reduced Google matrix algorithm, they presented the most preferable interlinks of the most valuable users.

Multiplex networks are actively used to simulate complex networks of different nature: from financial (stock market [340, 341, 338, 337], banks [342], guarantee market [343]) to social [344]. Particular attention should be paid to the work [340], in which the above multiplex measures are analyzed for the subject of correlations with known stock market crises.

8.1. Methods of converting time series into graphs

In recent years, interesting algorithms for the transformation of time series into a network have been developed, which allows extending the range of known characteristics of time series even to network ones. Recently, several approaches have been proposed to transform time sequences into complex network-like mappings. These methods can be conventionally divided into three classes [345].

The first is based on the study of the convexity of successive values of the time series and is called visibility graph (VG) [345, 346]. The second analyzes the mutual approximation of different segments of the time sequence and uses the technique of recurrent analysis [345]. The recurrent diagram reflects the existing repetition of phase trajectories in the form of a binary matrix whose elements are units or zeros, depending on whether they are close (recurrent) with given accuracy or not, the selected points of the phase space of the dynamic system. The recurrence diagram is easily transformed into an adjacency matrix, on which the spectral and topological characteristics of the graph are calculated. Finally, if the basis of forming the links of the elements of the graph is to put correlation relations between them, we obtain a correlation graph [345, 328]. To construct and analyze the properties of a correlation graph, we must form an adjacency matrix from the correlation matrix. To do this, you need to enter a value that for the correlation field will serve as the distance between the correlated agents. Such a distance may be dependent on the ratio of the correlation C_{ij} value $d_{ij} = \sqrt{2(1 - C_{ij})}$. So, if the correlation coefficient between the two assets is significant, the distance between them is small, and, starting from a certain critical value x_{crit}, assets can be considered bound on the graph. For an adjacency matrix, this means that they are adjacent to the graph. Otherwise, the assets are not contiguous. In this case, the binding condition of the graph is a prerequisite.

The main purpose of such methods is to accurately reproduce the information stored in the time series in an alternative mathematical structure, so that powerful graph theory tools could eventually be used to characterize the time series from a different point of view in order to overcome the gap between nonlinear analysis of time series, dynamic systems and the graphs theory.

The usage of the complexity of recurrent networks to prevent critical and crisis phenomena in stock markets has been considered by us in a recent paper [347]. Therefore, we will focus on algorithms of the VG and multiplex VG (MVG).

The recurrence diagrams for the visualization of phase space recurrences are based on Henri Poincare's idea of the phase space recurrence of dynamical systems. According to Takens' theorem [137], an equivalent phase trajectory that preserves the structure of the original phase trajectory can be recovered from a single observation or time series by the time delay method: the recurrence diagram is easily transformed into an adjacency matrix, by which the spectral and topological characteristics of the graph are calculated [328].

The algorithm of the VG is realized as follows [346].

Take a time series $\{x(t_i) \,|\, i = 1, \ldots, N\}$ of length N. Each point in the time series data can be considered as a vertex in an associative network, and the edge connects two vertices if two corresponding data points can "see" each other from the corresponding point of the time series (see Figure 26). Formally, two values $x(t_a)$ (at a point in time t_a) and $x(t_b)$ (at a point in time t_b) are connected, if, for any other value $(x(t_c), t_c)$, which is placed between them (i.e., $t_a < t_c < t_b$), the condition is satisfied:

$$x(t_c) < x(t_a) + (x(t_b) - x(t_a))\frac{t_c - t_a}{t_b - t_a}.$$

Note that the visibility graph is always connected by definition and also is invariant under affine transformations, due to the mapping method.

An alternative (and much simpler) algorithm is the horizontal visibility graph (HVG) [345], in which a connection can be established between two data points a and b if one can draw a horizontal line in the time series joining them that does not intersect any intermediate data by the following geometrical criterion: $x(t_a), x(t_b) > x(t_c)$ for all c such that $t_a < t_c < t_b$ (see figure 26).

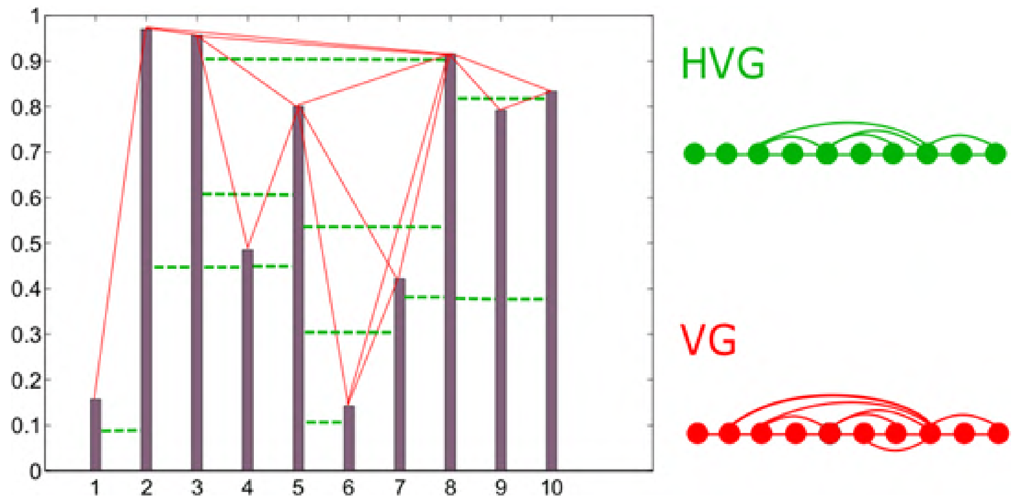


Figure 26: Illustration of constructing the visibility graph (red lines) and the horizontal visibility graph (green lines) [348].

In multiplex networks, there are two tasks [347]: (1) turn separate time series into the network for each layer; (2) connect the intra-loop networks to each other. The first problem is solved within the framework of the standard algorithms described above. For multiplex networks, the algorithm of the MVG for the three layers is presented in figure 27.

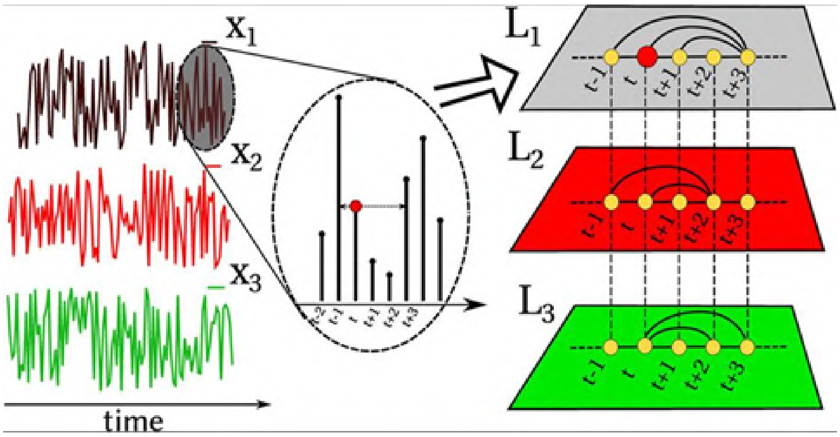


Figure 27: Scheme for forming bonds between three layers of the multiplex network [347].

The cross-recurrent multiplex network (MCRP) is formed from recursive diagrams of individual layers.

8.2. Spectral and topological graph properties

Spectral theory of graphs is based on algebraic invariants of a graph – its spectra [27]. The spectrum of graph G is the set of eigenvalues $S_p(G)$ of a matrix corresponding to a given graph. For adjacency matrix A of a graph, there exists a characteristic polynomial $|\lambda I - A|$, which is called the characteristic polynomial of a graph $P_G(\lambda)$. The eigenvalues of the matrix A (the zeros of the polynomial $|\lambda I - A|$) and the spectrum of the matrix A (the set of eigenvalues) are called respectively their eigenvalues.

Another common type of graph spectrum is the spectrum of the Laplace matrix L. The Laplace matrix is used to calculate the tree graphs, as well as to obtain some important spectral characteristics of the graph. It can be defined as $L = D - A$ where D – diagonal matrix of order n:

$$d_{ij} = \begin{cases} d_i, & i = j, \\ 0, & i \neq j, \end{cases} \tag{45}$$

with d_i – the degree of corresponding vertex in the graph.

The spectrum $S_{p_L}(G)$ of the matrix L is the root of the characteristic equation:

$$|\lambda I - L| = |\lambda I - D + A| = 0. \tag{46}$$

Comparing the spectra of S_p with S_{p_L}, it is easy to establish that

$$S_p(G) = [\lambda_1, \lambda_2, \dots, \lambda_n],$$
$$S_{p_L}(G) = [r - \lambda_n, \lambda_{n-1}, \dots, r - \lambda_1],$$

where $\lambda_1 = r$.

The number zero is the eigenvalue of the matrix L, which corresponds to an eigenvector whose coordinates are equal to unity. The multiplicity of the null eigenvalue is equal to the number of connected components of the graph. The rest of eigenvalues L are positive. The least of the positive eigenvalues λ_2 is called the index of *algebraic connectivity* of the graph. This value represents the "force" of the connectivity of the graph component and is used in the analysis of reliability and synchronization of the graph.

Important derivative characteristics are spectral gap, graph energy, spectral moments, and spectral radius. The spectral gap is the difference between the largest and the next eigenvalues of the adjacency matrix and characterizes the rate of return of the system to the equilibrium state. The *graph energy* is the sum of the modules of the eigenvalues of the graph adjacency matrix:

$$E(G) = \sum_{i=1}^{n} |\lambda_i| .$$

The spectral radius is the largest modulus of the eigenvalue of the adjacency matrix. Denote by N_c the value, which corresponds to an *average eigenvalue* of the graph adjacency matrix:

$$N_c = \ln\left(\frac{1}{n}\sum_{i=1}^{n} \exp(\lambda_i)\right)$$

and is called *natural connectivity*.

The k^{th} spectral moment of the adjacency matrix is determined by the expression

$$m_k(A) = \frac{1}{n}\sum_{i=1}^{n} \lambda_i^k,$$

with λ_i that represents eigenvalues of the adjacency matrix, and n is the vertex of G.

Among the topological measures, one of the most important is the *node degree* k – the number of links attached to this node. For non-directed networks, the node's degree k_i is determined by the sum $k_i = \sum_j a_{ij}$, where the elements a_{ij} of the adjacency matrix.

For characterizing the *linear size* of the network, there are useful concepts of average $\langle l \rangle$ and maximum l_{max} shortest paths. For a connected network of n nodes, the *average path length* (ApLen) is equal to

$$\langle l \rangle = \frac{2}{n(n-1)} \sum_{i>j} l_{ij}, \tag{47}$$

where l_{ij} – the length of the shortest path between the nodes. The *diameter* of the connected graph is the maximum possible distance between its two vertices, while the minimum possible is the radius of the graph.

If the average length of the shortest path gives an idea of the whole network and is a global characteristic, the next parameter – the clustering coefficient – is a local value and characterizes a separate node. For a given node i, the *clustering coefficient* C_i is defined as the ratio of the existing number of links between its closest neighbors to the maximum possible number of such relationships:

$$C_i = \frac{2E_i}{k_i(k_i - 1)}. \tag{48}$$

In equation (48), $k_i(k_i - 1)/2$ is the maximum number of links between the closest neighbors. The clustering coefficient of the entire network is defined as the average value of C_i for all its nodes. The clustering coefficient shows how many of the nearest neighbors of the given node are also the closest neighbors to each other. It characterizes the tendency to form groups of interconnected nodes – clusters. For the real-life networks, the high values of the clustering coefficient are high.

Another feature of the node is *betweenness*. It reflects the role of the node in establishing network connections and shows how many shortest paths pass through this node. Node betweenness $c(v)$) is defined as

$$c(v) = \sum_{i \neq j} \frac{\sigma(i,j|v)}{\sigma(i,j)}, \tag{49}$$

where $\sigma(i,j)$ – the total number of shortest paths between nodes i and j; $\sigma(i,j|v)$ – the number of those shortest paths between i,j passing through v. The value of (49) is also called the *load* or *betweenneess centrality*.

One of the main characteristics of the network is the degree distribution $P(d)$, which is defined as the probability that the node i has a degree $d_i = d$. Most natural and actual artificial networks follow a power law distribution

$$P(d) \sim 1/d^{\gamma},\ d \neq 0,\ \gamma > 0. \tag{50}$$

Also, important topological characteristic is the *vertex eccentricity* – the largest distance between i and any other vertex, that is, how far the vertex is far from the other vertices of the graph. The centrality of the vertex measures its relative importance in the graph. At the same time, the *farness* of a node is defined as the sum of its distances to all other nodes, and its *closeness* is defined as the backward distance. Thus, the centrality of the node is lower than its total distance to all other nodes.

Another important measure is the *link density* in the graph, which is defined as the number of links n_e, divided by the expression $n_n(n_n - 1)/2$, where n_n is the number of nodes of the graph.

From spectral measures, we consider the maximal node degree (d_{max} – figure 28a and figure 28b). From the topological measures, the average path length (APLen – figure 28c and 28d) is found, which is in accordance with equation (47).

Figure 28 demonstrates the asymmetric response of the spectral and topological measures of network complexity. For the complete series, the calculation parameters are as follows: window length of 100 days, step is of 1 day.

Figures above show that all of the above spectral measures have maximum values in pre-crisis periods. The complex system has the greatest complexity. With the approach of the crisis, the complexity of the system decreases, recovering after the crisis. Some of the topological, in particular, APLen, shows an opposite relationship. Indeed, in more complex systems you can always find shorter paths that connect any nodes. During the crisis (reducing complexity, increasing the chaotic component), the length of the corresponding path increases.

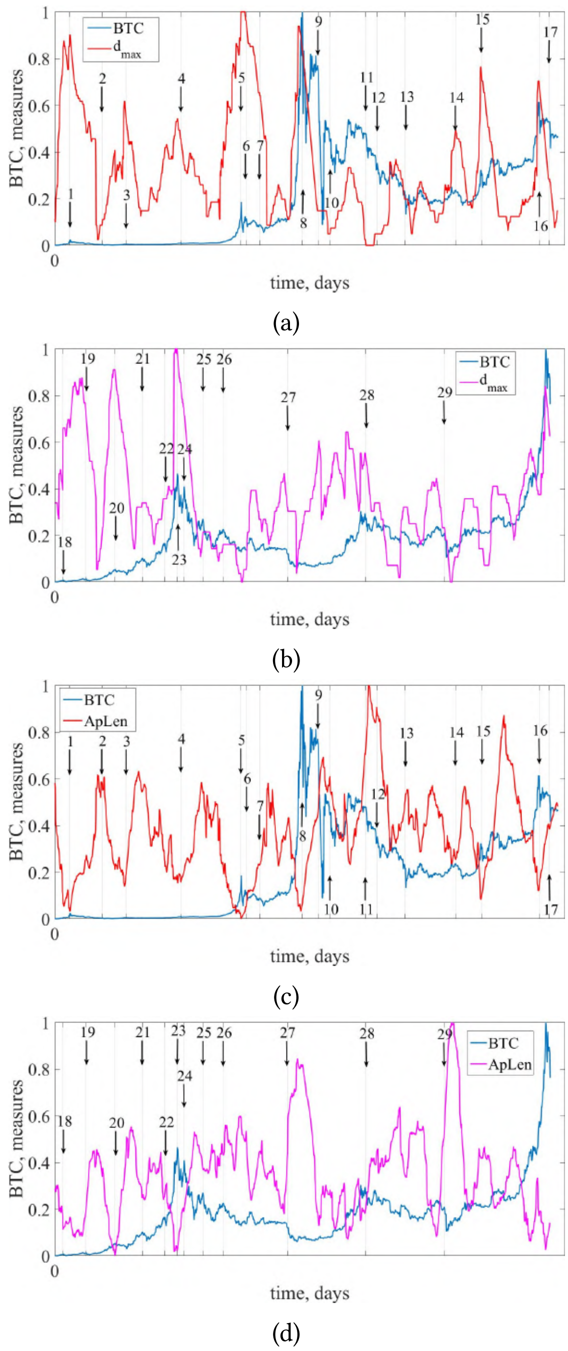


Figure 28: Maximal node degree d_{max} (a, b) and average path length $\langle l \rangle$ (c, d) for two periods of Bitcoin time series.

8.3. Multiplex complexity measures

A multilayer/multiplex network [349, 84] is a pair M=(G, C) where $\{G_\alpha \,|\, \alpha \in 1, \ldots, M\}$ is a family of graphs (whether directed or not, weighed or not) $G_\alpha = (X_\alpha, E_\alpha)$ that called layers and

$$C = \left\{E_{\alpha\beta} \subseteq X_\alpha \times X_\beta \,|\, \alpha, \beta \in 1, \ldots, M, \alpha \neq \beta\right\}$$

is a set of links between nodes of layers G_α and G_β where $\alpha \neq \beta$. The elements of each E_α are intralayer edges in M in contrast to the elements of each $E_{\alpha\beta}$ that called interlayer edges.

A set of nodes in a layer G_α is denoted as X_α=$\left\{x_1^\alpha, \ldots, x_{N_\alpha}^\alpha\right\}$, and an intralayer adjacency matrix as $A^{[\alpha]} = (a_{ij}^\alpha) \in \mathrm{Re}^{N_\alpha \times N_\alpha}$, where

$$\alpha_{ij}^\alpha = \begin{cases} 1, & (x_i^\alpha, x_j^\alpha) \in E_\alpha, \\ 0, & \text{otherwise.} \end{cases} \tag{51}$$

for $1 \leq i \leq N_\alpha$, $1 \leq j \leq N_\beta$ and $1 \leq \alpha \leq M$. For an interlayer adjacency matrix, we have $A^{[\alpha,\beta]}(a_{ij}^{\alpha\beta}) \in \mathrm{Re}^{N_\alpha \times N_\beta}$, where

$$\alpha_{ij}^{\alpha\beta} = \begin{cases} 1, & (x_i^\alpha, x_j^\beta) \in E_{\alpha\beta}, \\ 0, & \text{otherwise.} \end{cases} \tag{52}$$

A multiplex network is a partial case of interlayer networks, and it contains a fixed number of nodes connected by different types of links. Multiplex networks are characterized by correlations of different nature [331], which enable the introduction of additional multiplexes.

Let's evaluate the quantitative overlap between the various layers. The average edge overlap is defined as [341]

$$\omega = \frac{\sum_i \sum_{j>i} \sum_\alpha a_{ij}^{[\alpha]}}{M \sum_i \sum_j 1 - \delta_{0, \sum_\alpha a_{ij}^{[\alpha]}}} \tag{53}$$

and determines the number of layers in which this edge is presented. Its value lies on the interval $[1/M, 1]$ and equals $1/M$ if the connection (i, j) exists only in one layer. In other words, if there is a layer α such that $a_{ij}^{[\beta]} = 0 \,\forall\, \beta \neq \alpha$. If all layers are identical, then $\omega = 1$. Consequently, this measure can serve as a measure of the coherence of the output time series: high values of ω indicate a noticeable correlation in the structure of the time series.

The *total overlap* $O^{\alpha\beta}$ between the two layers α and β is defined as the total number of edges that are shared between the layers α and β:

$$O^{\alpha\beta} = \sum a_{ij}^\alpha a_{ij}^\beta, \tag{54}$$

where $\alpha \neq \beta$.

For a multiplex network, the node degree k is already a vector

$$k_i = (k_i^{[1]}, \ldots, k_i^{[M]}), \tag{55}$$

with the degree $k_i^{[\alpha]}$ of the node i in the layer α, namely

$$k_i^{[\alpha]} = \sum\nolimits_j a_{ij}^{[\alpha]},$$

while $a_{ij}^{[\alpha]}$ is the element of the adjacency matrix of the layer α. Specificity of the node degree in vector form allows to describe additional quantities. One of them is the *ovelapping degree* of node i

$$o_i = \sum_{\alpha=1}^{M} k_i^{[\alpha]}. \tag{56}$$

The next measure quantitatively describes the interlayer degree correlations of the selected node in two different layers. If a chosen pair (α, β) from M layers is characterized by the distributions $P(k^{[\alpha]}), P(k^{[\beta]})$, the so-called *interlayer mutual information* is determined as:

$$I_{\alpha,\beta} = \sum \sum P(k^{[\alpha]}, k^{[\beta]}) \log \frac{P(k^{[\alpha]}, k^{[\beta]})}{P(k^{[\alpha]}), P(k^{[\beta]})}, \tag{57}$$

where $P(k^{[\alpha]}, k^{[\beta]})$ is the probability of finding a node degree $k^{[\alpha]}$ in a layer α and a degree $k^{[\beta]}$ in a layer β. The higher the value of $I_{\alpha,\beta}$, the more correlated (or anti-correlated) is the degree distribution of the two layers and, consequently, the structure of a time series associated with them. We also find the mean value of $I_{\alpha,\beta}$ for all possible pairs of layers – the scalar $\langle I_{\alpha,\beta} \rangle$ that quantifies the information flow in the system.

The quantity that quantitatively describes the distribution of a node degree i between different layers is the *entropy of a multiplexed degree*:

$$S_i = -\sum_{\alpha=1}^{M} \frac{k_i^{[\alpha]}}{o_i} \ln \frac{k_i^{[\alpha]}}{o_i}. \tag{58}$$

Entropy is zero if all connected to i edges are in the same layer, and has the maximum value when they are evenly distributed between different layers. So, the higher value of S_i, the more evenly distributed edges connected to i between different layers.

A similar indicator is the *multiplex participation coefficient*:

$$P_i = \frac{M}{M-1} \left[1 - \sum_{\alpha=1}^{M} \left(\frac{k_i^{\alpha}}{o_i} \right)^2 \right]. \tag{59}$$

The P_i takes values on the interval $[0, 1]$ and determines how homogeneously are distributed the links of a node i among M layers. If a node is only active on one layer, $P_i = 0$; $P_i = 1$ if a node has an precisely defined number of incident links that are equally distributed across M layers.

Obviously, measures S_i and P_i are very similar.

We will show that some of these spectral and topological indicators serve as the measures of system complexity, and the dynamic of their changes allow us to build precursors of crashes and critical events in the cryptocurrency market.

As far as multiplex measures are concerned, they are very similar in their dynamic to the spectral and topological representations above (see figure 28). In the case of a shorter sample of a base of three layers – Bitcoin, Ethereum, Litecoin (see figure 29-31), we have the asymmetric behavior of the multiplex measures I, O, o (Equations (57), (54), (56) and S, P (Equations (58) and (59)) for different methods of building multiplex networks.

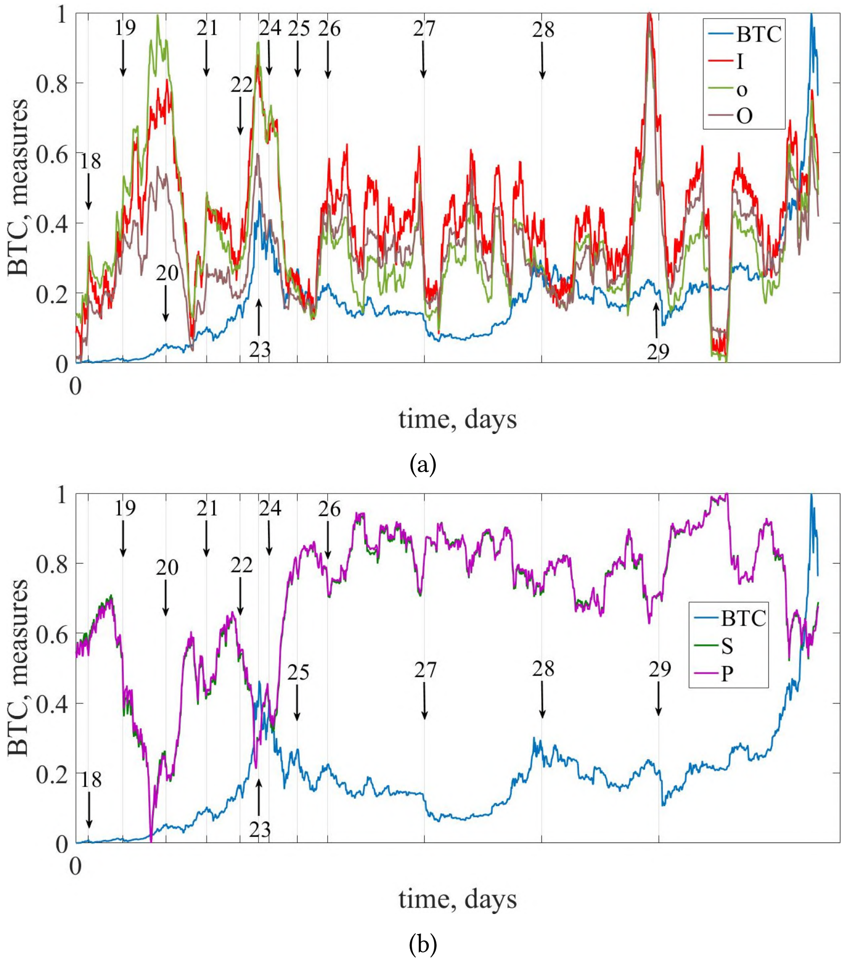


Figure 29: Dynamics of the second BTC period and multiplex measures for a base of three layers. The graph was built using the multiplex visibility graph.

Figures 29-31 show that the mentioned multiplex measures are excellent indicators that warn about the approaching crisis phenomenon, that is, are indicator-precursors.

9. Quantum precursors

Quantum econophysics, a direction distinguished by the use of mathematical apparatus of quantum mechanics as well as its fundamental conceptual ideas and relativistic aspects, developed within its boundaries just a couple of years later, in the first decade of the 21^{st} century

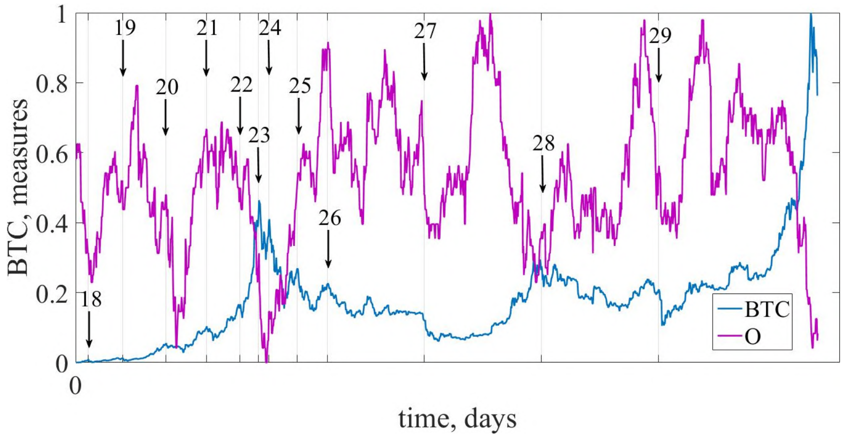


Figure 30: Dynamics of BTC daily prices and multiplex measures O. The graph was built using the MHVG.

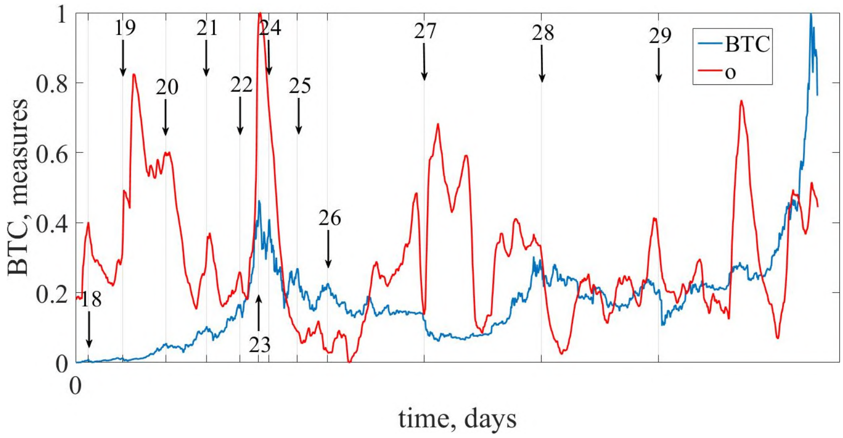


Figure 31: Dynamics of BTC daily prices and multiplex measures o. The graph was built using the MCRP.

[350, 351, 352, 42].

According to classical physics, immediate values of physical quantities, which describe the system status, not only exist but can also be exactly measured. Although non-relativistic quantum mechanics does not reject the existence of immediate values of classic physical quantities, it postulates that not all of them can be measured simultaneously (Heisenberg uncertainty ratio). Relativistic quantum mechanics denies the existence of immediate values for all kinds of physical quantities, and, therefore, the notion of system status seizes to be algoristic.

In this section, we will demonstrate the possibilities of quantum econophysics on the example of the application of the Heisenberg uncertainty principle and the Random Matrices Theory to the actual and debatable now market of cryptocurrencies [81, 353].

9.1. Heisenberg uncertainty principle and economic analogues of basic physical quantities

In our paper [43], we have suggested a new paradigm of complex systems modeling based on the ideas of quantum as well as relativistic mechanics. It has been revealed that the use of quantum-mechanical analogies (such as the uncertainty principle, the notion of the operator, and quantum measurement interpretation) can be applied to describing socio-economic processes. Methodological and philosophical analysis of fundamental physical notions and constants, such as time, space, and spatial coordinates, mass, Planck's constant, light velocity from modern theoretical physics provides an opportunity to search for adequate and useful analogs in socio-economic phenomena and processes.

The Heisenberg uncertainty principle is one of the cornerstones of quantum mechanics. The modern version of the uncertainty principle, deals not with the precision of a measurement and the disturbance it introduces, but with the intrinsic uncertainty any quantum state must possess, regardless of what measurement is performed [354, 355]. Recently, the study of uncertainty relations, in general, has been a topic of growing interest, specifically in the setting of quantum information and quantum cryptography, where it is fundamental to the security of certain protocols [356, 357].

To demonstrate it, let us use the known Heisenberg's uncertainty ratio which is the fundamental consequence of non-relativistic quantum mechanics axioms and appears to be (e.g., [358]):

$$\Delta x \cdot \Delta v \geqslant \frac{\hbar}{2m_0}, \tag{60}$$

where Δx and Δv are mean square deviations of x coordinate and velocity v corresponding to the particle with (rest) mass m_0, $\hbar$ – Planck's constant. Considering values Δx and Δv to be measurable when their product reaches their minimum, according to equation (60) we derive:

$$m_0 = \frac{\hbar}{2 \cdot \Delta x \cdot \Delta v}, \tag{61}$$

i.e., the mass of the particle is conveyed via uncertainties of its coordinate and velocity – time derivative of the same coordinate.

Economic measurements are fundamentally relative, local in time, space and other socio-economic coordinates, and can be carried out via consequent and/or parallel comparisons "here and now", "here and there", "yesterday and today", "a year ago and now", etc.

Due to these reasons constant monitoring, analysis, and time series prediction (time series imply data derived from the dynamics of stock indices, exchange rates, cryptocurrency prices, spot prices, and other socio-economic indicators) become relevant for the evaluation of the state, tendencies, and perspectives of global, regional, and national economies.

Suppose there is a set of K time series, each of N samples, that correspond to the single distance T, with an equally minimal time step Δt_{min}:

$$\mathrm{X}_i(t_n), \quad \text{for } n = 0, 1, \ldots, N-1; \text{ for } i = 1, 2, \ldots, K. \tag{62}$$

To bring all series to the unified and non-dimensional representation, accurate to the additive constant, we normalize them, have taken a natural logarithm of each term of the series. Then, consider that every new series $X_i(t_n)$ is a one-dimensional trajectory of a certain fictitious or abstract particle numbered i, while its coordinate is registered after every time span Δt_{min}, and evaluate mean square deviations of its coordinate and speed in some time window $\Delta T = \Delta N \cdot \Delta t_{min} = \Delta N$, $1 \ll \Delta N \ll N$. The "immediate" speed of i^{th} particle at the moment t_n is defined by the ratio:

$$v_i(t_n) = \frac{x_i(t_{n+1}) - x_i(t_n)}{\Delta t_{min}} = \frac{1}{\Delta t_{min}} \ln \frac{X_i(t_{n+1})}{X_i(t_n)} \tag{63}$$

with variance D_{v_i} and standard deviation Δv_i.

Keeping an analogy with equation (1), after some transformations, we can write an uncertainty ratio for this trajectory [43]:

$$\frac{1}{\Delta t_{min}} \left(\langle \ln^2 \frac{X_i(t_{n+1})}{X_i(t_n)} \rangle_{n,\Delta N} - \left(\langle \ln^2 \frac{X_i(t_{n+1})}{X_i(t_n)} \rangle_{n,\Delta N} \right)^2 \right) \sim \frac{h}{m_i}, \tag{64}$$

where m_i – economic "mass" of a X_i series, h – value which comes as an economic Planck's constant.

Since the analogy with physical particle trajectory is merely formal, h value, unlike the physical Planck's constant $\hbar$, can, generally speaking, depend on the historical period, for which the series are taken, and the length of the averaging interval (e.g., economical processes are different in the time of crisis and recession), on the series number i etc. Whether this analogy is correct or not depends on the particular series' properties.

In recent research [359, 30], we tested the economic mass as an indicator of crisis phenomena on stock index data. Here, we test the model for the cryptocurrency market on the example of Bitcoin [80, 81, 353, 360].

Obviously, there is a dynamic characteristic value m, depending on the internal dynamics of the market. In times of crashes and critical events marked by arrows, mass m is significantly reduced in the pre-crash and pre-critical periods.

Obviously, m remains a good indicator-precursor even in this case. Value m is considerably reduced before a special market condition. The market becomes more volatile and prone to changes.

Next method of quantum econophysics is borrowed from nuclear physicists and is called Random Matrix Theory.

9.2. Random matrix theory and quantum indicators-precursors

Random Matrix Theory (RMT) developed in this context the energy levels of complex nuclei, which the existing models failed to explain (Prevedel et al. [357], Benítez Rodríguez and Aguilar [354], Rozema et al. [355]). Deviations from the universal predictions of RMT identify specific, nonrandom properties of the system under consideration, providing clues about the underlying interactions.

Unlike most physical systems, where one relates correlations between subunits to basic interactions, the underlying "interactions" for the stock market problem are not known. Here, we analyze cross-correlations between stocks by applying concepts and methods of random matrix theory, developed in the context of complex quantum systems where the precise nature of the interactions between subunits are not known.

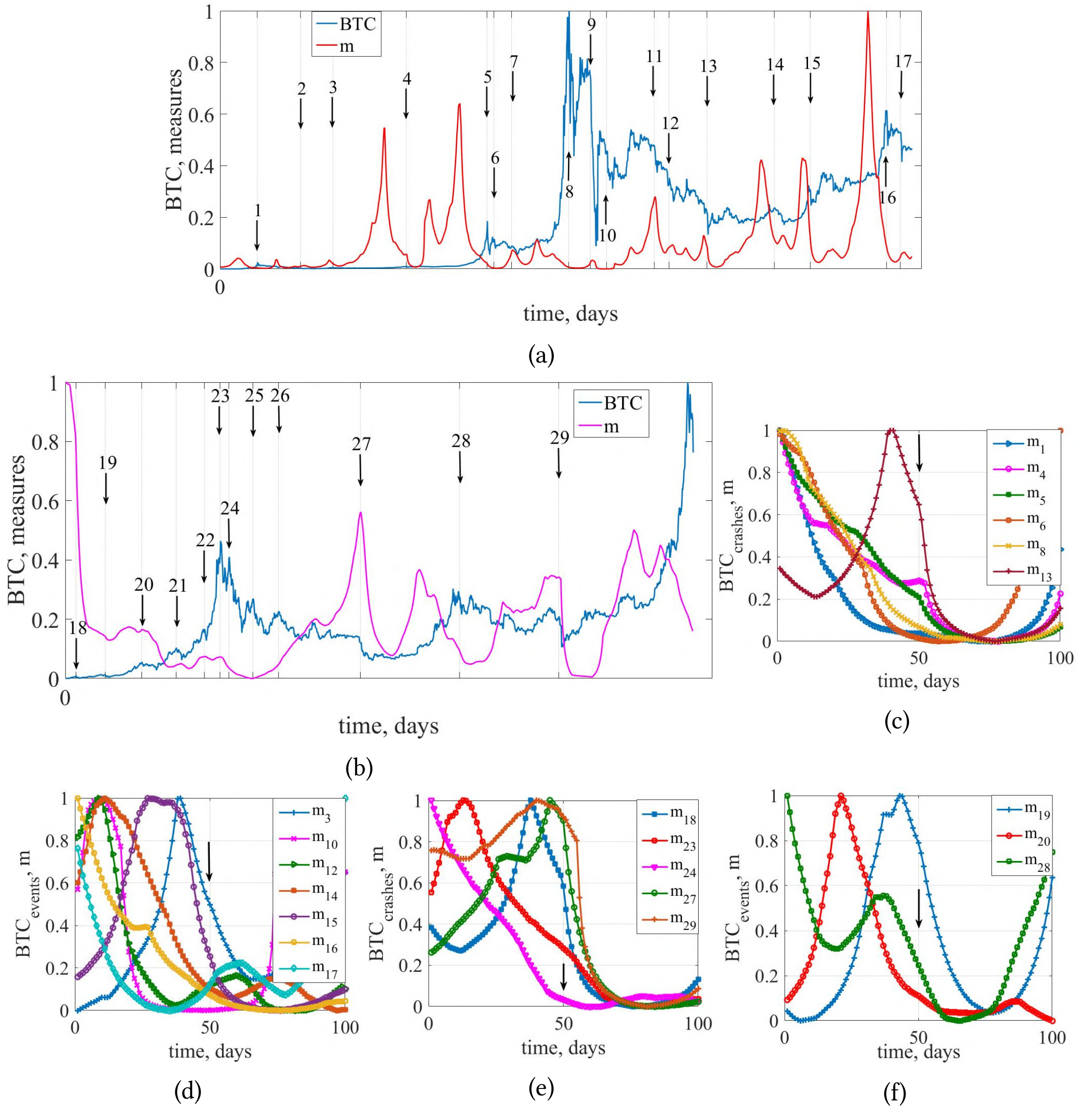
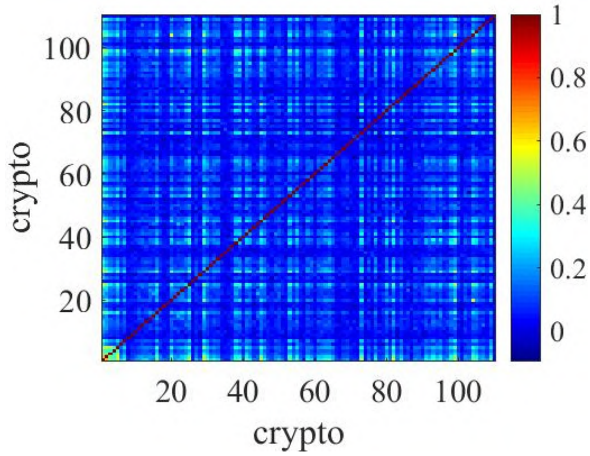


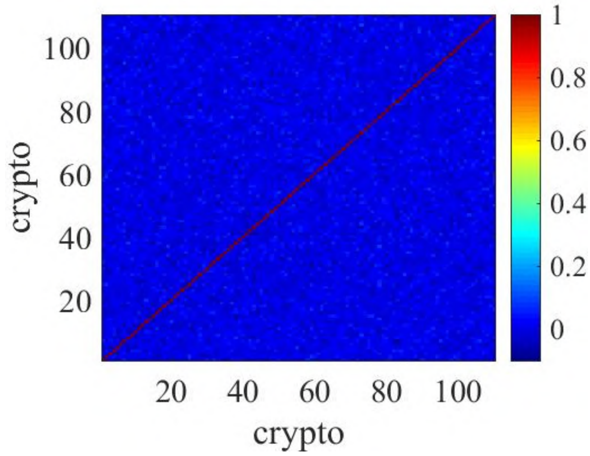
Figure 32: Dynamics of measure m for two entire periods, and its dynamics for their local crashes (c, e) and critical events (d, f) with the window size of 50 days and step of 1 day.

RMT has been applied extensively in studying multiple financial time series [361, 356, 362, 358, 43, 353, 359, 360, 363].

Special databases have been prepared, consisting of cryptocurrency time series for a certain time. The largest number of cryptocurrencies 689 contained a base of 456 days from 31.12.2017 to 31.05.2020, and the smallest (22 cryptocurrencies) contained a base, respectively, from 11.09.2013 to 31.05.2020 (https://coinmarketcap.com/all/views/all/). In order to quantify correlations, we first calculate the logarithmic returns of the cryptocurrencies price series over a time scale $\Delta t = 1$ day. We calculate the pairwise cross-correlation coefficients between any two cryptocurrency time series returns. for the largest database, a graphical representation of the pair correlation field is shown in figure 33a. For comparison, a map of correlations of randomly mixed time series of the same length shown in figure 33 b.



(a)



(b)

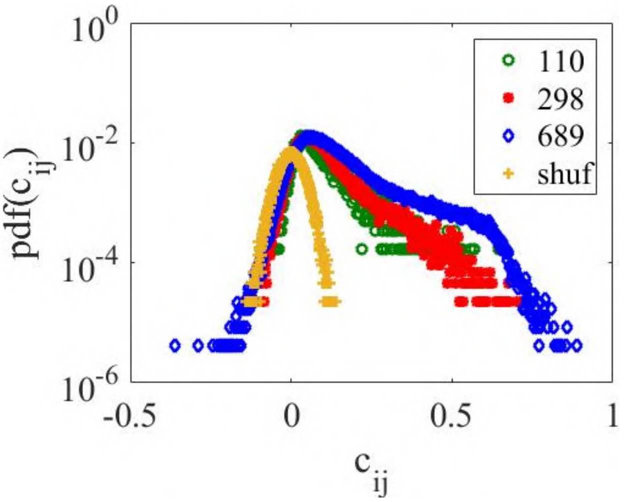
Figure 33: Visualization of the field of correlations for the initial (a, c) and mixed (b, d) matrix cryptocurrency.

For the correlation matrix C we can calculate its eigenvalues, $C = U\Lambda U^T$, where U denotes the eigenvectors, Λ is the eigenvalues of the correlation matrix, whose density $f_c(\lambda)$ is defined as $f_c(\lambda) = (1/N)dn(\lambda)/d\lambda$, where $n(\lambda)$ is the number of eigenvalues of C that are less than λ. In the limit $N \to \infty, T \to \infty$ and $Q = T/N \geqslant 1$ fixed, the probability density function $f_c(\lambda)$ of eigenvalues λ of the random correlation matrix M has a closed form:

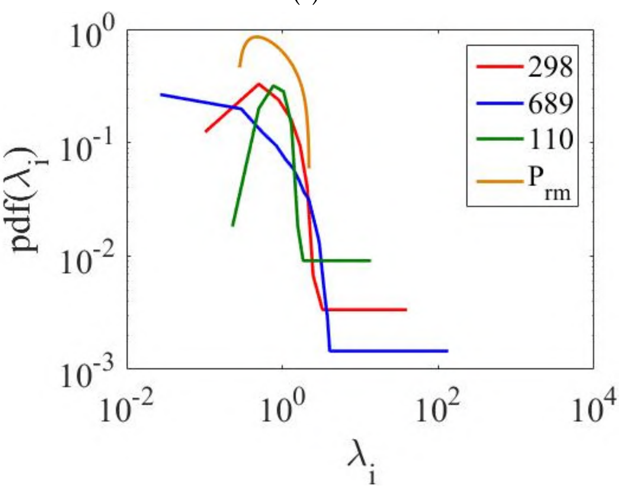
$$f_c(\lambda) = \frac{Q}{2\pi\sigma^2}\frac{\sqrt{(\lambda_{max} - \lambda)(\lambda - \lambda_{min})}}{\lambda}, \tag{65}$$

with $\lambda \in [\lambda_{min}, \lambda_{max}]$, where λ_{min}^{max} is given by λ_{min}^{max}=$\sigma^2(1 + 1/Q \pm 2\sqrt{1/Q})$ and σ^2 is equal to the variance of the elements of matrix M.

We compute the eigenvalues of the correlation matrix C, $\lambda_{max} = \lambda_1 > \lambda_2 > \cdots > \lambda_{15} = \lambda_{min}$. The probability density functions of paired correlation coefficients c_{ij} and eigenvalues λ_i for matrices of 110, 298, and 689 cryptocurrencies are presented in figure 34.



(a)



(b)

Figure 34: Comparison of distributions of the pair correlation coefficients (a) and eigenvalues of the correlation matrix (b) obtained for initial cryptocurrencies and their shuffled version.

From figure 34 it can be seen that the distribution functions for the paired correlation coefficients of the selected matrices differ significantly from the distribution function described by the RMT. It can be seen that the crypto market has a significantly correlated, self-organized system, and the difference from the RMT of the case, the correlation coefficients exceed the value of 0.6-0.8 on "thick tails". The distribution of eigenvalues of the correlation matrix also differs markedly from the case of RMT. In our case, only one-third of its values refer to the RMT region.

Eigenvectors correspond to the participation ratio PR and its inverse participation ratio IPR

$$I^k = \sum_{l=1}^{N} \left[u_l^l\right]^4, \tag{66}$$

where $u_k^l, l = 1, \dots, N$, are the components of the eigenvector u^k. So, PR indicates the number of eigenvector components that contribute significantly to that eigenvector. More specifically, a low IPR indicates that cryptocurrencies contribute more equally. In contrast, a large IPR would imply that the factor is driven by the dynamics of a small number of cryptocurrencies.

The irregularity of the influence of eigenvalues of the correlation matrix is determined by the absorption ratio (AR)

$$AR_n = \frac{\sum_{k=1}^{n} \lambda_l}{\sum_{k=1}^{N} \lambda_k}, \tag{67}$$

which is a cumulative risk measure and indicates which part of the overall variation is described from the total number of N eigenvalues.

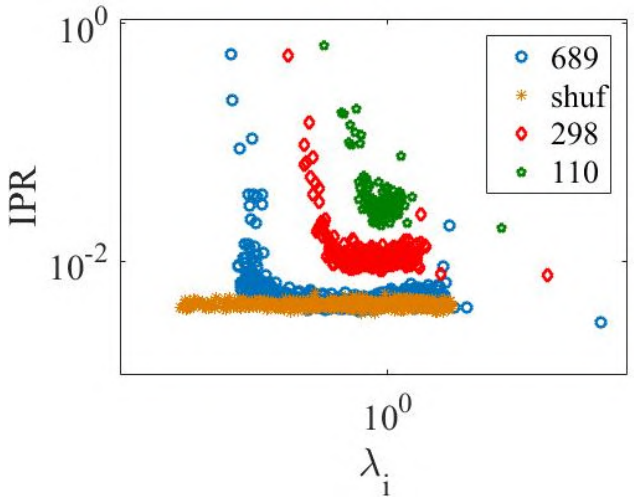
The difference in dynamics is due to the peculiarities of non-random correlations between the time series of individual assets. Under the framework of RMT, if the eigenvalues of the real-time series differ from the prediction of RMT, there must exist hidden economic information in those deviating eigenvalues. For cryptocurrencies markets, there are several deviating eigenvalues in which the largest eigenvalue λ_{max} reflects a collective effect of the whole market. As for PR, the differences from RMT appear at large and small λ values and are similar to the Anderson quantum effect of localization [364]. Under crash and critical event conditions, the states at the edges of the distributions of eigenvalues are delocalized, thus identifying the beginning of one of these events. This is evidenced by the results presented in figure 35b.

We find that both λ_{max} and $PR\lambda_{max}$ have large values for periods containing the crypto market crashes and critical events. At the same time, their growth begins in the pre-crashes periods. As well as the economic mass, they are quantum precursors of crashes and critical events phenomena.

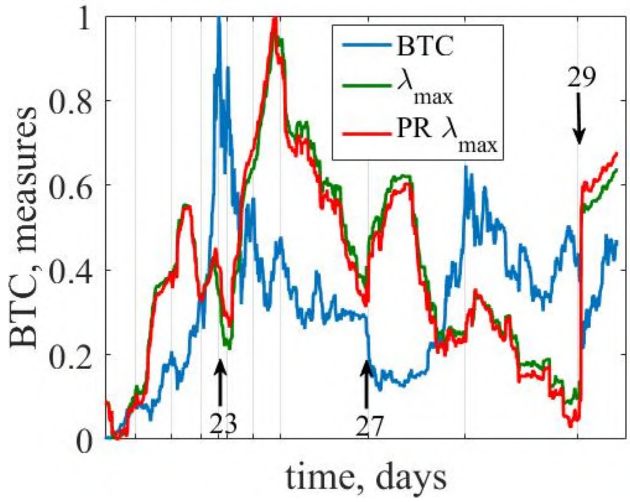
10. Conclusions

Definitely, the factors from within and outside of the cryptocurrencies universe are going to evolve all of them. The great influence will go from incumbents and policymakers, as well as challengers and users. Current mistrust on the part of the government may lead to the introduction of specific licensing requirements that may make these digital currencies less attractive. Similarly, the adaptation to them and acceptance of cryptocurrencies may lead to increasing demand for them. The current situation with coronavirus is of paramount importance and is of significant danger.

From the literature overview, we have understood that crashes and critical events do not disappear without a trace, but will also affect the fate of individuals. On the other hand, in the future, the influence of these events may attract users to alternative forms of currency such as cryptocurrencies. Increased trading activity on cryptocurrency exchanges could positively affect the popularity of stablecoins. Lastly, the overall monetary system may be fundamentally



(a)



(b)

Figure 35: Inverse participation ratio (a) and (b) quantum measures of complexity λ_{max} and its participation ratio.

changed through the introduction of a central bank digital currency, potentially upstaging stablecoins [365].

In order to give reliable, powerful, and simple indicators-precursors that are able to minimize further losses as a result of changes, we addressed the reach arsenal of the theory of complexity and the methods of nonlinear dynamics that can identify special trajectories in the complex dynamics and classify them. Following our research, we obtain informational, (multi-)fractal, chaos-dynamical, recurrent, irreversible, based on complex networks, and quantum measures of complexity.

The obtained quantitative methods were applied to classified crashes of the Bitcoin market, where it was seen that these indicators can be used in order to protect yourself from the upcoming critical change. To draw some conclusions about its evolution and factors that influence it, we pointed out the most influential critical changes in this market. Relying on different research articles and our previous experience, such changes were classified as crashes and critical events. Moreover, we assume crash as strong, time localized drop with high volatility whose percentage decline exceeds 30%. Critical events are those falls with less percentage decline and volatility. The analysis of the crypto market with the sliding (rolling or moving) window approach allowed us to draw some conclusions about its evolution and factors that influence it. Regarding empirical results, we have shown that some of the measures are very sensitive to the length of the sliding window and its time step. For example, if we consider two closest to each other events, a previous event that had much more volatility can have a great influence on the corresponding measure of complexity and spoil the identification of the next less influential, but important event. Thus, time localization is significant while calculating the measure of complexity. The less time localization and time step, the more corresponding changes are taken into account. For a much larger time window and its step, we can have less accurate estimations.

It turned out that most of the chosen measures of complexity respond in advance to the corresponding changes of complexity in the cryptocurrency market and can be used in the diagnostic processes. Such measures can be presented as indicators or even indicators-precursors of the approaching crashes and critical events.

Relying on the information theory and its powerful toolkit, we emphasized four measures of complexity, such as the measure of Lempel-Ziv, classical Shannon entropy, and its two modifications (Approximate entropy and Permutation entropy). We referred to the complexity of the systems, how it was described in different studies, and what methods were applied to quantify its degree. Our results show that in the pre-crash or pre-critical period, the complexity of Bitcoin starts to change that is it starts to decrease, indicating that such events presented to be more predictable and corresponding patterns are more structured. Thus, the degree of predictability increases in times of such events.

Along with information theory, we referred to the multifractal properties of the cryptocurrency market. As it was obtained with multifractal detrended fluctuation analysis, the scaling exponents remain non-linear, and the width of singularity spectrum changes in time that gives evidence that at different times (scales) BTC time series exhibits more or less complex behavior, indicating that cryptocurrency exhibits multifractal properties. Applying the width of multifractality as an indicator of possible critical states we found that before crash or critical event, this measure starts to decrease that tell us that the series is expected to be more predictable and stable, while its dynamics after such events is increasing that present system to be more susceptible to fluctuations.

Chaos-dynamical measures were applied to study bitcoin from the perspective of Chaos theory. Transitions between chaotic and non-chaotic behavior were identified with Lyapunov exponents where there was selected the largest Lyapunov exponent. The beginning of each crash or critical event can be characterized by the convergence of the two considered, initially close trajectories where the Largest Lyapunov exponent reflects the rate of such convergence, while the high-price level regimes are characterized by the divergence of the initially close trajectories. Whereas decreasing largest Lyapunov exponent indicates that patterns in the system tend to be predictable (trajectories converge), increasing indicates that the system goes into an unstable regime. Our empirical results obtained with the sliding window presented the largest Lyapunov exponent to be an indicator of such regimes. Moreover, we considered that such extreme events can be related to the fat-tails and better described with non-Gaussian distributions, particularly, described by Lévy alpha-stable distribution and its four parameters. As it is still debatable whether the stable distribution is completely applicable or not, we addressed to its group of stable parameters, and during tests, we emphasized that the characteristic exponent that serve to describe the thickness of tails is the best for serving as an indicator-precursor of possible crashes and critical events. Thus, is shown that such a complex system as the bitcoin market, with growth and preferential attachments, is characterized by power-laws.

The analysis of the crypto market with the measures from the recurrence quantification analysis revealed that its toolkit is suitable for distinguishing diverse market periods. Such measures as recurrence rate, determinism, the entropy diagonal line histogram, recurrence periodic density entropy, recurrence entropy, and divergence, which methodology is based on clusters of isolated points, vertical/diagonal lines, etc., are presented to be great for detection of the periods of instability or relaxation.

Also, applying the concept of time irreversibility (asymmetry), we found it changes for the period of crashes and critical events, and corresponding measures detect such phenomena. Thus, time irreversibility based on permutation patterns can serve as a good base for further models of financial control. However, much more measures can be added to this list [108, 366, 367, 317, 368].

Moreover, we have demonstrated the possibility of studying complex cryptosystems within the network paradigm. The time series can be presented as an economic network (visibility graph) and multiplex network with a set of both spectral and topological characteristics, which are sensitive to the critical changes in the BTC market.

Addressing to quantum econophysics and its apparatus where appropriate measures of complexity were obtained. Such quantitative methods as Heisenberg uncertainty and the Random matrix approach have confirmed their effectiveness for studying the cryptocurrency market. We found that economic "mass" along with λ_{max} and $PR\ \lambda_{max}$ are presented to be effective due to their robustness, computational efficiency, and simplicity.

Apparently, the impact of different crashes and critical events was reflected in the cryptocurrency market, as well as the coronavirus pandemic and therefore, the dynamics of past events, as well as of the subsequent could be identified in advance using the appropriate indicators of the theory of complexity. In our further studies, we are going to aim our view on exploring and analyzing of other methods from the theory of complexity. Being emerging currency that still needs to become trustworthy among as many people as possible, bitcoin's dynamics is the subject not only to high fluctuations but also to various attacks [369, 370, 371, 372, 373] of blockchain [374] and Proof-of-Work protocol [375]. Thus, particular interest presents research on implementing the theory of complex systems and its enormous toolkit for identifying such abnormal activity when transactions between users happen to make appropriate actions in advance. Equally interesting is the development of the theory of quantum computation and quantum information [376, 377, 378, 379] where it would be interesting to see their influence on blockchain and cryptocurrencies. Moreover, the research in the field of artificial intelligence, machine, and deep learning does not remain without attention [380, 381, 52, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391].

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Irreversibility of financial time series: a case of crisis

Andrii O. Bielinskyi[1], Serhii V. Hushko[2], Andriy V. Matviychuk[3], Oleksandr A. Serdyuk[4], Serhiy O. Semerikov[1,5,6,7] and Vladimir N. Soloviev[1]

[1]*Kryvyi Rih State Pedagogical University, 54 Gagarin Ave., Kryvyi Rih, 50086, Ukraine*
[2]*State University of Economics and Technology, 16 Medychna Str., Kryvyi Rih, 50005, Ukraine*
[3]*Kyiv National Economic University named after Vadym Hetman, 54/1 Peremogy Ave., Kyiv, 03680, Ukraine*
[4]*The Bohdan Khmelnytsky National University of Cherkasy, 81 Shevchenka Blvd., 18031, Cherkasy, Ukraine*
[5]*Kryvyi Rih National University, 11 Vitalii Matusevych Str., Kryvyi Rih, 50027, Ukraine*
[6]*Institute of Information Technologies and Learning Tools of the NAES of Ukraine, 9 M. Berlynskoho Str., Kyiv, 04060, Ukraine*
[7]*University of Educational Management, 52A Sichovykh Striltsiv Str., Kyiv, 04053, Ukraine*

Abstract

The focus of this study to measure the varying irreversibility of stock markets. A fundamental idea of this study is that financial systems are complex and nonlinear systems that are presented to be non-Gaussian fractal and chaotic. Their complexity and different aspects of nonlinear properties, such as time irreversibility, vary over time and for a long-range of scales. Therefore, our work presents approaches to measure the complexity and irreversibility of the time series. To the presented methods we include Guzik's index, Porta's index, Costa's index, based on complex networks measures, Multiscale time irreversibility index and based on permutation patterns measures. Our study presents that the corresponding measures can be used as indicators or indicator-precursors of crisis states in stock markets.

Keywords

irreversibility, stock markets, crisis states

1. Introduction

Complex systems are open systems that exchange energy, matter, and information with the environment. Investigating complex systems in the natural sciences, Prigogine [1] made

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krivogame@gmail.com (A. O. Bielinskyi); dep.director_edu@kneu.edu.ua (S. V. Hushko); editor@nfmte.com (A. V. Matviychuk); serdyuk@ukr.net (O. A. Serdyuk); semerikov@gmail.com (S. O. Semerikov); vnsoloviev2016@gmail.com (V. N. Soloviev)
https://www.researchgate.net/profile/Andrii-Bielinskyi (A. O. Bielinskyi); https://www.duet.edu.ua/ua/persons/12 (S. V. Hushko); https://kneu.edu.ua/en/depts9/k_ekon_matematychn_modeljuvannja/vykladachi_kmm/Matvijchuk.A.V/ (A. V. Matviychuk); http://iot.cdu.edu.ua/pidrozdily/kafedra-pm-ta-informatyky/personal-nij-sklad/serdiuk-oleksandr-anatoliiovych.html (O. A. Serdyuk); https://kdpu.edu.ua/semerikov (S. O. Semerikov); https://kdpu.edu.ua/personal/vmsoloviov.html (V. N. Soloviev)
0000-0002-2821-2895 (A. O. Bielinskyi); 0000-0002-4833-3694 (S. V. Hushko); 0000-0002-8911-5677 (A. V. Matviychuk); 0000-0002-1230-0305 (O. A. Serdyuk); 0000-0003-0789-0272 (S. O. Semerikov); 0000-0002-4945-202X (V. N. Soloviev)

a fundamental generalization, indicating the need for consideration of the phenomena of irreversibility and non-equilibrium as principles of selection of space-time structures that are implemented in practice. Later it became clear that this generalization extends to complex systems of another nature: social, economic, biomedical, etc. [2]. Prigogine believed that the most important changes in the modern scientific revolution are related to the removal of previous restrictions in the scientific understanding of time. The nonlinear world is characterized by features of temporality, i.e., irreversibility and transience of processes and phenomena. Self-organization is considered as a spontaneous process of formation of integrating complex systems. It is due to the ambiguity of choice at bifurcation points that time in theories of self-organization becomes truly irreversible. In contrast to linear dynamic theories – classical, relativistic, quantum (where time is reversed), in the thermodynamics of dissipative structures created by Prigogine, time ceases to be a simple parameter and becomes a concept that expresses the pace and direction of events.

Thus, the irreversibility of time is a fundamental property of non-equilibrium dissipative systems, and its loss may indicate the development of destructive processes [3, 2].

Considering the statistical properties of a signal under study, its evolution could be called irreversible if there is the lack of invariance, i.e., the same signal would have been obtained if we measured it in the opposite direction. The function f could be applied to find characteristics that differ forward and backward versions, i.e., time series would be irreversible if $f(\mathrm{X}^d) \neq f(\mathrm{X}^r)$. The main idea of this definition there is no any restrictions on f.

Our study implies that a stationary process X is called statistically inverse in time if the probability distributions of the forward and backward in time systems are approximately the same [4, 5, 6]. The irreversibility of time series indicates the presence of nonlinear dependencies (memory) [7] in the dynamics of a system far from equilibrium, including non-Gaussian random processes and dissipative chaos. Since the definition of the irreversibility of the time series is formal, there is no a priori optimal algorithm for its quantification. Several methods for measuring the irreversibility of time have been proposed [8, 3, 2, 9, 10, 4, 11, 12, 13, 14]. Such methods significant as their purpose to deal with signals that exclude linear Gaussian random processes and, there by, allow to quantify the degree of predictability in the system.

In the first group of methods, the symbolization of time series is performed, and then the analysis is performed by statistical comparison of the appearance of a string of symbols in the forward and reverse directions [10]. Sometimes additional compression algorithms are used [9]. An important step for this group is the symbolization – the conversion of the time series into a character series requires additional special information (e.g., division of the range or size of the alphabet) and, therefore, contains the problem of the algorithm's dependence on these additional parameters. The second problem arises when considering the large-scale invariance of complex signals. Since the procedures of typical symbolizations are local, taking into account different scales can cause some difficulties [3].

Another group of methods in formalizing the index of irreversibility does not use the symbolization procedure but is based on the use of real values of the time series or returns.

One such approaches is based on the asymmetry of the distribution of points of the Poincare map, built on the basis of the values of the analyzed time series [11, 14].

Recently, a fundamentally new approach to measuring the irreversibility of time series has been proposed, which uses the methods of complex network theory [4, 13] and which combines two tools: the algorithm for visibility of time series recovery into a complex network and the Kullbak-Leibler divergence algorithm [13]. The first forms a directional network according to the geometric criterion. The degree of irreversibility of the series is then estimated by the Kullbak-Leibler divergence (i.e., the resolution) between the distribution of the input and output stages of the associated count. This method is computationally efficient, does not require any special symbolization of the process, and, according to the authors, naturally takes into account multiscale.

In this study, we apply irreversibility analysis and construct indicators or indicators-precursors of crashes and critical events, which dynamics is associated with luck of irreversibility the system. To these measures we include Guzik's index, Porta's index, Costa's index, based on complex networks, multiscale time irreversibility index with measure based on ordinal patterns.

For analyzing and explaining basic characteristics of stock market with time irreversibility measures, we have chosen Dow Jones Industrial Average index (DJIA) as the most quoted financial barometer in the world. In order to have better look on its intraday dynamics, we have separated its time series into two parts: from 2 January 1920 to 3 January 1983 and second part from 4 January 1983 to 3 March 2021. Both periods of daily values have been obtained through Yahoo Finance (http://finance.yahoo.com/) and Investing.com (https://www.investing.com/).

Regarding our previous studies [15, 16, 17, 18, 19, 20, 21, 22, 23, 24], we have emphasized 30 crisis events that were classified as *crashes* and *critical events.* According to classification:

- Crashes are short, time-localized drops, with strong losing of price each day.
- Critical events are those falls that, during their existence, have not had such serious changes in price as crashes.

Table 1 shows the major crashes and critical events related to our classification.

As it is seen from the Table, during DJIA existence, many crashes and critical events shook it. According to our classification, events with number (1, 10, 13, 15, 20) are crashes, all the rest – critical events.

The calculations of indicators for them will be carried out within the sliding window approach. According to the procedure, we emphasize the frame of a predefined length in which the calculation of the corresponding measure is obtained. For this fragment measure of irreversibility is obtained regarding normalized returns, where returns are calculated as

$$G(t) = \ln x(t + \Delta t) - \ln x(t) \cong [x(t + \Delta t) - x(t)] / x(t) \tag{1}$$

and normalized (standardized) returns as

$$g(t) \cong [G(t) - \langle G \rangle] / \sigma, \tag{2}$$

where $\sigma \equiv \sqrt{\langle G^2 \rangle - \langle G \rangle^2}$ is the standard deviation of G, Δt is the time shift (in our case $\Delta t = 1$), and $\langle \ldots \rangle$ is the average over studied time period.

Then, the time window is shifted along the time by a predefined value, and the procedure is repeated until the entire series is exhausted. Comparing the calculated measure of irreversibility (asymmetry) and the actual time series of DJIA, we can analyze changes of complexity in the system. Our measures can be called indicators or precursors if they behave in a definite way for

Table 1
Major Historical Corrections of the DJIA price since 1920

№	Interval	Days in correction	Decline, %
1	03.09.1929-29.10.1929	41	39.64
2	01.03.1938-31.03.1938	23	24.15
3	08.04.1940-05.06.1940	42	25.1
4	21.08.1946-10.09.1946	14	16.35
5	30.07.1957-22.10.1957	60	17.51
6	19.03.1962-28.05.1962	50	19.91
7	18.07.1966-07.10.1966	59	12.84
8	09.04.1970-26.05.1970	34	20.35
9	24.10.1974-04.10.1974	52	27.45
10	02.10.1987-19.10.1987	12	34.16
11	17.07.1990-23.08.1990	28	17.21
12	01.10.1997-21.10.1997	15	12.43
13	17.08.1998-31.08.1998	11	18.44
14	14.08.2002-01.10.2002	34	19.52
15	16.10.2008-15.12.2008	42	30.21
16	09.08.2011-22.09.2011	32	11.94
17	18.08.2015-25.08.2015	6	10.53
18	29.12.2015-20.01.2016	16	11.02
19	03.12.2018-24.12.2018	15	15.62
20	04.03.2020-23.03.2020	13	31.38

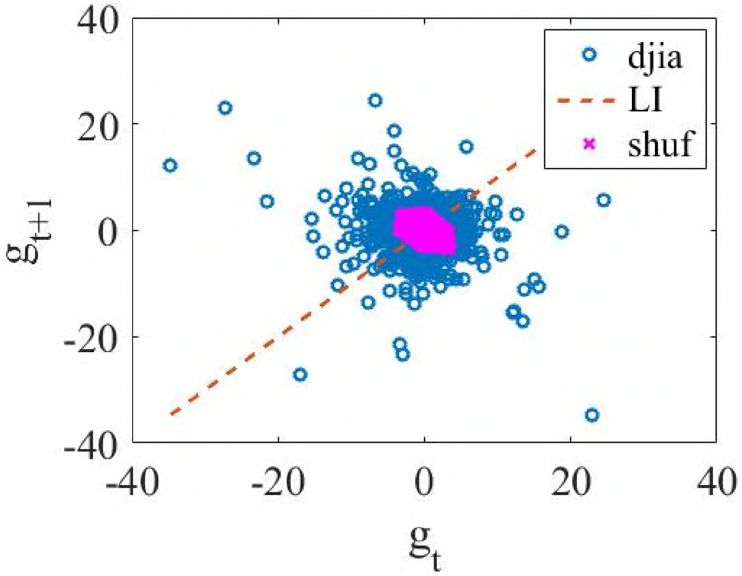
all periods of crashes, for example, decreases or increases during the pre-crash or pre-critical period. For our calculations time frame with the length 500 and step 1 are seemed to be the most reasonable parameters.

2. Assessing financial crises throughout irreversibility analysis

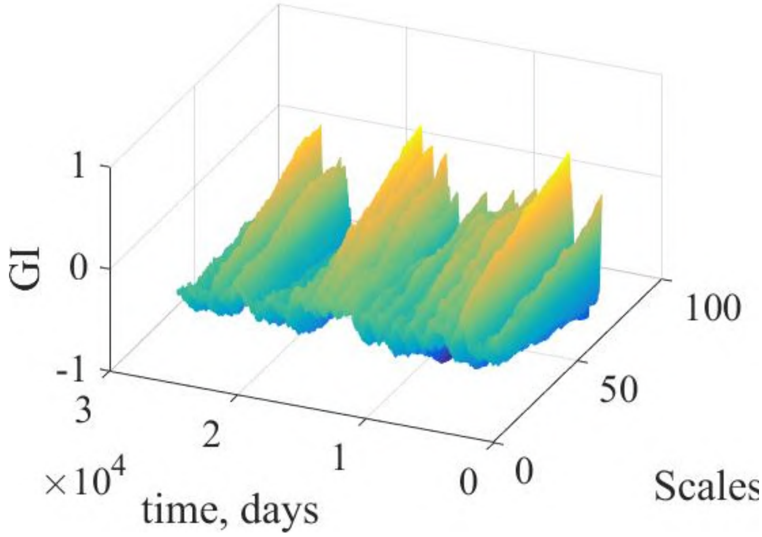
2.1. Irreversible complexity measures based on Poincaré diagrams

The Poincaré diagram for the time series is a graph on the x axis of which the normalized returns for current time $g(t)$ are plotted, and subsequent values $g(t+1)$ on the y axis. In Figure 1 the Poincaré diagram for the initial and shuffled series of the DJIA is shown.

All consequent values that are equal to each other ($g(t) = g(t+1)$) are located on the line of identity (LI). Intervals, representing increasing in returns, above LI ($g(t) < g(t+1)$), whereas shortenings of two succeeding returns represent points below this line ($g(t) > g(t+1)$). By assessing the asymmetry of points in the diagram, further, we will present quantitative measures for varying degree of irreversibility in the DJIA.



(a)



(b)

Figure 1: The Poincaré diagram (a) and the dependence of the Costa's index that will be described further on time and scale (b).

2.1.1. Guzik's index

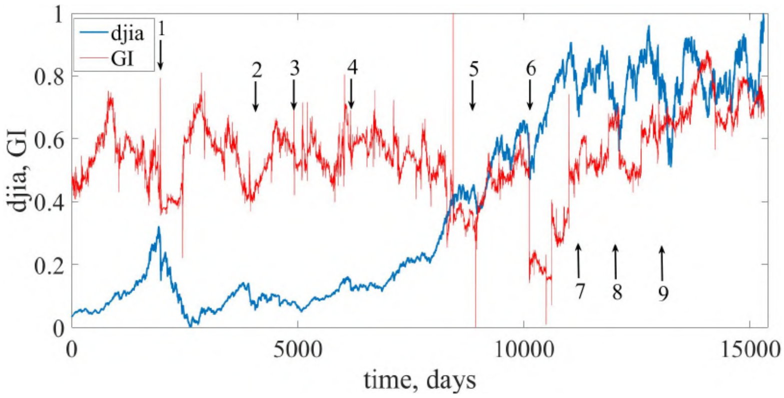
Guzik's index (GI) was defined as the distance of points above LI to LI divided by the distance of all points in Poincaré plot except those that are located on LI [25, 11]. Specifically,

$$GI = \frac{\sum_{i=1}^{a} \left(D_i^+\right)^2}{\sum_{i=1}^{m} \left(D_i\right)^2}, \tag{3}$$

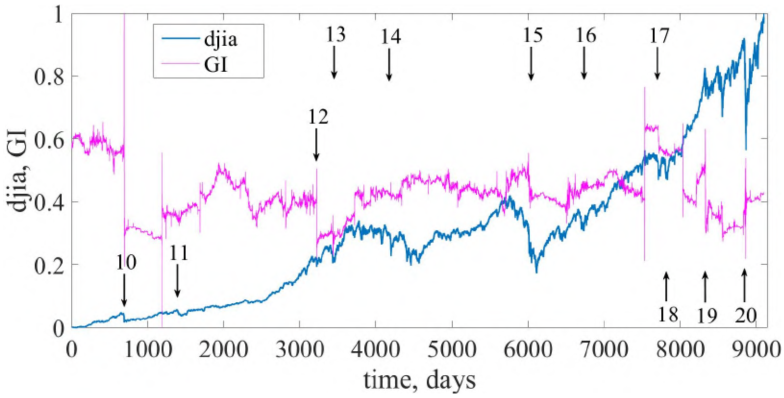
where $a = C(P_i^+)$ means the number of points above LI; $m = C(P_i^+) + C(P_i^-)$ means the number of points in Poincaré plot except those which are not on LI; D_i^+ is the distance of points above the line to itself, and D_i is the distance of point $P_i\,(g(i), g(i+1))$ to LI which can be defined as

$$D_i = \frac{|g(i+1) - g(i)|}{\sqrt{2}}. \tag{4}$$

In fugure 2 is illustrated GI for two periods of the DJIA.



(a)



(b)

Figure 2: Guzik's index with corresponding first (a) and second (b) periods of the DJIA time series.

As we can see from illustration above, GI for crashes and critical events noticeably falling before deviant event and rising during emerging crises, which makes it as an excellent indicator-precursor of abnormal events.

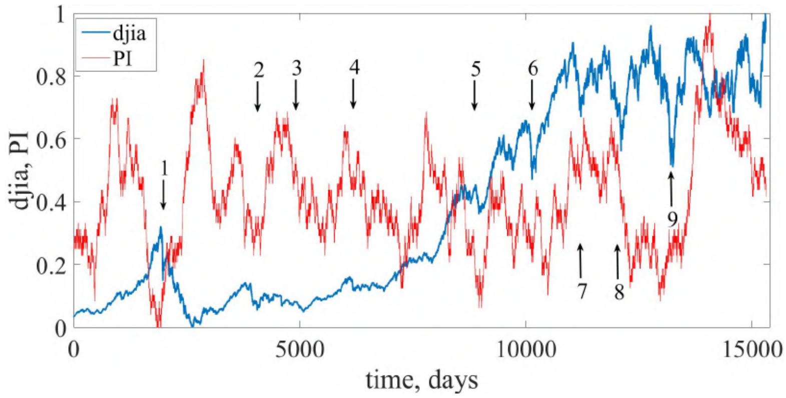
2.1.2. Porta's index

Porta's index (PI) [14] was defined as the number of points below LI divided by the total number of points in Poincaré plot except those that are located on LI, specifically

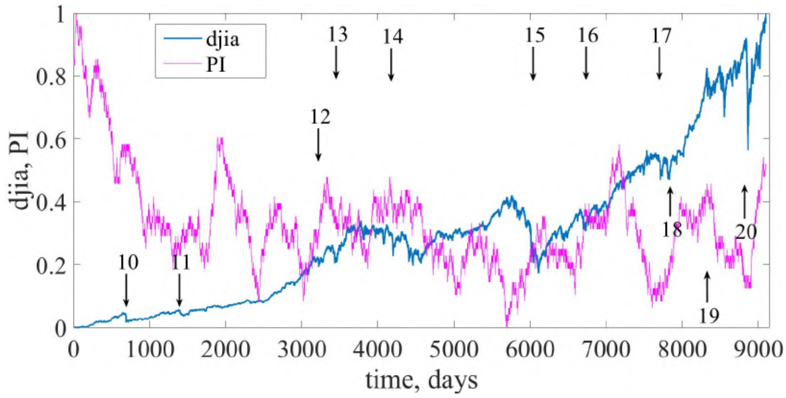
$$PI = \frac{b}{m}, \tag{5}$$

where $b = C(P_i^-)$ is the number of points below LI, and $m = C(P_i^+) + C(P_i^-)$ is the total number of points below and above LI.

In figure 3 is illustrated PI for two periods of DJIA.



(a)



(b)

Figure 3: Dynamics of Porta's index for first (a) and second (b) periods of DJIA time series.

As we can see, according to Porta's index, irreversibility decreases during crash and critical events similarly to previous index which makes it appropriate indicator.

2.1.3. Costa's index

Costa's index represents a simplified version of [25] where number of increments $(x(i+1)-x(i) > 0)$ and decrements $(x(i+1)-x(i) < 0)$ are taken into account. They are presented to be symmetric if equal to each other. The procedure is implemented for coarse-grained time series. For scale τ, we consider the time series $G_\tau = \{g(i)\}, g(i) = x(i+\tau) - x(i), 1 \leq i \leq N-\tau$. The Costa's index [3], which displays the asymmetry of the probability distribution of positive and negative returns, is calculated by the formula:

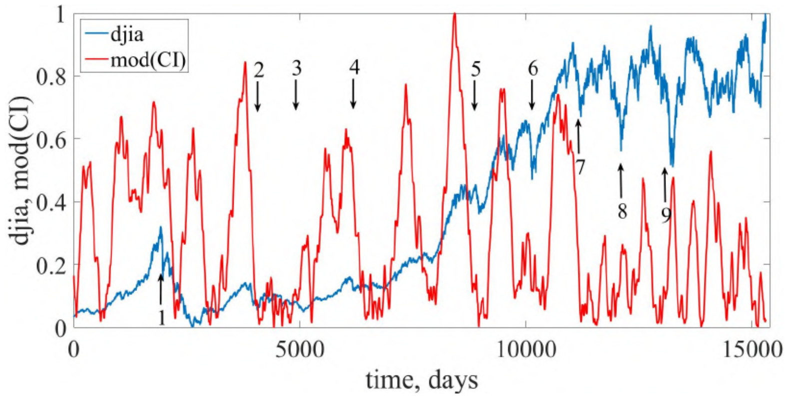
$$CI_\tau = \frac{\sum_{i=1}^{N-\tau} \mathscr{H}[-g(i)] - \sum_{i=1}^{N-\tau} \mathscr{H}[g(i)]}{N-\tau}. \tag{6}$$

The generelized Costa's index according to can be defined as

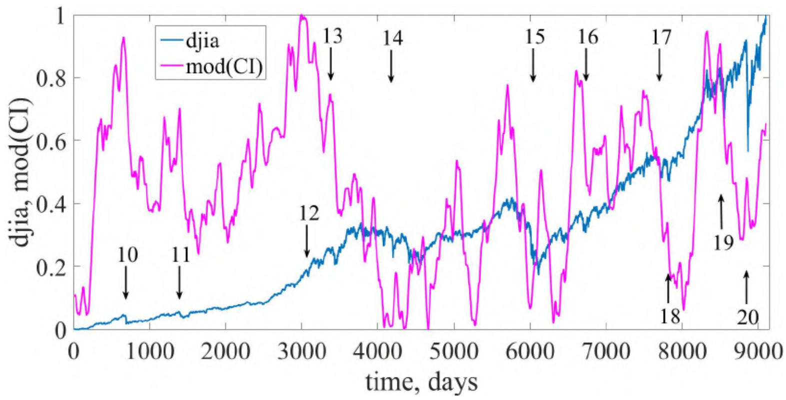
$$CI = \frac{1}{L}\sum_{\tau=1}^{L}|CI_{\tau}|, \tag{7}$$

where L is the maximal scale.

In figure 4 CI presents the similar pehavior for the two periods of DJIA as in previous two measures.



(a)



(b)

Figure 4: Dynamics of Costa's index for first (a) and second (b) periods of DJIA time series.

2.2. Complex network methods

Visibility graphs (VGs) are based on a simple mapping from the time series to the network domain exploiting the local convexity of scalar-valued time series $\{x_i \,|\, i = 1, \ldots, N\}$ where each observation x_i is a vertex in a complex network. Two vertices i and j are linked by an edge (i, j)

if for all vertices k with $t_i < t_k < t_j$ the following condition is applied [26]:

$$x_k < x_j + (x_i - x_j)\frac{t_j - t_k}{t_j - t_i}. \tag{8}$$

This is, the adjacency matrix (A_{ij}) of the following undirected and unweighted VG is presented as:

$$A_{ij}^{(VG)} = A_{ji}^{(VG)} = \prod_{k=i+1}^{j-1} \mathscr{H}\left(x_k < x_j + (x_i - x_j)\frac{t_j - t_k}{t_j - t_i}\right), \tag{9}$$

where $\mathscr{H}(\cdot)$ is the Heaviside function.

Horizontal visibility graphs (HVGs) provide a simplified version of this algorithm [27]. For a given time series, the vertex sets of VG and HVG are the same, whereas the edge set of the HVG maps the mutual horizontal visibility of two observations x_i and x_j, i.e., there is an edge (i, j) if $x_k < \min(x_i, x_j)$ for all k with $t_i < t_k < t_j$, so that

$$A_{ij}^{(VG)} = A_{ji}^{(VG)} = \prod_{k=i+1}^{j-1} \mathscr{H}(x_i - x_k)\mathscr{H}(x_j - x_k). \tag{10}$$

VG and HVG capture essentially the same properties of the system under study (e.g., regarding fractal properties of a time series), since the HVG is a subgraph of the VG with the same vertex set, but possessing only a subset of the VG's edges. Note that the VG is invariant under a superposition of linear trends, whereas the HVG is not.

Since the definition of VGs and HVGs takes the timing (or at least time-ordering) of observations explicitly into account, the direction of time is intrinsically interwoven with the resulting network structure. To account for this fact, we define a set of novel statistical network quantiers based on two simple vertex characteristics:

(i) As the number of edges incident to a given vertex i can be defined as $k_i^r = \sum_j A_{ij}$, for a (H)VG, we rewrite this quantity for a vertex of time t_i, regarding its past and future vertices (prices):

$$k_i^r = \sum\nolimits_{j<i} A_{ij}, \tag{11}$$

$$k_i^a = \sum\nolimits_{j>i} A_{ij}, \tag{12}$$

where $k_i = k_i^r + k_i^a$, and k_i^r with k_i^a referred to as the *retarded* and *advanced* degrees. As it is defined in [13], following measures correspond to the in- and out-degrees of time-directed (H)VGs.

(ii) The local clustering coefficient $C_i = \binom{k_i}{2}^{-1} \sum_{j,k} A_{ij}A_{jk}A_{ki}$ is another vertex property of higher order characterizing the neighborhood structure of vertex i [28]. Similarly to (11) and (12), for studying the connectivity due to past and future prices, we rewrite the standard coefficient as the *retarded* and *advanced local clustering coefficients*

$$C_i^r = \binom{k_i^r}{2}^{-1} \sum\nolimits_{j<i,k<i} A_{ij}A_{jk}A_{ki}, \tag{13}$$

$$C_i^a = \binom{k_i^a}{2}^{-1} \sum\nolimits_{j>i,k>i} A_{ij}A_{jk}A_{ki}, \tag{14}$$

According to graph-based method, we will utilize the probability density functions (PDFs) of (11)-(14). If our system is presented to be time-reversible, we conjecture that probability distributions of forward and backward in time characteristics should be the same. For irreversible processes, we expect to find statistical non-equivalence. According to Lacasa et al. [13], this deviation will be defined through Kullback-Leibler divergence:

$$D_{KL}(p \parallel q) = \sum_{i=1}^{N} p(x_i) \cdot \log \frac{p(x_i)}{q(x_i)}, \tag{15}$$

where, in our case, p responds to a distribution of the *retarded* characteristics and q is of the *advanced.*

Figure 5 presents D_{KL} measure for the distribution of degrees and local clustering coefficients.

As it can be seen for figure 5 and b, both irreversibility measures for degrees and local clustering decrease during crashes and critical events which tells about luck of irreversibility during them. Also, it is shown in figure 4 that the first period of the DJIA is presented to be more reversible as the distance between distribution of degrees is close to zero for almost the entire period. Local clustering coefficient is seemed to be more robust and informative comparing to degree.

2.3. Multiscale time irreversibility index

For the following procedure [25], first of all, we need to construct goarse-grained time series which can be defined as

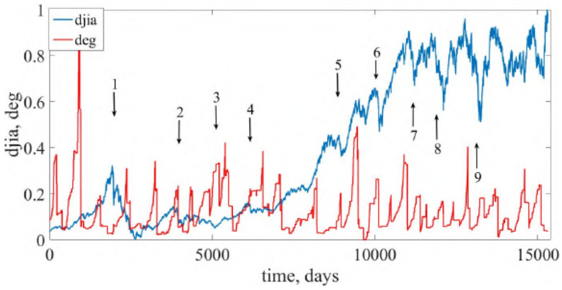
$$y_\tau(j) = \frac{1}{\tau} \sum_{i=(j-1)\tau+1}^{j\tau} g(i), \quad \text{for } 1 \le j \le \frac{N}{\tau}. \tag{16}$$

Then, using a statistical physics approach, we make the simplifying assumptions that each transition (increase or decrease of $y_\tau(j)$) is independent and requires a specific amount of "energy" E. The probability density function of this class of system [29] can be assumed to follow $\rho \propto \exp(-\beta E - \gamma Q)$ where Q represents the non-equilibrium heat flux across the boundary of the system, and β and γ are the Lagrange multipliers derived from the constraints on the average value of the energy E per transition and the average contribution of each transition to the heat flux Q.

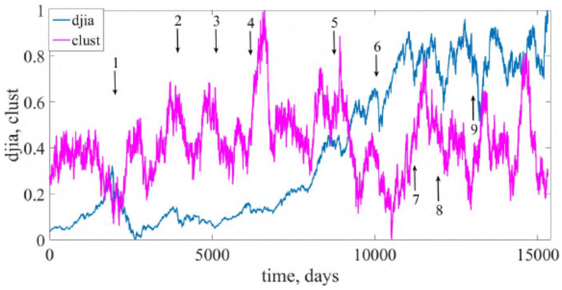
Since the time reversal operation on the original financial index time series inverts an increase to a decrease and vice versa, the difference between the average energy for the *activation* of information rate, i.e., $\langle \beta E + \gamma Q \rangle_{y_\tau > 0}$, and the *relaxation* of information rate, i.e., $\langle \beta E + \gamma Q \rangle_{y_\tau < 0}$, can be used as measurement of time reversal asymmetry.

Taking into consideration that the assumption of the distribution function ρ links the energy to the empirical distribution, we, following , define the next measure of temporal irreversibility:

$$a(\tau) = \frac{\int_{0}^{\infty} [\rho(y_\tau) \ln \rho(y_\tau) - \rho(-y_\tau) \ln \rho(-y_\tau)]^2 \, dy_\tau}{\int_{-\infty}^{\infty} \rho(y_\tau) \ln \rho(y_\tau) dy_\tau} \tag{17}$$



(a)



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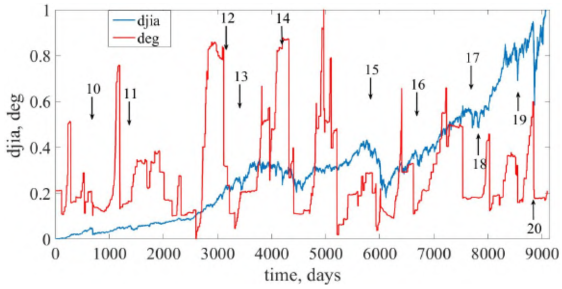
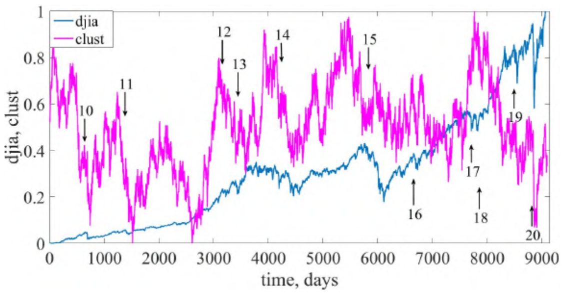


(c)



(d)

Figure 5: Dynamics of graph-based time irreversibility measures for the first (a, b) and second (c, d) periods of DJIA time series.

The time series is called reversivle if $a(\tau) = 0$.

Sometimes it is important for us to know not only the degree of irreversibility but also whether it reversed in time or not. For this purpose, we will replace equation (17) by the following one:

$$A(\tau) = \frac{\int\limits_{0}^{\infty} [\rho(y_\tau)\ln\rho(y_\tau) - \rho(-y_\tau)\ln\rho(-y_\tau)]\, dy_\tau}{\int\limits_{-\infty}^{\infty} \rho(y_\tau)\ln\rho(y_\tau) dy_\tau} \tag{18}$$

The time series is said to be irreversible for all scale τ if $A(\tau) > 0$. In case when $A(\tau) = 0$, the time series may be reversible or not for scale τ.

For the analysis of discrete values, equation (18) can be presented as:

$$\hat{A}(\tau) = \frac{\sum_{y_\tau>0} \Pr(y_\tau)\ln[\Pr(y_\tau)]}{\sum_{y_\tau} \Pr(y_\tau)\ln[\Pr(y_\tau)]} - \frac{\sum_{y_\tau<0} \Pr(y_\tau)\ln[\Pr(y_\tau)]}{\sum_{y_\tau} \Pr(y_\tau)\ln[\Pr(y_\tau)]}. \tag{19}$$

The generalized multiscale asymmetry index (A_I) is defined as the summation of $\hat{A}(\tau)$ obtained for a predefined range of scales, i.e.,

$$A_I = \sum_{\tau=1}^{L} \hat{A}(\tau). \tag{20}$$

The figures illustrate that time series are significantly irreversible. For initial time series (for approximately 5-10 scales), the *transition* of prices is presented to be reversible (symmetric). After it, *transitions* presented to be asymmetric. Draws attention and noticeable unevenness introduced measures, which correlate with the fluctuations of the input time series. Identifying significant changes in the time series and comparing them with the corresponding changes of non-reversible measures of complexity, it is possible to construct the corresponding indicators.

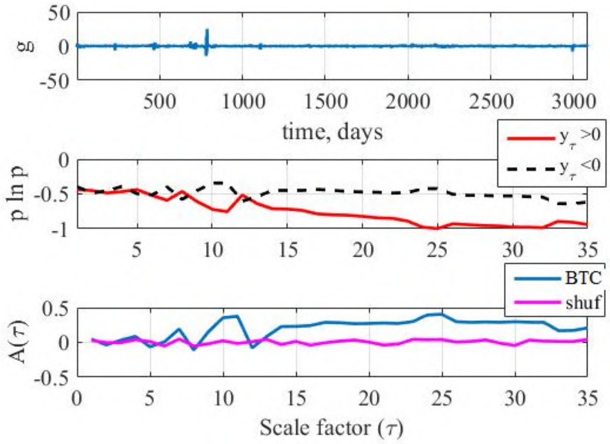
2.4. Time series irreversibility measure based on permutation patterns

The idea of analyzing the permutation patterns (PP) was initially introduced by Bandt and Pompe [30] to provide researchers with a simple and efficient tool to characterize the complexity of the real systems dynamics. With respect to other approaches, as entropies, fractal dimensions, or Lyapunov exponents, it avoids amplitude threshold and instead dealing with casual values inhereted from time series dynamics, deals with ordinal permutation patterns [31]. Their frequencies allow us to distinguish deterministic processes from completely random.

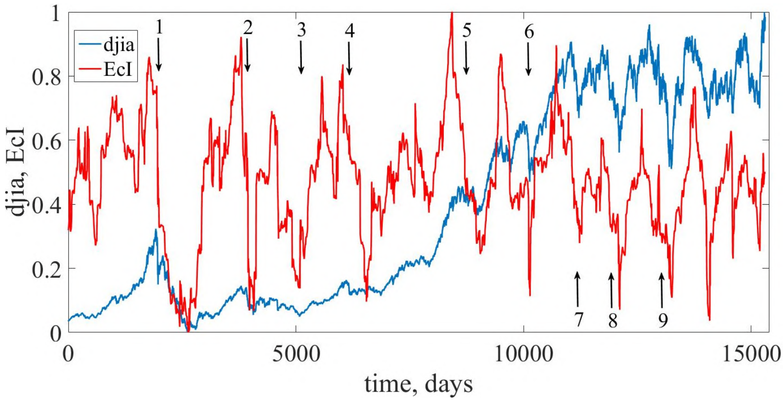
The calculations of PP assume that the time series is partitioned with the *embedding dimension* d_E (number of elements to be compared) and the *embedding delay* τ (time separation between elements). In our opinion, $d_E \in \{3, 4\}$ and $\tau \in \{2, 3\}$ are the best parameters that encapsulate all the necessary quantitative information.

Further, all embedded patterns are assigned to their ordinal *rankings*. As an example, let us consider a fragment of the DJIA time series for period 18.08.2015-26.08.2015:

$$\begin{aligned} \mathrm{X} = \{&17511.34, 17348.73, 16990.69, 16459.75, \\ &15871.35, 15666.44, 16285.51\}. \end{aligned}$$



(a)



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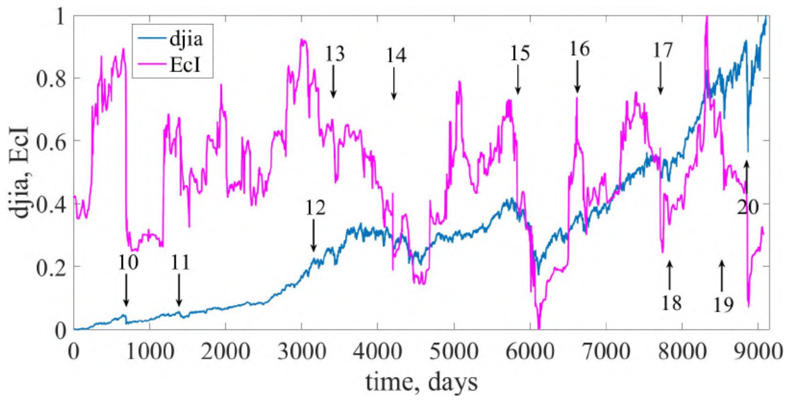

(c)

Figure 6: Dynamics of asymmetry index for first (a) and second (b) periods.

According to mentioned steps, we will construct embedded matrix of overlapping column vectors with $d_E = 3$ and $\tau = 2$. Our sampled data is partitioned as follows:

$$\mathrm{X}_t^{d_E,\tau} = \begin{bmatrix} 17511.34 & 16990.69 & 15871.35 \\ 17348.73 & 16459.75 & 15666.44 \\ 16990.69 & 15871.35 & 16285.51 \end{bmatrix}. \tag{21}$$

After it, our time-delayed vectors are mapped to *permutations* or *ordinal patterns* of the same size. Our example consists $3! = 6$ different ordinal patterns in total:

$$\begin{aligned} \pi_1 &= \{0, 1, 2\} \\ \pi_2 &= \{0, 2, 1\} \\ \pi_3 &= \{1, 0, 2\} \\ \pi_4 &= \{1, 2, 0\} \\ \pi_5 &= \{2, 0, 1\} \\ \pi_6 &= \{2, 1, 0\} \end{aligned}$$

As an example, the corresponding permutation of the first column from (21) would be $\phi([17511.34, 17348.73, 16990.69]) = 210$ since $x(3) \leq x(2) \leq x(1)$. Therefore, after mapping from the time-series data into a series of permutations ($\phi : \mathbb{R}^{d_E} \rightarrow S_{d_E}$), we obtain the ordinal matrix:

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}. \tag{22}$$

Finally, the probability of each pattern is calculated as

$$p(\pi) = \frac{\#\{t \leq N - (d_E - 1)\tau, \phi(\mathrm{X}_t^{d_E,\tau}) = \pi\}}{N - (d_E - 1)\tau}, \tag{23}$$

where $\#\{\cdot\}$ denotes the cardinality of a set, and *permutation entropy* is calculated regarding a probability distribution P, whose elements $p_i \equiv p(\pi_i)$ are the probabilities associated with the i^{th} permutation pattern, $i = 1, \dots, d_E!$:

$$S[P] = -\sum_{i=1}^{d_E!} p_i \log_2 p_i. \tag{24}$$

Interesting for us time irreversibility of permutation patterns is not related on (24), but on the probability distribution of ordinal patterns. That is, we find probabilities of finding corresponding ordinal patterns for both initial and reversed times series. Correspondingly, if both types have approximately the same probability distributions of their patterns, time series is presented to be reversible and the opposite conclusion for the other case.

The difference between distributions of direct time series (P^d) and reversed (P^r) can be estimated with equation (15).

From the presented figures it can be seen that as financial crisis comes, the distance between two distributions becomes more close to zero, denoting that those period is less irreversible and efficient. Moreover, in this case we see that D_{KL} for permutaiton patterns acts as a measure of complexity. The dynamics before crisis events starts do decrease, presenting trend to be more predictable, and after them it increases, demonstrating the increasing complexity.

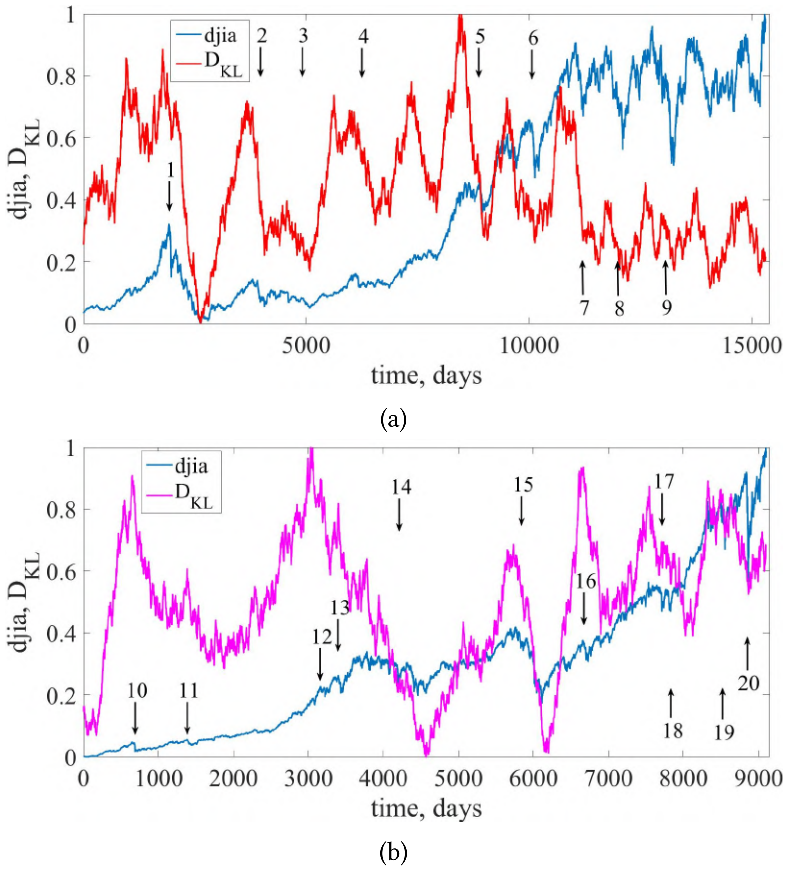

Figure 7: Dynamics of permutation-based time irreversibility measure for first (a) and second (b) periods.

3. Conclusions

Financial systems does not always evolve with precisely the same values. Instead, their prices increase or decrease over time due to different market conditions, political, and economical situations in concrete countries or in the word.

In this work we have presented how to deal with (statistical) time irreversibility, varying over time. Using the time series of Dow Jones Industrial Average index and the sliding window procedure, first of all, we have presented our classification of crisis events in DJIA index, and we have constructed econophysical and econometrical indicators of financial crashes and critical events. Our study affirms ranging degrees of irreversibility in DJIA stock index. Some of its periods of existence are presented to be more irreversible comparing to others. Namely, periods of financial stress are characterized by higher irreversibility and, thus, by increasing predictability and less efficiency.

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Big Data based marketing forecasting

Sergey M. Ivanov, Mykola M. Ivanov

Zaporizhzhia National University, 66 Zhukovsky Str., Zaporizhzhia, 69063, Ukraine

Abstract

In the paper discusses the use of big data as a tool to increase data transfer speed while providing access to multidimensional data in the process of forecasting product sales in the market. In this paper discusses modern big data tools that use the MapReduce model. The big data presented in this article is a single, centralized source of information across your entire domain. In the paper also proposes the structure of a marketing analytics system that includes many databases in which transactions are processed in real time. For marketing forecasting of multidimensional data in Matlab, a neural network is considered and built. For training and building a network, it is proposed to construct a matrix of input data for presentation in a neural network and a matrix of target data that determine the output statistical information. Input and output data in the neural network is presented in the form of a 5x10 matrix, which represents static information about 10 products for five days of the week. The application of the Levenberg-Marquardt algorithm for training a neural network is considered. The results of the neural network training process in Matlab are also presented. The obtained forecasting results are given, which allows us to conclude about the advantages of a neural network in multivariate forecasting in real time.

Keywords

Big Data, marketing forecasting, MapReduce model, Levenberg-Marquardt algorithm

1. Introduction

As Flors [1] explained, the success of digital marketing is determined by the success of digital marketing, thus how they are measured and used. However, attention is paid to marketing forecasting in the digital economy, taking into account intelligent systems [2].

Intelligent systems and the use of multidimensional communication determined the emergence of a new concept by Schwab [3]. According to this concept, it is argued that we are in the era of the fourth industrial revolution (Industry 4.0), when the virtual world is combined with the physical world using information technology. The fourth industrial revolution is characterized by a change in economic relations and the widespread use of intelligent technologies (big data [4], artificial neural networks [5], and others).

It should be noted that with the use of digital marketing, D2C models have come to be used. The D2C (Direct to Consumer) model represents a direct selling system, where companies

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flydaiver@gmail.com (S. M. Ivanov); nn_iva@ukr.net (M. M. Ivanov)

https://sites.znu.edu.ua/cms/index.php?action=news/view_details&news_id=37238&lang=eng&news_code=Ivanov-Serhii-Mykolaiovych (S. M. Ivanov); https://sites.znu.edu.ua/cms/index.php?action=news/view_details&news_id=40272&lang=eng&news_code=Mykola-Ivanov (M. M. Ivanov)

0000-0003-1086-0701 (S. M. Ivanov); 0000-0002-1908-0763 (M. M. Ivanov)

themselves manufacture, promote, sell and deliver their product without the involvement of intermediaries. So, the article provides an analysis that shows that, unlike traditional promotion through retail chains, companies using the D2C model develop faster with their own distribution channels [6]. Thanks to their good positioning, these companies not only have a competitive advantage in the market, but also have their own structure on the Internet. These companies have changed the producer-consumer relationship and are reducing the distance between them. Today, any customer can contact the manufacturer directly, ask their question and make a purchase, avoiding extra charges and saving time. Renowned manufacturers have recognized the need to develop their own D2C strategies based on marketing analytics. The authors of the article acknowledge that the use of D2C opens up additional opportunities for companies. According to the authors, Nike is a prime example, with D2C sales accounting for a third of total revenues by the end of 2020 based on its Consumer Direct strategy.

Given the widespread use of digital marketing, Kats [7] explored social commerce, which fostered development along the D2C model. Today, social commerce is used to increase the reach of consumers, those who may know about direct contact with the manufacturer.

So in the presented report by Enberg [8], a summary of the main events, their analysis for marketing management, solving problems strategic development of companies. Report author Jasmine Enberg determines that the global forecast for monthly social media users in 2020 has increased due to the effects of the pandemic. However, no platform will be able to maintain the growth it picked up at the beginning of the year. Therefore, in 2021, the growth rate will begin to normalize. Recent product launches, including Facebook live shopping and Instagram shopping tags, show that e-commerce continues to be a priority for the two platforms. Snapchat and Twitter were more focused on effective marketing, namely the release of new sets of promotional offers with direct consumer response and others.

Therefore, the modern market is based on direct selling (D2C) models. allow you to get images of buyers and segment them. In addition to these tasks, it is necessary to solve forecasting the market, which changes every year.

It should be noted that worldwide targeted statistics for the entire sales system. This information is stored in cloud storage (Big Data). The information used includes data from the time of attracting a new consumer to the required resource information about the number, including repeated ones.

In this article, we propose a marketing analytics method based on artificial neural networks.

Marketing forecasting is part of any firm's overall marketing analytics. An important role in the forecasting method is played by the multidimensionality of information and methods of their processing. The use of Big Data with OLAP technologies requires new approaches to processing and applying large amounts of data. This is due to the wide range of communication systems used in the e-commerce market. Therefore, marketing forecasting based on Big Data is an urgent task, which is discussed in this article.

The article is devoted to marketing forecasting based on the use of neural networks. The information base of marketing forecasting is Big Data. The process of constructing training matrices, training a neural network and making the predictions are presented in the result of paper.

2. Literature review

Modern companies use many available forecasting methods, they not only improve the quality of their products, but also get information about the needs of customers. Marketing forecasting models are a great way to predict customer preferences and apply new ways to stand out from the competition. Using practical forecasting models today is the best way to get the most effective and complete data to improve marketing decisions. In this case, forecasting methods in digital marketing should include not only customer surveys, their age, interests and price, but also the characteristics of the product, brand, logistics and others.

In the work of Bard [9], the problem of fitting mathematical models to numerical data was considered. Such a fit is often performed by the least squares method, regardless of previous knowledge of parameter values or the statistical nature of measurement errors.

According to the results of a study of the opinions of experts, Ashton [10] considered a scenario in which the results of the opinions of experts differ significantly from the polls of intentions. For this case, the author considered the problem of predicting market behavior.

Further development of the principle of intentions was found by Morwitz [11]. The author proposed the principles of using intentions when solving the forecasting problem. These tasks included researching people and how they would behave in different situations. For this, the method of polling the intentions of people was used. Intent surveys are widely used in marketing when sales data is unknown, for example, to forecast new products.

A continuation of these works found themselves in the work of Armstrong [12], where the role of a person as a dominant factor was considered. This task was solved as a role-playing game for making predictions of the behavior of people who interact with others. A key tenet of this approach is to provide realistic simulation of interactions.

This forecasting method is currently rarely used. Rowe and Wright [13] considered the application of the Delphi method as a procedure. The authors found that the accuracy of expert predictions can be improved through the use of Delphi structured methods. One of the principles of the method is that experts' forecasts should not depend on each other. Expert groups sometimes violate this principle; as a result, the data should not be used in forecasting.

The task "Intentions" was considered by Wittink and Bergestuen [14]. This paper examines the intention as an indicator of a consumer to purchase a product under the influence of various factors. The consumer can declare his intentions to make a purchase of various goods. This method is based on the following principles, namely, using a new design to create an acceptable situation.

The formation of a digital marketing strategy was considered by Mandal and Joshi [15]. The authors of the article emphasize that digital technologies make marketing more effective, since they allow to identify individual consumer interests, better manage campaigns and improve the product. In this article, the authors propose a flowchart for developing marketing strategies.

The analysis of the state of the digital economy and digital marketing is discussed by Ivanov [16]. The author in the article shows that the dynamics of processes in the economy is quite high and requires a quick analysis of multidimensional data. The author proposes a conceptual model and a method for assessing consumer demand in the target market, aimed at the prospective management of trading floors using Big Data.

The analysis of the overview of macroeconomic forecasting are discussed by Carnot et al.

[17, 18], Ivanov [19], Lim [20]. The authors take the focus on a wide range of theories as well as empirical methods: business cycle analysis, time series methods, macroeconomic models, medium and long-run projections, fiscal and financial forecasts, and sectoral forecasting.

The research of the technological sources of the next long wave of growth is made by Vázquez [21], Lee and Lee [22]. The authors study the impacts of national innovation systems (NIS) and economic complexity index (ECI) on economic growth.

The use of Exponential smoothing (ES) forecasting methods is discussed by Chatfield et al. [23], Hyndman et al. [24]. The authors research revolution of exponential smoothing, which has been improved with the introduction of a complete modeling framework incorporating innovations state space models, likelihood calculation, prediction intervals and procedures for model selection.

3. Marketing forecasting modeling

The modern economy is characterized by rapid dynamics of economic processes. Under these conditions, marketing forecasting models acquire new meanings in managerial decision-making. The process of the importance of making management decisions in digital marketing systems is shown in figure 1.

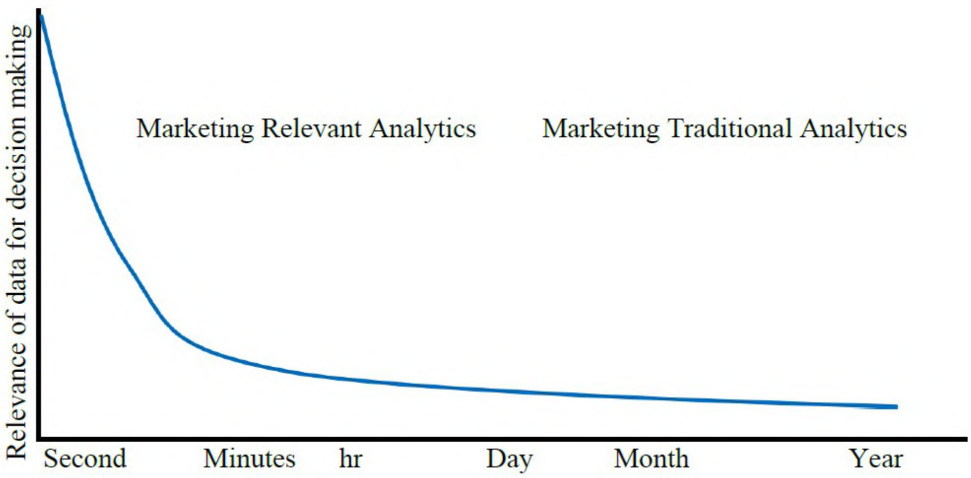

Figure 1: The importance of management decision making in digital marketing.

Traditional Marketing Analytics (MTA) is based on the use of classical approaches and forecasting methods. MRA is aimed at solving economic important forecasting problems from the moment information appears to several hours.

These tasks are solved using neural networks and marketing robots.

Traditional Marketing Analytics is based on the use of classical approaches and forecasting methods. Marketing Actual Analytics (MRA) is aimed at solving economic important forecasting problems from the moment information appears to several hours. These tasks are solved using neural networks and marketing robots. Today the number of information sources of data in the world is growing rapidly. Therefore, storage technologies and their processing of information are becoming more and more in demand. By storing information, one can single out the use of Big Data for which the basic principles of work can be formulated:

1. Horizontal scalability, which takes into account that the data can be arbitrarily large from any system. They have the ability to handle big data.
2. Tolerance to failures, which use the principles of horizontal scalability and apply methods of clustering systems; locality of data, which allows in large distributed systems to separate data from a large number of data centers. All modern tools for working with big data, one way or another, follow these three principles. The first principle is based on the MapReduce model. The MapReduce model provides for distributed data processing proposed by Google and is shown in figure 2.

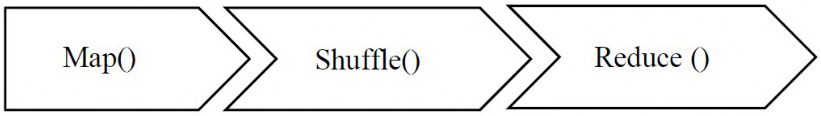

Figure 2: Data processing according to the MapReduce model.

MapReduce provides that data is organized as relational or multidimensional data (OLAP). The data processing method takes place in three stages. The first stage is aimed at executing the Map() function. At this stage, the data is preprocessed using the Map() function, which is defined by the user. The work of this stage is to preprocess and filter the data. The second stage of the model is performed by the Shuffle() function. This stage goes unnoticed by the user. At this stage, the Map() function performs the data immersion procedure similarly to the formation of data marts (Data Mart), that is, one Map() data output corresponds to each mart. In the future, these showcases will serve as an input for the Reduce() function. The third stage of the model is aimed at executing the Reduce() function. Each data mart, which is formed in the second stage, transfers information to the input of the Reduce() function. The Reduce() function is user defined and calculates the result for individual storefronts. The set of all values returned by Reduce() is the result in this method. Therefore, Big Data technology is consider as a tool that allows you to increase the speed of data transfer while providing a large capacity of information carriers. In addition, this technology can improve the availability of cloud applications and data services. Thus, digital marketing is shape around the mainstream e-commerce models. The interconnection of the main models (B2B, B2A, D2C, C2A and C2C) of e-commerce systems based on the systems for collecting, storing and analyzing information in real time. Which based on subsequent storage in historical data layers. For the implementation of systems that perform Marketing Relevant Analytical tasks using data, OLAP data systems are used, which are structured according to the principle of multidimensional information presentation [16]. Reducing the cost of creating multidimensional warehouses can be achieve by using Data Mart. A data mart can only contain thematically aggregated data. Big Data is today a single, centralized source of information for the entire subject area. The structure of the marketing analytical system can be represent as follows (figure 3).

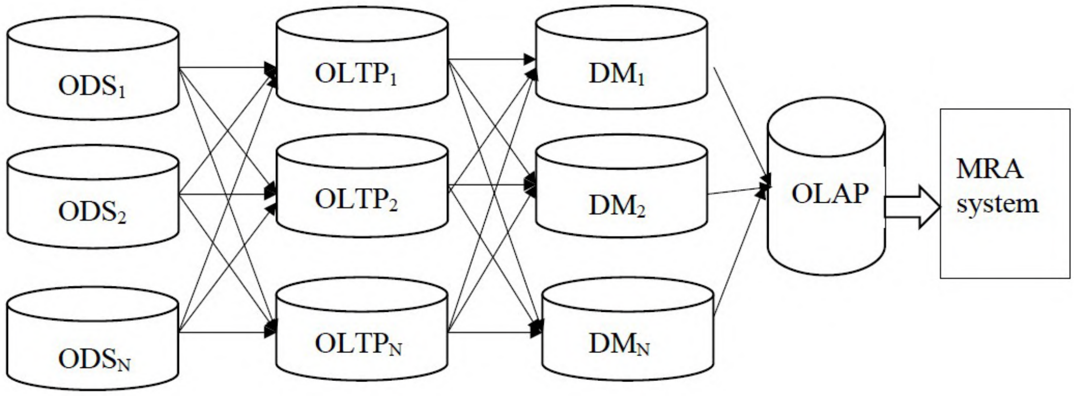

Figure 3: The structure of the marketing analytical system.

In a marketing analytical system, there are many databases. where transaction processing is done in real time. Therefore, online data source systems (ODS) provide information for processing in OLTP. OLTP systems provide storage and processing of information in real time. The processed data in OLTP is transferred to the Data Mart systems with the subsequent construction of multidimensional OLAP data cubes.

This multidimensional data is aimed at presenting information on thematic sections both on marketing information and other information from different areas of the economy. The marketer has the ability to access multidimensional data in the repository, as well as complete economic information for conducting an MRA. The advantages of this approach are:

- simplicity of creating and filling OLAP, since filling comes from reliable sources of data marts;
- reducing the load on working with multidimensional data, namely, one multidimensional query processes multiple OLAP layers.

The data coming from the OID is transferred to the OLTP and the data marts are moved. OLAP stores data as multidimensional layers of measures and dimensions [16]. For marketing forecasting, a multidimensional query is formed to multidimensional data, which allows obtaining the following information input stream (Inflow) – formed by data from OLTP and DMN subsystems:

$$I = \left\{p_j = (g_j, in_j, mi_j)\right\},\ j = 1, .., N, \tag{1}$$

where g_j is the product included in the analyzed sets of N – the object of research,

in_j – indicator of income j of the product,

mi_j – product marketing indicators j.

Datasets from set I stored in OLAP and on demand allow you to obtain and conduct marketing analysis with the subsequent storage of data, which are called transactions. Description of a transaction to set I as follows:

$$T = \left\{in_j | in_N \in I\right\}. \tag{2}$$

Such transactions for retail outlets on the Internet correspond to the nomenclature of goods that the consumer buys and the data stored in OLAP as multidimensional data cubes (OLAP).

Then solving the problem of marketing forecasting based on a neural network, data arrays of its training are formed. The forecasting technique using a neural network is formalized through the problem of pattern recognition. Data on the predicted economic indicators of a product for a certain period of time form an image, the class of which is determined by the values of the predicted indicators.

In the proposed methodology, the dimension of the multidimensional array will determine both the forecasting interval and the number of predicted indicators. Each next line of the array is formed as a result of a shift by one interval equal to the prediction interval.

The neural network is trained on the generated training array of product indicators and adjusts its weights accordingly. As a result, the neural network is trained to solve the forecasting problem for a certain forecasting horizon. It should be noted that two forecasting approaches are used: one-step and multi-step. One-step forecasting is used for short-term forecasts and multistep forecasting is used for long-term forecasting.

Let the time interval $[t_0, t_k]$ be given, the indicators g_j, in_j, pr_j of the product are defined, where t_0 is the initial time value, t_k is the current time value. To find the predicted values on the prediction interval Δ, a method is proposed that includes the following stages:

1. Analytical analysis of marketing indicators and the formation of a learning matrix from selected values from historical slices of multidimensional databases (OLAP technology).

A learning matrix (ML - matrix learning) can be written as input data for representation in a neural network (equation (3)).

$$ML_{Input} = \begin{bmatrix} in_{01} = f_{01}(t_0) & in_{02} = f_{02}(t_0 + \Delta) & \dots & in_{0m} = f_0 m(t_0 + (k-1)\Delta) \\ in_{11} = f_{11}(t_0) & in_{12} = f_{12}(t_0 + \Delta) & \dots & in_{1m} = f_1 m(t_0 + (k-1)\Delta) \\ \dots & \dots & \dots & \dots \\ in_{N1} = f_{N1}(t_0) & in_{N2} = f_{N2}(t_0 + \Delta) & \dots & in_{Nm} = f_N m(t_0 + (k-1)\Delta) \end{bmatrix}, \quad m = 1, \dots, k \tag{3}$$

where Δ – is the horizon (time interval) of forecasting.

$$TVM_{Output} = \begin{bmatrix} in_{01} = f_{01}(t_0 + \Delta) & in_{02} = f_{02}(t_0 + 2\Delta) & \dots & in_{0m} = f_{0m}(t_0 + k\Delta) \\ in_{11} = f_{11}(t_0 + \Delta) & in_{12} = f_{12}(t_0 + 2\Delta) & \dots & in_{1m} = f_{1m}(t_0 + k\Delta) \\ \dots & \dots & \dots & \dots \\ in_{N1} = f_{N1}(t_0 + \Delta) & in_{N2} = f_{N2}(t_0 + 2\Delta) & \dots & in_{Nm} = f_{Nm}(t_0 + k\Delta) \end{bmatrix}, \quad m = 1, \dots, k \tag{4}$$

The input data in the neural network in Matlab is presented in the form of a 5x10 matrix, which represents static information on 10 products for five days of the week and has the following form (figure 4).

Target data that determine the output statistical information for the neural output can be represented in the form of learning value matrix (TVM_{Output}, equation (4)).

The target data, which determines the output statistical information in Matlab for neural output, is presented in figure 5.

2. Neural network training. The process of training a neural network is to match to each ML_Input element the value of the TVM_Output matrix corresponding to the mapping in the value

Variables - dataInp_1

dataInp_1

5x10 double

	1	2	3	4	5	6	7	8	9	10
1	123654	635345	135604	125149	82954	18335	20700	99954	119200	26509
2	124546	640234	149526	122305	84201	23654	20700	11048	113405	28079
3	154321	643121	142989	120203	85601	15187	20700	75419	114036	27517
4	142352	639432	145902	129785	84302	22443	20700	98621	114483	28153
5	124567	645354	140923	127115	85106	22950	20700	121815	118207	27546

Figure 4: DataInp matrix view.

Variables - dataout_1

dataInp_1 dataout_1

5x10 double

	1	2	3	4	5	6	7	8	9	10
1	124546	640234	149526	122305	84201	23654	20700	11048	113405	28079
2	154321	643121	142989	120203	85601	15187	20700	75419	114036	27517
3	142352	639432	145902	129785	84302	22443	20700	98621	114483	28153
4	124567	645354	140923	127115	85106	22950	20700	121815	118207	27546
5	173876	647811	142989	121069	83241	23154	20650	97103	119905	27561

Figure 5: DataOut matrix view.

of the elements of the weight matrix w_j:

$$w_j \; : \; ML_{Input} \rightarrow TVM_{Output}. \tag{5}$$

In the process of training the neural network, the task of minimizing the objective function is solved. With this approach, an algorithm is used for training, which is the most efficient not only in terms of errors, but also in time. The neural network in Matlab is trained using the Levenberg-Marquardt error backpropagation algorithm. The Levenberg-Marquardt algorithm uses a scalable conjugate gradient backpropagation. Therefore, the training of the neural network is represented in time, and the network is tuned in accordance with its error. The magnitude parameter is used to measure the generalization of the neural network and stop learning when the generalization stops improving. The test score itself does not affect training and provides an independent assessment of the performance of the neural network during and after training. The choice of the algorithm, as well as the learning process of the neural network, is shown in figure 6 and 7.

The number of neural network training epochs can be written as follows:

$$epochs = \left| \frac{t_k - t_0}{\Delta} \right|. \tag{6}$$

In the process of forecasting by a neural network, it is necessary to take into account the forecasting horizon. In the Matlab system, the sim (net,[;;]) function is implemented, which allows you to supply a variety of input values and get a solution at the output of a neural network. The forecast results for the sale of 10 goods are shown in figure 8.

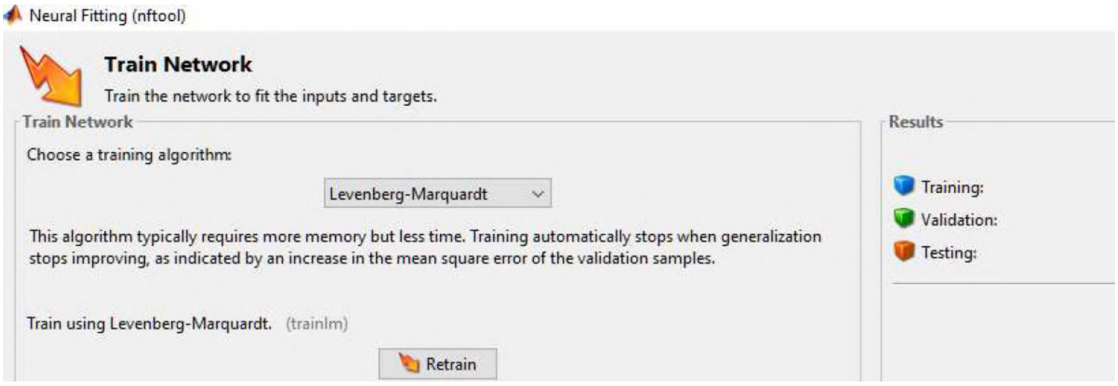

Figure 6: The choice of the learning algorithm.

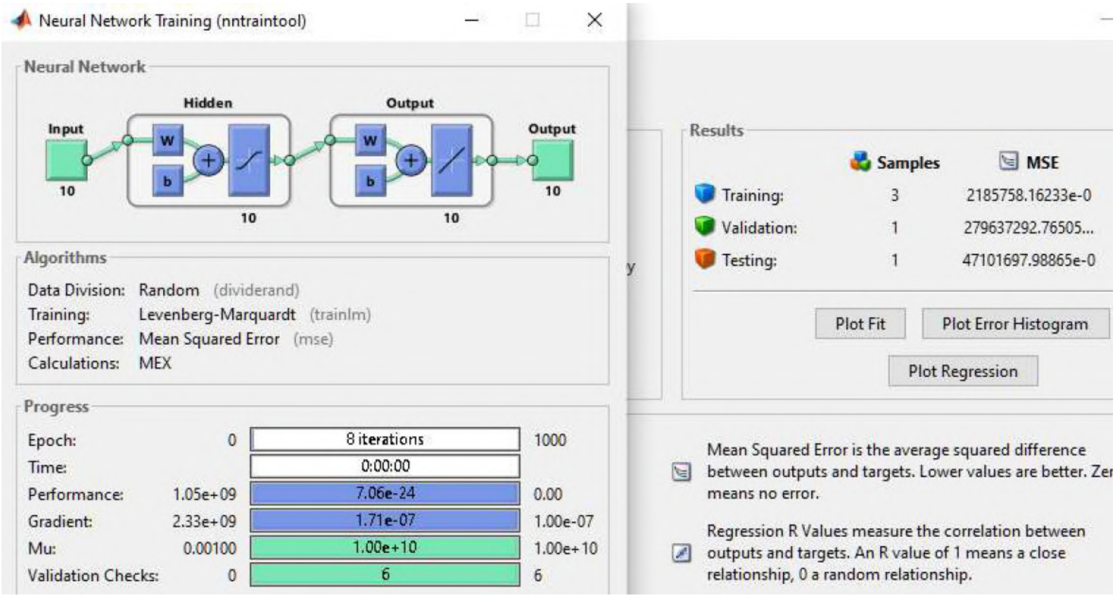

Figure 7: The process of learning the neural network.

Therefore, the created neural network does indeed make multiple predictive decisions. It allows you to solve the sales function of marketing and consider the dynamics of the sale of many products in real time. Marketing forecasting looks at the number of future periods that the forecast will cover. That is, you may need a forecast 7 days ahead, with data for every day. In this case, the period is a day, and the horizon is 7 days. Finally, the prediction interval is the frequency with which a new prediction is made. Often the prediction interval coincides with the prediction period. The choice of the forecasting period and horizon is usually dictated by the conditions for making marketing decisions. Choosing these two parameters is one of the hardest parts of marketing forecasting. For forecasting to be meaningful, the forecasting horizon must be no less than the time required to implement the decision made on the basis of

Figure 8: The result of forecasting the sale of 10 products.

the forecast.

Thus, forecasting is highly dependent on the nature of the decision being made.

4. Conclusion

It is known that making management decisions in digital marketing, taking into account the time of receipt and a large amount of information are an urgent task today.

In addition, in the article, the authors consider the use of Big Data as a tool to increase the data transfer speed while providing access to multidimensional data (OLAP).

The article proposes the structure of the marketing analytics system. It includes many databases, where transactions are processed in real time. Consequently, online data source systems (ODS) provide information for processing in OLTP. This ensures prompt processing of information in real time. For marketing forecasting of multidimensional data, a neural network in Matlab is built. To solve the problem of improving forecasts, the authors have proposed building input data matrices for presentation in a neural network and target data matrices that determine the output statistical information.

Also in the paper, the results of the neural network training process in Matlab are represented on a chosen learning algorithm. The obtained forecasting results allow us to draw a conclusion about the advantages of a neural network in multivariate forecasting in real time. Multidimensional data and the level of their detail are important to solving the problem of forecasting in real time.

The wider use of digital-marketing systems gives an opportunity for applying the proposed approach to marketing forecasting.

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Fuzzy model for complex risk assessment of an enterprise investment project

Inna I. Chaikovska[1,2], Pavlo M. Hryhoruk[2] and Maksym Yu. Chaikovskyi[2]

[1]Leonid Yuzkov Khmelnytskyi University of Management and Law, 8 Heroiv Maidanu str., Khmelnytskyi, 29000, Ukraine
[2]Khmelnytskyi National University, 11 Instytutska Str., Khmelnytskyi, 29016, Ukraine

Abstract
The article proposes an economic-mathematical model for determining a comprehensive risk assessment of the investment project of the enterprise which are based on the approaches of A. Nedosekin. The model is built using fuzzy logic and takes into account the probability of occurrence of each of the identified risks and the level of impact of each of them on the project. The probability of risk is set by experts in the form of points and converted into linguistic terms, and the level of influence of each of them on the project – the ratio of benefits and is determined using Fishburne scales. The proposed Project Risk Model consists of the following stages: formation of initial data using expert opinions; construction of a hierarchical project risk tree; determination of weight coefficients (Fishburne weights) of project risks; selection and description of membership function and linguistic variables; conversion of input data provided by experts from a score scale into linguistic terms; recognition of qualitative input data on a linguistic scale; determination of a complex indicator of investment project risks; interpretation of a complex indicator. The developed model allows managing the risks of the project to maximize the probability of its successful implementation, to compare alternative projects and choose less risky, to minimize the level of unforeseen costs of the project.

Keywords
fuzzy model, complex risk assessment, enterprise investment project

1. Introduction

1.1. Problem description

The following subsystems are distinguished in the project management system: time management [1], labor resources, cost, information and communications, quality, project risks, etc. Project risk management is one of the most important subsystems of project management because it allows at the planning stage of the project to identify problematic issues for its successful implementation. The comprehensive risk assessment of the project allows you to take

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inna.chaikovska@gmail.com (I. I. Chaikovska); violete@ukr.net (P. M. Hryhoruk); larumlab@gmail.com (M. Yu. Chaikovskyi)
https://scholar.google.com/citations?user=PL1fn8IAAAAJ (I. I. Chaikovska);
http://asme.khnu.km.ua/sklad-kafedry/grygoruk-p-m/ (P. M. Hryhoruk)
0000-0001-7482-1010 (I. I. Chaikovska); 0000-0002-2732-5038 (P. M. Hryhoruk); 0000-0002-9596-6697 (M. Yu. Chaikovskyi)

into account the most significant risks for the project and quantify them to make an effectively-informed management decision. Quite often when assessing the risks of enterprise projects, the information is non-numerical, it is necessary to take into account both quantitative and qualitative information in one system and the formation of a single quantitative comprehensive indicator. To form a comprehensive risk assessment of the project, it is advisable to use fuzzy logic and obtain a comprehensive numerical risk assessment of the project.

If we consider the methods of risk identification, there will be many approaches only to the classification of risks. According to the common in foreign countries approach Construction Risk Management System (CRMS), proposed by American analysts, the process of risk identification consists of six stages: detection of uncertainties; compiling a preliminary checklist; consequence scenarios; a reflection of risks; systematic list of risks; total risk. Therefore, determining the total risk (complex value of project risks) is an urgent task. Besides, a comprehensive assessment should take into account both the probability of a risk event and the level of its impact on project implementation (weights). The obtained comprehensive assessment will determine whether it is appropriate to implement the project, what risks are most likely, and the level of impact on the successful implementation of the project, reduce the cost of the project, as well as make an effectively-informed management decision [2].

1.2. Literature review

Peskova et al. [3] identifies the need for effective methods of risk management in enterprises in modern conditions because the profitability of the enterprise largely depends on the level of risk. Also, risk allows you to assess the internal reserves of the enterprise and the level of risk depends on the feasibility of a particular financial and economic transaction. Risk management makes it possible to establish forecast quantitative estimates of economic performance of the enterprise.

Waszkiewicz and Grzeszczyk [4] states that in modern conditions it is necessary to use not only general methods of evaluation of investment projects. The use of various methods of evaluation of investment projects in one system will allow to make an informed management decision on the feasibility of their further implementation and ensure their flexibility.

Shchur et al. [5] specifies that an investment project is a set of measures, works, and documents, the financial result of which is profit (income). The material result of the investment project is new or reconstructed fixed assets (objects). The project may also result in the acquisition and use of financial instruments, intangible assets followed by income or social impact. An investment project is an activity that involves the implementation of any action to achieve specific goals.

Bogomolova [6] emphasizes that risk management requires a systematic assessment of the severity of risks affecting the project. The article notes the need to use different qualitative and quantitative methods of risk assessment of investment projects and their distinctive features, advantages, and disadvantages. The necessity of choosing the most expedient method of risk assessment for a specific project is determined.

Zhang and Yang [7] assesses the risks to real estate with the construction of a fuzzy mathematical model. The authors emphasize the need to use several methods of quantitative risk assessment due to a certain subjectivity of the developed model.

Voronov et al. [8] considers the CAPM model for emerging capital markets and the DCF method, which allows assessing the attractiveness of both the business in general and the investment project of the enterprise in particular. The application of risk assessment methods to an investment project helps to assess its feasibility, the period when it will start to make a profit, and its level in the future.

Nizamova et al. [9] offered to use a method of expert estimations for estimation of risks of the investment project. This approach allows you to quantify the risks of the project, rank them and obtain a comprehensive indicator of risk assessment of the investment project.

Galevskiy [10] proposes a methodology that modifies the capital pricing model (CAPM) using the discounted cash flow method. This approach allows you to assess the effectiveness of investment projects, risks, and management based on the information obtained.

Bayguzina et al. [11] analyzes the investment project for the construction of a greenhouse complex for growing vegetables. To achieve the planned key economic indicators of the project, it is necessary to reduce the negative impact of risk factors. Since the project did not provide for the analysis of project risks, namely there was no system of identified risks, qualitative and quantitative risk assessment, sensitivity analysis, break-even point, etc., the authors note the need for project risk management, namely mandatory funding conditions for project analysis risks.

Griffis and Whipple [12] proposes a mechanism for assessing and prioritizing risks. This will allow managers to make informed management decisions regarding risk prevention and speed up the recovery time from real risks.

Vitlinskyi and Glushchevsky [13] offered to consider risk in system stratification metamodeling system. The authors propose to increase the investment attractiveness, profitability, and competitiveness of the enterprise by reducing various types of costs by implementing a BPM-system based on the proposed stratification metamodeling system. A mandatory component of this system is a subsystem that takes into account the risk.

Efimova and Koroleva [14] presents a model for assessing the risks of investment projects using Bayesian networks. The application of this approach is explained by the presence of different types of uncertainty and the need to formalize and process information taking into account uncertainty.

Chong [15] examines international risks and their impact on the implementation of the investment project. The authors compare the traditional NPV model and the NPV model taking into account the weight of entropy as a risk assessment of the investment project. The second model showed the best results and is offered as a basis for decision making.

Yang et al. [16] uses a comprehensive assessment based on fuzzy logic to assess the security risk of an energy investment project. The model includes the definition of the factor characteristics of the object of evaluation, the establishment of a set of estimates, the establishment of a relationship matrix, the calculation of the weight of the index, fuzzy complex assessment. The practical implementation of the model demonstrated a risk value of 3.1154 (average risk).

Three models were developed by Mousalami [17] to accurately predict the quality of project planning, based on both deterministic and fuzzy concepts, and the results show that the fuzzy model is more accurate and realistic than the deterministic one. Thus, the correct use of fuzzy theory will develop more accurate, realistic, and reliable models than deterministic ones.

Despite the significant achievements of scientists in the direction of risk assessment of investment projects of the enterprise, the further study requires a comprehensive assessment of investment risks using economic and mathematical modeling, namely fuzzy logic. After all, it is necessary to take into account both the probability of occurrence of each of the risks and their weights, the ability to take into account both qualitative and quantitative information in one system, comparing several alternative projects. This is necessary to manage the investment risks of the project, minimize unplanned costs and increase the level of competitiveness of the enterprise. In this regard, it is proposed to conduct a comprehensive risk assessment of the investment project using fuzzy set theory according to the methodology of determining a comprehensive assessment of the risk of bankruptcy of the enterprise Nedosekin [18], which is adapted to determine a comprehensive risk assessment of the investment project.

Fuzzy logic and fuzzy set theory have received a wide range of successful practical applications in various spheres of life. For example, Hryhoruk et al. [19] proposed to choose a solution in the presence of a large number of efficiency criteria based on fuzzy preference relations. This approach allows you to form a set of alternatives based on selected performance criteria, choose the best and make an effective management decision.

In the paper [20], economic and mathematical models for diagnosing the bankruptcy of the enterprise using the methods of fuzzy logic and developed a comprehensive analysis of the financial condition of the enterprise. Also, the application of fuzzy logic in the financial sector is presented by Diaz et al. [21].

Special attention in today's conditions deserves research, which reflects the use of fuzzy logic to stabilize the epidemiological situation with COVID-19 [22].

1.3. The aim and objectives of the research

The study aims to develop an economic-mathematical model for determining a comprehensive risk assessment of an investment project based on fuzzy logic, taking into account the probability of each of the identified risks and the level of impact of each of them on the project.

To achieve this goal the following tasks were solved:

1) set the task and develop a descriptive model for determining a comprehensive risk assessment of the project;
2) to build a mathematical model of the problem of determining a comprehensive risk assessment of the project and to develop an algorithm for its solution;
3) generate input data using the opinions of experts;
4) build a hierarchical project risk tree;
5) determine the weights (Fishburn weights) of project risks;
6) select and describe membership functions and linguistic variables;
7) convert the initial data provided by experts from a score scale into linguistic terms;
8) to recognize qualitative input data on a linguistic scale;
9) determine a comprehensive risk indicator of the investment project;
10) to analyze the obtained complex indicator.

2. Research methodology

The proposed model can be implemented with the following input data:

- a set of expert assessments of the probability of project risks;
- hierarchy of existing project risks (a hierarchical tree of logical conclusion);
- system of relations of advantages of some risks over others (for one level of hierarchy).

The initial data of the model are:

- comprehensive quantitative risk assessment of the project;
- interpretation of the obtained complex project risk indicator.

The model involves the use of elements of fuzzy logic (logical inference tree, membership function, linguistic terms). Expert knowledge and the Fishburn method of scales were also used to determine the project risk weights.

Problem statement and development of a descriptive model for determining a comprehensive risk assessment of the project.

The task of determining a comprehensive risk assessment of the project within this study is implemented by a construction company, which assesses the risks of a new investment project for the construction of a residential complex. The experts identified the following types of risk, which are typical for the project of residential complex construction, and their components that affect the results of the project and selected for the formation of a comprehensive assessment (R_0):

1. Technical R_1 (risks of reassessment of project sustainability $R_{1.1}$; risks associated with the reassessment of additional opportunities for project development $R_{1.2}$).
2. External R_2 (risks of incorrect assessment of demand for the project $R_{2.1}$; risks associated with the nature of competition in the market $R_{2.2}$; risks associated with the solvency of the customer $R_{2.3}$; risks of the uncertainty of the external environment of the project $R_{2.4}$).
3. Organizational R_3 (risks for estimating the costs of project commercialization $R_{3.1}$; risks of potential losses from project implementation $R_{3.2}$; risks of underestimation of project development costs $R_{3.3}$; risks of the uncertainty of the internal project environment $R_{3.4}$).

Each of the risks has its probability of occurrence and level of impact on the successful implementation of the project.

It is necessary to determine a comprehensive indicator of the risk level of the project to make a management decision on the feasibility of this investment project and the necessary actions to increase the probability of successful project implementation, taking into account existing risks.

Mathematical model of the problem of determining a comprehensive risk assessment of the project and the algorithm for its solution.

In this situation, for a comprehensive assessment of project risks, it is advisable to consider the PRM model (Project Risk Model):

$$PRM =< F, A, R >, \tag{1}$$

where F – the hierarchy of existing project risks (a hierarchical tree of logical conclusion);

A – a set of qualitative assessments of each factor in the hierarchy (linguistic terms);

R – a system of relations of advantages of some risks over others (for one level of hierarchy).

In this case:

$$A = ((L), (LM), (M), (HM), (H)), \tag{2}$$

where L - Low, LM – LowMedium, M – Medium, HM – HighMedium, H – High.

$$R = Ri(r)Rj|r \in (\succ, \approx), \tag{3}$$

where $\succ$ – the ratio of preference;

where $\approx$ – equilibrium ratio.

The proposed model consists of the following stages:

Stage 1. Formation of initial data with the use of expert opinions.

Experts estimate the probability of risks using a scale from 0 to 100 points. For each indicator, points are added and the average is determined C_i:

$$C_i = \frac{\sum_{i=1}^{n} C_{ij}}{N}, \tag{4}$$

where N – the number of interviewed experts;

C_{ij} – the sum of points for each indicator.

Next, the concordance coefficient is determined by the formula:

$$W = \frac{\sigma_f^2}{\sigma_{max}^2} = \frac{\sum_{i=1}^{m} \left(a_i - \frac{1}{2} n \cdot (m+1)\right)^2}{\frac{1}{12} n^2 \cdot m \cdot (m^2 - 1)}, \tag{5}$$

where σ_f^2 – the actual variance (SD) of the final (ordered, ranked) estimates provided by experts;

σ_{max}^2 – dispersion of final (ordered) assessments, provided that the opinions of experts completely coincide;

a_i – the total estimate obtained by the i-th object;

m – the number of studied objects;

n – the number of experts.

The materiality of the concordance coefficient is checked using χ^2 with $(m-1)$ the number of degrees of freedom. Statistical characteristics are calculated by the formula:

$$\chi^2 = W \cdot n(m-1), \tag{6}$$

Stage 2. Construction of a hierarchical risk tree of the project using the system of relations of advantages.

Stage 3. Determination of weights (Fishburne weights). For a system of declining benefits N alternatives:

$$p_i = \frac{2(N - i + 1)}{(N+1)N}, \qquad i = 1..N. \tag{7}$$

A system of equivalent N alternatives – a set of identical weights:

$$p_i = N^{-1}, \qquad i = 1..N. \tag{8}$$

Stage 4. Selection and description of membership function and linguistic variables. Selected triangular membership function:

$$\mu(x) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \\ 0, & c \le x \end{cases}, \tag{9}$$

where $\mu(x)$ – the membership function of linguistic terms (0 – does not belong, 1 – belongs to all 100%);

a, b, c are some numerical parameters that take arbitrary actual values and are ordered by the relation $a \le b \le c$. Parameters a and c characterize the base of the triangle, and parameter b is its vertex.

Stage 5. Transformation of the initial data provided by experts on the probability of occurrence of each of the risks, from a score scale to linguistic terms. The obtained average scores of the probability of occurrence of each of the risks are translated into linguistic terms according to the selected membership function.

Stage 6. Recognition of qualitative input data on a linguistic scale:

$$Z^*(a) = (\mu_1^*(a), \mu_2^*(a), \mu_3^*(a), \mu_4^*(a), \mu_5^*(a)), \tag{10}$$

where a – the value of the factor to be recognized;

$\mu_i^*(a)$ is membership function with linguistic terms i;

$\mu_i^*(a)$ is determined by the formula (9).

Stage 7. Determination of a complex indicator.

First, you need to convert all vectors $Z^*(x^*)$ in the hierarchy F with weight P according to the formula:

$$\sum_{i=0}^{N} p_i \cdot (\mu_{i\cdot1}, \mu_{i\cdot2}, \mu_{i\cdot3}, \mu_{i\cdot4}, \mu_{i\cdot5}) = (\sum_{i=1}^{N} p_i \cdot \mu_{i\cdot1}, \sum_{i=1}^{N} p_i \cdot \mu_{i\cdot2}, \sum_{i=1}^{N} p_i \cdot \mu_{i\cdot3} \sum_{i=1}^{N} p_i \cdot \mu_{i\cdot4}, \sum_{i=1}^{N} p_i \cdot \mu_{i\cdot5}) \tag{11}$$

where i – possible options for determining the membership function for each linguistic term.

Next, the vector can be determined, which will characterize the complex risk assessment of the project (taking into account the selected triangular membership function):

$$A\_N = \sum_{i=1}^{5} (0.25i - 0.25) \cdot \mu_{0i}, \tag{12}$$

where ($0.25i$–0,25) = (0; 0.25; 0.5; 0.75; 1) – nodal points of the triangular membership function, in which it is equal to 1 on a scale from 0 to 1 (nodal points at which the membership function refers to a certain linguistic term for 100 %) according to the formula (14 – 18) and figure 2.

Stage 8. Interpretation of the obtained complex indicator.

3. Results and discussion

Consider the implementation of the model according to the problem.

Table 1
Expert assessment of the probability of occurrence of each of the risks of the project in points

Risk	Ex1	Ex2	Ex3	Ex4	Ex5	Ex6	Ex7	Ex8	Ex9	Ex10	Values sum	Average value
$R_{1.1}$	40	45	55	60	50	60	40	55	45	60	510	51.0
$R_{1.2}$	60	65	75	70	80	70	75	65	55	60	675	67.5
$R_{2.1}$	15	20	20	30	30	35	40	45	45	35	315	31.5
$R_{2.2}$	60	80	75	65	85	100	85	90	95	100	835	83.5
$R_{2.3}$	10	15	15	25	40	35	30	45	25	15	255	25.5
$R_{2.4}$	60	45	55	50	60	40	40	55	60	70	535	53.5
$R_{3.1}$	70	80	90	60	55	85	70	65	60	80	715	71.5
$R_{3.2}$	55	60	60	45	80	65	45	75	80	85	650	65.0
$R_{3.3}$	25	15	15	10	20	30	35	15	40	40	245	24.5
$R_{3.4}$	95	90	100	70	80	65	55	85	90	100	830	83.0

Stage 1. Formation of initial data with the use of expert opinions.

Table 1 provides information provided by experts regarding the assessment of the probability of occurrence of each of the selected risks of the project on a scale from 0 to 100 points (ascending).

The concordance coefficient according to formula (5) is 0.7048, which indicates that the opinions of experts are consistent.

Checking the materiality of the concordance coefficient according to the formula (6) $\chi^2 = 63.44$.

The data in table (χ^2) for (10-1) degrees of freedom and confidence probability (= 0.95, = 0.99, = 0.999) show that the calculated value of the Pearson criterion χ^2 is greater than the tabular (respectively 16.92; 21.67, and 27.88), which confirms the conclusion that experts agree.

Stage 2. Construction of a hierarchical tree of risks of the project using the system of relations of advantages.

The experts provided the following information on the risk-benefit ratios (figure 1).

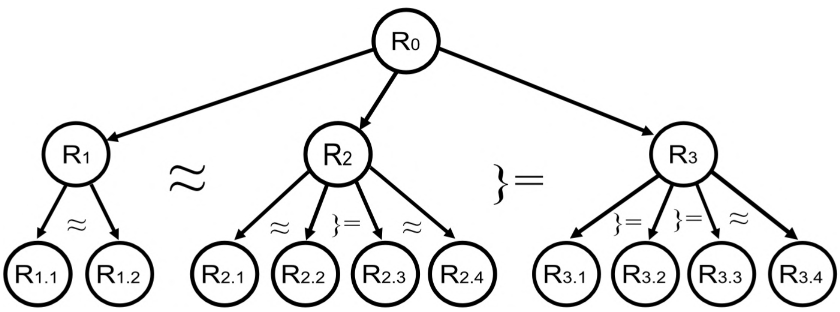

Figure 1: Hierarchical risk tree of the project with the indication of the system of relations of advantages.

Figure 1 corresponds to the system of relations R (formula 13).

$$R = (R_1 \approx R_2 \succ R_3; R_{1.1} \approx R_{1.2}; R_{2.1} \approx R_{2.2} \succ R_{2.3} \approx R_{2.4}; R_{3.1} \succ R_{3.2} \succ R_{3.3} \approx R_{3.4} \tag{13}$$

The obtained system of preference ratios can be used to determine the risk weights of Fishburne weights.

Stage 3. Determination of weights (Fishburne weights)

According to formulas (7) and (8), as well as the system of preference relations (figure 1) formed a system of Fisher scales (table 2).

Table 2
Fishburne system of scales

№	R	p_1	p_2	p_3	p_4
2	$R_{1.1} \approx R_{1.2}$	1/2	1/2	-	-
3	$R_1 \approx R_2 \succ R_3$	2/5	2/5	1/5	-
4	$R_{2.1} \approx R_{2.2} \succ R_{2.3} \approx R_{2.4}$	2/6	2/6	1/6	1/6
4	$R_{3.1} \succ R_{3.2} \succ R_{3.3} \approx R_{3.4}$	3/7	2/7	1/7	1/7

After convolving the weights of different levels of the hierarchy, the following generalized weights (level of influence) were obtained: $R_{1.1} = 0.2000$; $R_{1.2} = 0.2000$; $R_{2.1} = 0.1333$; $R_{2.2} = 0.1333$; $R_{2.3} = 0.0667$; $R_{2.4} = 0.0667$; $R_{3.1} = 0.0857$; $R_{3.2} = 0.0571$; $R_{3.3} = 0.0286$; $R_{3.4} = 0.0286$.

The highest weights are in risks $R_{1.1}$, $R_{1.2}$, the lowest in $R_{3.3}$, $R_{3.4}$.

Stage 4. Selection and description of membership function and linguistic variables

The linguistic variable "Risk Level" was formed with a term set of values of A (formula 2). The triangular membership function (figure 2) with the following linguistic terms was chosen as membership functions: Low (L), Low Medium (LM), Medium (M), High Medium (HM), High (H), distributed on a scale from 0 to 100 points.

The following system of equations corresponds to this membership function with linguistic terms:

$$L : \mu_1(x) = \begin{cases} \dfrac{25 - x}{25}, & 0 \leq x \leq 25 \\ 0, & 25 \leq x \end{cases} . \tag{14}$$

$$LM : \mu_2(x) = \begin{cases} \dfrac{x}{25}, & 0 \leq x \leq 25 \\ \dfrac{50 - x}{25}, & 25 \leq x \leq 50 \\ 0, & 50 \leq x \end{cases} \cdot \tag{15}$$

$$M : \mu_3(x) = \begin{cases} 0, & x \leq 25 \\ \dfrac{x - 25}{25}, & 25 \leq x \leq 50 \\ \dfrac{75 - x}{25}, & 50 \leq x \leq 75 \\ 0, & 75 \leq x \end{cases} . \tag{16}$$

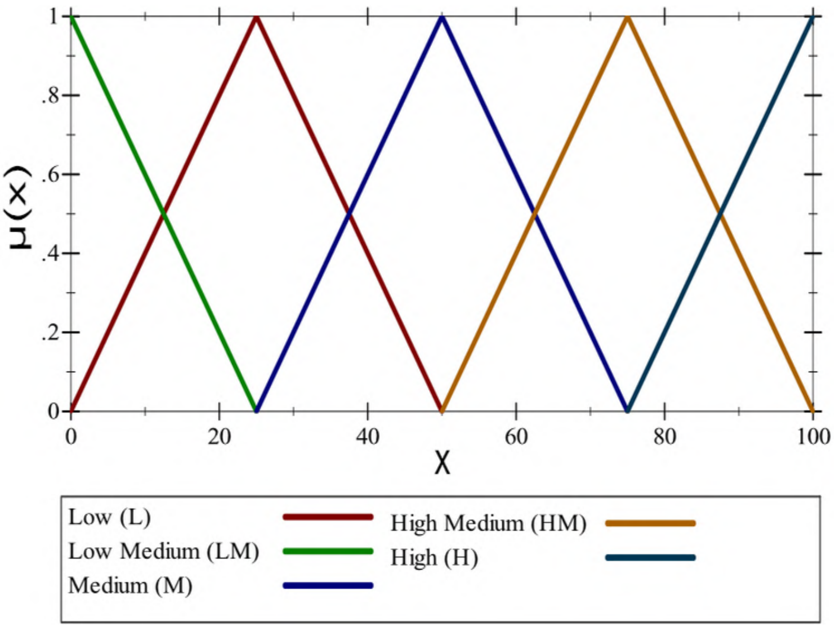

Figure 2: Triangular membership functions with linguistic terms Low (L), Low Medium (LM), Medium (M), High Medium (HM), High (H).

$$HM : \mu_4(x) = \begin{cases} 0, & x \leq 50 \\ \dfrac{x-50}{25}, & 50 \leq x \leq 75 \\ \dfrac{100-x}{25}, & 75 \leq x \leq 100 \end{cases} . \tag{17}$$

$$H : \mu_5(x) = \begin{cases} 0, & x \leq 75 \\ \dfrac{x-75}{25}, & 75 \leq x \leq 100 \end{cases} . \tag{18}$$

Formulas (14-18) are based on formula 9 and figure 2.

Stage 5. Transformation of the initial data provided by experts on the probability of occurrence of each of the risks, from the score scale to the linguistic terms.

The scores of the probability of occurrence of each of the risks from table 1 are translated into linguistic terms (according to figure 2): Low (L), Low Medium (LM), Medium (M), High Medium (HM), High (H) (table 3).

According to the table, it can be seen that among the risks are risks, both with a low probability of occurrence and with a very high probability. The indication of two linguistic terms for risk indicates that the obtained average scores of experts are on the border of two terms.

Stage 6. Recognition of qualitative input data on a linguistic scale

According to formulas (14-18) the input data was recognized according to the linguistic scale (table 4).

In the table, the cells corresponding to the values of linguistic variables obtained using risk

Table 3
Risks and their levels in linguistic terms

Risk number	Risk designation	Name of risk	Risk level (probability of occurrence)
1	$R_{1.1}$	Risks of reassessment of project sustainability	Medium, High Medium
2	$R_{1.2}$	Risks associated with the reassessment of additional project development opportunities	Medium, High Medium
3	$R_{2.1}$	Risks of incorrect assessment of demand for the project	Low Medium, Medium
4	$R_{2.2}$	Risks associated with the nature of competition in the market	High Medium, High
5	$R_{2.3}$	Risks associated with the solvency of the customer	Low Medium, Medium
6	$R_{2.4}$	Risks of the uncertainty of the external environment of the project	Medium, High Medium
7	$R_{3.1}$	Risks for estimating the costs of project commercialization	Medium, High Medium
8	$R_{3.2}$	Risks of potential losses from project implementation	Medium, High Medium
9	$R_{3.3}$	Risks of underestimation of project development costs	Low, Low Medium
10	$R_{3.4}$	Risks of the uncertainty of the internal environment of the project	High Medium, High

Table 4
Matrix of the actual distribution of values by fuzzy sets

Risk	Weights (level of influence)	x	Membership functions μ				
			Low ($\mu_1(x)$)	Low Medium ($\mu_2(x)$)	Medium ($\mu_3(x)$)	High Medium ($\mu_4(x)$)	High ($\mu_5(x)$)
$R_{1.1}$	0.2000	5.01	0	0	0.96	0.04	0
$R_{1.2}$	0.2000	67.5	0	0	0.30	0.70	0
$R_{2.1}$	0.1333	31.5	0	0.74	0.36	0	0
$R_{2.2}$	0.1333	83.5	0	0	0	0.66	0.34
$R_{2.3}$	0.0667	25.5	0	0.98	0.02	0	0
$R_{2.4}$	0.0667	53.5	0	0	0.86	0.14	0
$R_{3.1}$	0.0857	71.5	0	0	0.14	0.86	0
$R_{3.2}$	0.0571	65.0	0	0	0.40	0.60	0
$R_{3.3}$	0.0286	24.5	0.02	0.98	0	0	0
$R_{3.4}$	0.0286	83.0	0	0	0	0.68	0.32

analysis models are given the recognized value according to formulas (14-18). In other cells "0" is put.

Thus, the probability of risk R1.1 in accordance with expert opinions and linguistic terms refers to the level of Medium (96%), High Medium (4%); $R_{1.2}$ – Medium (30%), High Medium (70%); $R_{2.1}$ – Low Medium (74%), Medium (26%); $R_{2.2}$ – High Medium (66%), High (34%); $R_{2.3}$ – Low Medium (98%), Medium (2%); $R_{2.4}$ – Medium (86%), High Medium (14%); $R_{3.1}$ – Medium (14%), High Medium (86%); $R_{3.2}$ – Medium (40%), High Medium (60%); $R_{3.3}$ – Low (2%), Low Medium (98%); $R_{3.4}$ – High Medium (68%), High (32%).

Stage 7. Determination of a complex indicator

At the next stage, a comprehensive risk indicator of the investment project is determined, based on fuzzy sets. According to formula (11) found the vector:

$$\sum_{i=0}^{N} p_i \cdot (\mu_{i \cdot 1}, \mu_{i \cdot 2}, \mu_{i \cdot 3}, \mu_{i \cdot 4}, \mu_{i \cdot 5}) =$$
$$= (0.0006; 0.1920; 0.3802; 0.3727; 0.0545).$$

But according to the selected membership function, the nodal points in which the membership function is 1 are equal to (0; 25; 50; 75; 100). To find the integral exponent, use formula (12) and multiply the corresponding exponents of both vectors and find their sum.

$$A\_N = 51.8194 = R_0.$$

Thus, the complex risk indicator of the investment project is 51.8194 points.

Stage 8. Interpretation of a complex indicator.

According to formulas (14) – (18) and figure 2, the value of the complex risk indicator is average (93%) on the border of above average (7%). $R_{1.1}$ and $R_{1.2}$ risk have the greatest impact (according to weighting factors) (figure 3).

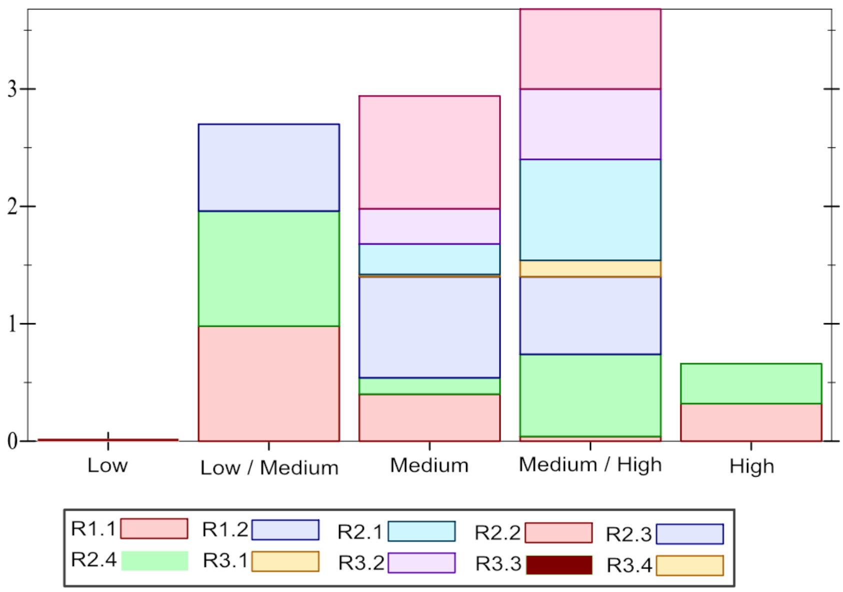

Figure 3: Components of a complex indicator of investment project risks and their compliance with linguistic terms.

According to figure 3, the highest risks belong to the level of High Medium, in second place the level of Medium, and in third place – Low Medium.

Figure 4 shows a risk map of the project (according to table 4). The y-axis reflects the probability of risk (in ascending order), and the x-axis shows the risk weight (in ascending order). The bold line shows the critical limit of the level of risk. Those risks that are above this

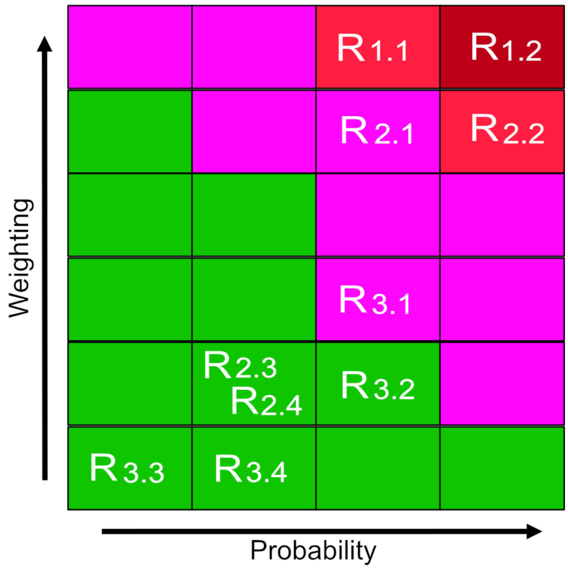

Figure 4: Risk map of the investment project.

line are critical for the project and require a priority management decision to transfer them from the critical (red-purple) zone to the green zone.

Therefore, the most critical for the project are risks $R_{1.2}$ (risks associated with the reassessment of additional opportunities for project development (errors in the assessment of alternative technologies and choice of technology and equipment for the project, failure to develop project capacity), risk $R_{1.1}$ project sustainability: risk of confidence that the new project is guaranteed success due to its unique qualities, even when imitating it, risk of confidence in the company's potential for exclusive cooperation), $R_{2.2}$ (risks associated with the nature of competition in the market, namely risks strong competitive influence in the target markets of the enterprise), $R_{2.1}$ (risks of incorrect estimation of demand for the project), $R_{3.1}$ (risks for estimation of expenses of commercialization of the project). To translate them into an acceptable green zone requires in-depth comprehensive marketing research, development, and implementation of additional measures to ensure the quality of raw materials; application of sanctions to suppliers, up to replacement of equipment; staff training during construction; search and implementation of reserves to reduce production costs; transition to alternative sources; reduction of the share of imported materials and spare parts due to the maximum use of domestic, primarily local, etc.

This approach to obtaining a comprehensive assessment should also be used to select one of several alternative projects because the best will be the one where the overall risk assessment of the project is less.

If we compare the obtained complex indicator with the complex indicator obtained by ordinary convolution (the sum of weights per indicator level), ie the average scores from table 1 and the corresponding weights, the complex indicator will be 57.2123. Thus, it is possible to notice the difference in obtaining a complex indicator by different methods, but the application of fuzzy logic allows to work with both quantitative and qualitative input indicators and to use easy-to-understand linguistic terms.

4. Conclusions

1. The task of determining a comprehensive risk assessment of the project within this study is implemented by a construction company, which assesses the risks of a new investment project for the construction of a residential complex. The experts identified the types of risk and their components that affect the results of the project and selected for the formation of a comprehensive assessment. These include technical (risks of reassessment of project sustainability; risks associated with the reassessment of additional opportunities for project development), external (risks of incorrect assessment of project demand; risks associated with the nature of competition in the market; risks associated with the solvency of the customer; risks of the uncertainty of the external environment of the project), organizational (risks of estimating the costs of project commercialization; risks of potential losses from project implementation; risks of underestimation of project development costs; risks of the uncertainty of the internal environment of the project).
2. The mathematical model of the problem of determining a comprehensive assessment of project risks consists of the following components: hierarchy of existing project risks (a hierarchical tree of logical conclusion); a set of qualitative assessments of each factor in the hierarchy (linguistic terms); system of relations of advantages of some risks over others (for one level of hierarchy). The proposed Project Risk Model consists of the following stages: formation of initial data using expert opinions; construction of a hierarchical project risk tree; determination of weighting factors (Fishburne weights) of project risks; selection and description of membership function and linguistic variables; conversion of input data provided by experts from a score scale into linguistic terms; recognition of qualitative input data on a linguistic scale; determination of a complex indicator of investment project risks; interpretation of a complex indicator.
3. Initial data were formed using the opinions of experts: the experts provided an estimate of the probability of occurrence of each of the selected risks of the project on a scale from 0 to 100 points (ascending). The risks related to the nature of competition in the market and the risks of uncertainty in the internal environment of the project are most likely.
4. A hierarchical tree of risks of the project is constructed with the indication of the system of relations of advantages that gives the chance to form weighting factors using Fishburne weights.
5. According to the system of relations of preferences, Fisher's system of weights is formed. The highest weights are the risks associated with the revaluation of additional project development opportunities and the risks of revaluation of project sustainability.
6. The linguistic variable "Level of risk" with the term set of values A was formed. The triangular membership function with the following linguistic terms was chosen as membership functions: Low (L), Low Medium (LM), Medium (M), High Medium (HM), High (H), distributed on a scale from 0 to 100 points.
7. The initial data provided by experts on the probability of occurrence of each of the risks were converted from a score scale to the linguistic terms Low (L), Low Medium (LM), Medium (M), High Medium (HM), High (H).
8. Recognition of qualitative input data on a linguistic scale is carried out. The highest total value according to the triangular membership function in the linguistic term High

Medium.

9. Determination of a complex indicator of the level of project risks. Its value is 51.8194 points.
10. The analysis of the obtained complex indicator shows that the total level of risk of the investment project is average (93%) on the verge of above average (7%). The constructed risk map shows that the most critical for the project implementation are the risks associated with the reassessment of additional opportunities for project development; risks of reassessment of project sustainability; risks associated with the nature of competition in the market; risks for estimating the costs of project commercialization. To reduce them requires in-depth comprehensive marketing research, development, and implementation of additional measures to ensure the quality of raw materials; application of sanctions to suppliers, up to replacement of equipment; staff training during construction; search and implementation of reserves to reduce production costs; transition to alternative sources; reduction of the share of imported materials and spare parts due to the maximum use of domestic, primarily local, etc.

The presented approach allows determining a comprehensive assessment of the risks of the investment project of the enterprise, which indicates its universality and creates conditions for its acceptability for different enterprises.

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Modeling structural changes in the regional economic development of Ukraine during the COVID-19 pandemic

Pavlo M. Hryhoruk, Nila A. Khrushch and Svitlana S. Grygoruk

Khmelnytskyi National University, 11 Instytutska Str., Khmelnytskyi, 29016, Ukraine

Abstract

The paper investigates the issues of evaluating structural changes in the regions' economic development based on the comprehensive index assessment technology. The impact of the COVID-19 pandemic on regional development and changes in the regional structure is considered. The authors propose the use of block convolution to design a comprehensive index based on a set of metric initial indicators that characterize the regions' economic development. Grouping the set of initial indicators is carried out based on the method of an extreme grouping of parameters and the method of principal components. A weighted linear additive convolution was used to develop partial composite indices and an economic development comprehensive index. The practical approbation was carried out for the regions of Ukraine according to the data of 9 months of 2019 and the same period of 2020. To establish the regions' structure, we used the division of the comprehensive index values into intervals and further distributing regions into classes according to the level of economic development. There is a general decrease in the value of the integrated indicator in 2020, caused by the impact of the COVID-19 pandemic. However, no significant changes in the structure of the regions were detected, which indicates an equally negative impact of the pandemic for all regions of Ukraine.

Keywords

COVID-19 pandemic, block convolution, economic development comprehensive index

1. Introduction

One of the most significant problems of regional development is to ensure sustainable economic growth. The economic system of any country is a multifunctional regional entity, so the definition of long-term priorities of strategic planning of regional development should be based on comprehensive assessments of the level of their economic development. They allow tracking the dynamics and asymmetry of development, to establish inequalities and gaps in the

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violete@ukr.net (P. M. Hryhoruk); nila.ukr@gmail.com (N. A. Khrushch); grygoruk.svitlana@gmail.com (S. S. Grygoruk)
http://asme.khnu.km.ua/sklad-kafedry/grygoruk-p-m/ (P. M. Hryhoruk); https://fbs.khmnu.edu.ua/page.aspx?l=en&r=4&p=101 (N. A. Khrushch); https://scholar.google.com.ua/citations?user=z5mZVpsAAAAJ (S. S. Grygoruk)
0000-0002-2732-5038 (P. M. Hryhoruk); 0000-0002-9930-7023 (N. A. Khrushch); 0000-0003-3047-2271 (S. S. Grygoruk)

region's structure, to provide an analytical basis for the preparation of strategic decisions on the transformation of socio-economic development policy of individual regions.

Global problems related to climate changes, financial crises, intensified competition in global and domestic markets, deepened in 2020 due to another global challenge – the COVID-19 pandemic [1, 2]. Its destructive impact has been reflected in all spheres of public life, destroying established socio-economic processes and relationships. Measures, severe restrictions, lock-downs aimed at curbing the spread of the pandemic, were reflected in the negative effects of slowing down the socio-economic development of both regions and the world economy as a whole. They were a prerequisite for a new financial and economic crisis. This is evidenced by the results of analytical studies and forecast estimates of basic macroeconomic indicators provided by global institutions, in particular, the World Bank (WB), the International Labor Organization (ILO), the World Health Organization (WHO), the United Nations (UN), the European Bank for Reconstruction and Development (EBRD) and others.

In particular, according to the ILO, the loss of labor income for the three quarters of 2020 compared to the corresponding period of 2019 is estimated at 10.7%, or 3.5 trillion USD [3]. The baseline forecast calculated by World Bank analysts [4] predicts a reduction in world GDP by 5.2% in 2020. And although the world economy is expected to grow by 4.3% in 2021, the pandemic may hold back economic activity and income growth for a long time [5]. UBS Chairman Axel Weber also made a cautious forecast about the pace of global economic recovery, noting that "it would be at least a year to go back to pre-crisis levels of GDP. It'll take another year or two to be anywhere near getting unemployment and pre-crisis growth back and so it would be quite a long recovery that we're facing" [6].

The consequences of the pandemic were especially acute in developing economies countries, particularly in Ukraine. Thus, according to the State Statistics Service of Ukraine [7], real GDP in the third quarter of 2020 compared to the third quarter of 2019 decreased by 3.5%. The financial result before taxation of large and medium-sized enterprises in the III quarter of 2020 amounted to 93.3 billion UAH of profit, while for the corresponding period of 2019 – UAH 342.8 billion in profit, which is 73% less. Exports of goods for the period under review decreased by 3.6%, and imports – by 14.3%.

The main forecast macroeconomic indicators for the end of 2020, presented by the Cabinet of Ministers of Ukraine, envisage a fall in GDP by 4.8%, the inflation rate – 11.6%; unemployment rate – 9.4%; reduction of the average salary – 4.5%; decrease in exports – 5.5%, imports – 10% [8]. According to the EBRD, by the end of 2020 GDP was expected to decline by 5.5%, but in 2021 it is predicted to grow by 3% [9]. The most optimistic about the resumption of production are construction companies, the most pessimistic – service companies that have suffered the most from the introduction of quarantine restrictions.

The decline in macroeconomic indicators is directly caused by negative changes in regional development. To reduce the negative socio-economic consequences of the COVID-19 pandemic, it is necessary to identify trends, assess different scenarios of regional development, identify existing structural changes and develop a system of measures within regional development strategies to stabilize the situation. The presented macroeconomic forecasts necessitate research aimed at estimating the real losses from COVID-19 pandemic in terms of socio-economic development of regions, identifying areas of rational use of endogenous factors to ensure their sustainable economic growth, which will contribute to the achievement of the goals reflected in the State Strategy for Regional Development for 2021–2027 [8].

2. Literature review

Currently, there is a large number of different scholar's approaches to assess the economic development level and the establishment of regional differences and imbalances.

These studies are based mainly on the use of quantitatively measurable indicators that allow sound mathematical processing to shape conclusions. One of the most commonly used approaches is research based on the analysis of the GDP indicator and indicators derived from it like the Hoover Concentration Index, the Theil index, the Herfindahl index, etc. [10, 11, 12, 13, 14]. In particular, the authors also use the Klassen typology to track the dynamics and nature of changes in regional development.

Given the natural multidimensionality of regions' economic development description, widely used methods of multidimensional statistical analysis for their structuring by the level of this characteristic and determination of disparities between regions, in particular, cluster analysis, factor analysis, multidimensional scaling, structural equation method, Solow-Swan, and Mankiw-Romer–Weil growth models [12, 15, 16, 17, 18, 19, 20, 21, 22, 23], which allows grouping regions into homogeneous aggregates based on various quantifiable indicators, to identify gaps in the development of individual regions. Among the shortcomings of these approaches, in our opinion, it is worth noting the difficulty of taking into account the importance of individual indicators. The authors of the study, who used these tools, also noted that the grouping results are significantly influenced by clustering methods, which is also a disadvantage. The further development of multidimensional statistics' methods is reflected in the application of fuzzy clustering methods for structuring regions and identifying imbalances in their economic development, which is presented in [24, 25, 26, 27].

Another way to take into account the multidimensionality for the description of regional development processes is to use the technology of comprehensive index assessment [28, 29, 30, 31, 32, 33]. The vast majority of scientists' approaches in the presented studies are focused on designing a composite indicator of economic development by linear convolution of a set of quantitatively measured indicators. The differences are in the information base chosen for the study and how the results are interpreted. Among the shortcomings, it is worth noting the lack of consideration of the weight of the initial indicators or proper justification of the proposed weights, which in most cases it is proposed to determine the expert method. Besides, either a linear relationship between the values of the composite indicator and these levels, or a desirability scale without proper conversion of the original data is usually used to interpret the results and establish levels of economic development [31].

The study of issues related to assessing the impact of the COVID-19 pandemic on the economic development of economic systems both at the global level and at the level of individual national economies is currently one of the most relevant and is quite intensively studied by scientists. The vast majority of researchers are inclined to believe that overcoming the crisis is possible only after a few years, even with the total vaccination of the population, which should curb the spread of viral infection. Such conclusions are supported by the results of economic and mathematical modeling and evaluation of current and future trends in the economic system development.

Issues related to the application of mathematical modeling to assess the impact of a pandemic on economic development are reflected, in particular, in publications [34, 35, 36, 37]. However, it should be noted that the authors of these studies provide short-term forecast estimates of macroeconomic indicators at the level of national economies. with an emphasis on trends and potential scenarios for their development. The main attention is paid to the assessment of GDP change as one of the most important macroeconomic indicators. In our point of view, insufficient attention is currently paid to research to identify changes in the trends of economic development of certain regions of the country.

Our study aims to develop an approach to building an economic development comprehensive index for analyzing the impact of COVID-19 on Ukraine's regions development and identifying structural changes by combining the technology of comprehensive index assessment, multidimensional statistical analysis, and projection of results on the desirability scale [38].

3. Problem description and methodology

The economic development of the regions is characterized by a large number of indicators. They usually reflect the quantitative results of the activities of regional business entities and therefore have a metric origin, i.e., measured on one of the quantitative scales. This significantly simplifies their further analytical processing, because for indicators of this nature it is quite correct to use mathematical operations.

One of the difficulties that arise in the process of processing such data and interpretation of results is their internal inconsistency, diversity, and inequality of impact on the studied quality. To concentrate the information contained in the initial indicators and reduce the dimension of characteristics' space, various computing technologies are used. One of them is the technology of comprehensive index assessment, which allows reducing the description of the studied phenomenon, in this case, the economic development of the regions, to a single comprehensive indicator. This is usually done by weighted convolution of the initial units. At the same time, there are several methodological problems to realize this process. First, the economic development of regions, as a complex phenomenon, requires the use of a large number of baselines for their description. Thus, the relative impact of each indicator on the final result is reduced. Secondly, there is a problem of reasonable determination of the weight of each component when they are integrated into a composite indicator.

A possible solution to these obstacles is the use of block convolution. Under such conditions, the initial set of indicators is divided into subsets that don't intersect. A partial composite index is constructed for each subset. The final result is settled by convolution of the constructed partial composite indices taking into account the weight of each obtained subset.

One of the approaches that allow getting a solution to this problem is the method of an extreme grouping of parameters. It is based on the hypothesis that the set of initial characteristics can be divided into groups, each of which reflects the effect of a certain factor – the latent characteristics of the group. Therefore, the method focuses on the selection of groups of parameters such that the relationships between the parameters within the group are maximum under the assumption that the number of such groups is fixed. It is assumed that the relationships within the group are explained by the relationship between some generalized latent characteristic of the group (generalized index) and the initial indicators included in this group. Direct relationships between initial indicators are unknown and may be absent. Since the indicators within each of these groups must be more closely related than the indicators of different groups, the task is to identify highly correlated groups of indicators.

Denote by G=$X_1, X_2, \dots, X_n$ the set of initial indicators. The initial data for the method's computational procedure is the correlation matrix R of these indicators. Let $G_1, G_2, \dots, G_s$ be subsets into which the set of initial indicators is divided:

$$\bigcup_{i=1}^{s} G_i = G, \tag{1}$$

$$G_i \cap G_j = \emptyset, \tag{2}$$

$i \neq j, i, j = 1, 2, \dots, s.$

Denote by $H_1, H_2, \dots, H_s$ – the corresponding latent characteristics (indicators) of each group. The criterion that allows you to determine the best grouping of indicators has the form:

$$\sum_{i=1}^{s} \sum_{X_j \in G_i} |r_{X_j, H_i}|, \tag{3}$$

where r_{X_j,H_i} is the correlation coefficient between initial indicator X_j, which belongs to subset G_i, and common indicator H_i of subset G_i.

To obtain a division of the original set of indicators into subgroups, you can use the method of principal components. It is known that the model of transition from the system of initial indicators to the set of latent characteristics, which are the principal components, is reflected by the dependence:

$$Z^T = WF^T, \tag{4}$$

where Z^T – transposed matrix of standardized initial indicators' values, F^T – transposed matrix of principal components' values, W – matrix of principal components factor loadings:

$$Z = \begin{pmatrix} z_{11} & z_{12} & \cdots & z_{1n} \\ z_{21} & z_{22} & \cdots & z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ z_{m1} & z_{m2} & \cdots & z_{mn} \end{pmatrix}, \tag{5}$$

$$F = \begin{pmatrix} f_{11} & f_{12} & \cdots & f_{1n} \\ f_{21} & f_{22} & \cdots & f_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ f_{m1} & f_{m2} & \cdots & f_{mn} \end{pmatrix}, \tag{6}$$

$$W = \begin{pmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \cdots & w_{nn} \end{pmatrix}, \tag{7}$$

where m – the volume of the sample, which is used to measure the initial set of indicators.

The relationship between the values of indicators and principal components (factors) can be written as follows:

$$z_{ji} = \sum_{k=1}^{n} w_{ik} f_{jk} \tag{8}$$

where z_{ji} – i-th component (value) of Z_j, w_{ik} – factor loadings for F_k, f_{jk} – j-th components of F_k, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$.

Let us calculate the correlation coefficient between the initial indicator Xi and the principal component F_j, taking into account the fact that the principal components are non-correlated:

$$r_{F_i, F_j} = 0, \tag{9}$$

$i \neq j$.

As a result, we obtain:

$$r_{X_i, F_j} = r_{Z_i, F_j} = \frac{1}{m} \left(\sum_{k=1}^{n} W_{ik} F_k \right) F_j = w_{ij}. \tag{10}$$

Therefore, the correlation coefficient between the initial indicator and the principal component is equal to the factor load of this component for the corresponding indicator. This fact allows us to conclude that to get the desired grouping of indicators it is necessary to analyze the values of the factor loadings of the principal components for each initial indicator. In this case, as the latent characteristic H_j of the group G_j, we choose the corresponding principal component F_j. To avoid the formation of empty groups or all groups, each of which will contain only one initial indicator, for grouping, we choose not all the principal components, but only the first s most influential, which explain the given share of variance of initial indicators. The value of s is defined as the smallest value of the number of principal components for which the inequality is met:

$$\frac{\sum_{i=1}^{s} \lambda_i}{n} \geq \gamma \tag{11}$$

where λ_i – eigenvalues, ordered by decreasing their values, γ – a predetermined explanation fraction of the initial indicators' variance by the principal components. Typically, this value is selected from 0.70 to 0.80.

In the group of homogeneous indicators, it is expedient to include those initial indicators for which the corresponding values of factor loadings for the principal components on absolute value will have the greatest values. To construct a partial composite index I_j for each formed group G_j, we use one of the formulas for weighted convolution [39]:

$$I_j = \sum_{i \in K_j} \alpha_i^{(j)} U_i^{(j)} \tag{12}$$

$$I_j = \prod_{i \in K_j} \left(U_i^{(j)} \right)^{\alpha_i^{(j)}} \tag{13}$$

$$I_j = -1 + \prod_{i \in K_j} \left(1 + U_i^{(j)} \right)^{\alpha_i^{(j)}} \tag{14}$$

where $U_i^{(j)}$ – normalized values of those indicators X_i, that belong to subset G_j, $a_i^{(j)}$ – weight coefficients of appropriate indicators, K_j – set of indices for those indicators X_i, that belong to subset G_j, $j = 1, 2, \dots, s$.

The initial indicators are transformed to normalize form according to the formula (15) or formula (16) [17]:

$$u_{ij} = 1 - \frac{|x_{ij} - x_j^*|}{x_{jmax} - x_{jmin}} \tag{15}$$

$$u_{ij} = \begin{cases} \frac{x_{ij}}{x_{jmax}}, & \text{when } X_j \text{ is an incentive;} \\ \frac{x_{jmin}}{x_{ij}}, & \text{when } X_j \text{ is a disincentive;} \end{cases} \tag{16}$$

where u_{ij} – normalized values of indicators, x_{ij} – initial values of indicators, $x_{jmin} = \min x_{ij}$, $x_{jmax} = \max x_{ij}$,

$$x_i^* = \begin{cases} x_{jmax}, & \text{when } X_j \text{ is an incentive;} \\ x_{jmin}, & \text{when } X_j \text{ is a disincentive;} \end{cases} \tag{17}$$

i=1, 2, … , m, j=1, 2, … , n, m – number of units under study, n – number of initial indicators.

The normalization procedure is necessary to extract the units of measurement of the original indicators and reducing their values to a scale from 0 to 1. This step is aimed at simplifying the further interpretation of the calculation result. To calculate the weight coefficients, we propose to use the components of eigenvector V_j:

$$\alpha_i^{(j)} = \frac{\left(v_i^{(j)}\right)^2}{\sum_{i \in K_j} \left(v_i^{(j)}\right)^2}, \tag{18}$$

where $a_i^{(j)}$ – weight coefficients of appropriate indicators, $v_i^{(j)}$ – components of j-th eigenvector V_j, that correspond to initial indicators X_i from the G_j , K_j – set of indices for those indicators X_i, that belong to subset G_j.

Equation (18) meets the condition, that the sum of weight coefficients should be equal to 1. This condition with the normalization procedure provides the location of partial composite indicators values in the range [0; 1].

We propose to calculate the final economic development comprehensive index I_{COM} using partial composite indicators I_j based on one of the convolution's forms like (12), (13), (14). For example, for linear weighted convolution appropriate expression has a form:

$$I_{COM} = \sum_{j=1}^{s} \beta_j I_j. \tag{19}$$

Weight coefficients β_j are calculated in proportion to the eigenvalues λ_j that correspond with G_j, $j = 1, 2, \dots, s$:

$$\beta_j = \frac{\lambda_j}{\sum_{k=1}^{s} \lambda_k}. \tag{20}$$

Under such conditions, the values of the I_{COM} will also be in the range from 0 to 1. This approach to calculations simplifies the interpretation of the result.

To assess the studied objects' structure, the range of values of the comprehensive index should be divided into ranges. Dividing the range [0; 1] of values of the comprehensive index into intervals of the same length to achieve this goal is impractical.

First, ranges can be formed that don't cover any of the objects under study.

Second, the latent characteristic under study is usually nonlinear, and the use of intervals of the same length can disrupt the true structure of objects.

Third, such a division can be led to a situation where one group includes objects that have significant differences in the values of the integrated indicator, while two neighboring objects belonging to different groups may have a slight deviation of the values of the comprehensive index.

To solve the problem of grouping, you can also use the approach presented in [17], in which the definition of the boundaries of the ranges is carried out by calculating the ratios of two adjacent values of the integrated indicator:

$$\delta_j = \frac{I_j}{I_{j-1}}, \tag{21}$$

$j = 2, 3, \ldots, m$.

The basis for the transition to a new range of values of the comprehensive index is a significant rise in the change of values of δ_j. The grouping objects is executed according to the level of the corresponding values of the comprehensive index. This approach also has drawbacks. Given the slight difference in the values of the integrated indicator, which are in the middle of the range of all its possible values, one of the groups can have a very large number of objects, which will be significantly different from the content of other groups. Besides, in the case of a slight discrepancy in the values of the comprehensive index for neighboring objects, a significant rise in the values of δ_j may not be observed. Thus, all objects can belong to one group. It is also necessary to take into account the fact that the value of δ_j is also affected by the level of values of the comprehensive index for which this value is calculated. And the closer these values are to 0, the smaller should be the hike in the change of values of δ_j, which decides on the formation of a new range.

The iterative procedure presented in [31] can be used to determine the limit values of the comprehensive index' ranges. Its advantage is the "adjustment" of grouping ranges to the value of a specific sample, which makes its application more practical. However, the disadvantage of this approach is the use of a training sample.

Another approach that allows you to solve this problem is the use of desirability scales, which allow you to match the quantitative and qualitative levels and group objects according to the level of studied quality. One such scale is the Harrington scale. The use of this scale involves the transformation based on Harrington's function [40]:

$$H(Z_i) = exp(-exp(-Z_i)), \tag{22}$$

where Z_i is the value of the indicator on the scale of partial indicators Z. The values $d = H(Z)$ of the Harrington's function form the desirability scale.

The correspondence between the values of Z_j and the values of the initial indicators I_j is determined by the formula:

$$Z_j = (Z^* - Z_*)\frac{I_{COM_j} - I_{min}}{I_{max} - I_{min}} + Z_*, \tag{23}$$

where Z_j – current value of the Z-scale, corresponds with the value of I_{COM_j}; I_{COM_j} – current value of comprehensive index I_{COM}; Z_* and Z^* – low and high bounds of Z-scale, which define the workspace of Z_j; I_{min}, I_{max} – minimum and maximum of I_{COM}; $j = 1, 2, \ldots, m$.

Transformation (23) is required to match the value of the comprehensive index I_{COM} and Z-scale with the correspondence of the minimum and maximum values of both indicators.

Next step, we identify the value of $d_j = H(Z_j)$, $j = 1, 2, \ldots, m$, and distribute objects under study into five groups by qualitative development level of the group (table 1).

Table 1
The relationships between the quantitative values of the desirability scale and qualitative development levels of group

Qualitative levels of development	The range of quantitative values on the desirability scale
relatively high	0.80..1.00
above average	0.63..0.80
average	0.37..0.63
below average	0.20..0.37
relatively low	0.00..0.20

This approach allows taking into account the nonlinear nature of the studied characteristic, in this case, the economic development level, as well as to investigate changes in the structure of the objects under study by the values of the comprehensive index calculated for different periods.

4. Findings

Let us consider the practical testing of the proposed approaches to the calculation of t economic development comprehensive index for Ukraine's regions, grouping regions based on their values, and the study of structural changes in the resulting grouping caused by the COVID-19 pandemic. We choose the data of the State Statistics Service of Ukraine [7] and the Ministry of Development of Communities and Territories of Ukraine [41] for the period of the first 9 months of 2019 and the first 9 months of 2020 as the information base for the calculations. We choose the following initial indicators:

X_1 – Volume of sold industrial products per capita, UAH;

X_2 – Volume of agricultural production per capita of the rural population, UAH;

X_3 – Volume of construction works performed per capita, UAH;

X_4 – Volume of capital investments per capita cumulatively since the beginning of the year, UAH;

X_5 – Exports of goods per capita, USD;

X_6 – An unemployment rate of the population aged 15-70 years (according to the ILO's Methodology), %;

X_7 – Employment rate of the population aged 15-70 years (according to the ILO's Methodology), %;

X_8 – Index of real wages, %;

X_9 – The volume of housing commissioned per 10 thousand people, sq. meters of the total area;

X_{10} – The volume of freight turnover of road and rail transport, thousand ton-kilometers per 1000 population, thousand ton-km.

We assigned to each of Ukraine regions' names the corresponding code which we used for the designation of each of them to further use (table 2).

Table 2
The relationships between the quantitative values of the desirability scale and qualitative development levels of group

Code	Region	Code	Region
r-01	Vinnytsia	r-13	Mykolaiv
r-02	Volyn	r-14	Odesa
r-03	Dnipro	r-15	Poltava
r-04	Donetsk	r-16	Rivne
r-05	Zhytomyr	r-17	Sumy
r-06	Zakarpattia	r-18	Ternopil
r-07	Zaporizhzhia	r-19	Kharkiv
r-08	Ivano-Frankivsk	r-20	Kherson
r-09	Kyiv	r-21	Khmelnytskyi
r-10	Kyrovohrad	r-22	Cherkasy
r-11	Luhansk	r-23	Chernivtsi
r-12	Lviv	r-24	Chernihiv

The values of initial indicators to provide calculations are shown in tables 3 and 4.

Let's group the initial indicators by the method of an extreme grouping of parameters. To determine the correlations between the initial indicators and the latent characteristics of each group in the context of maximizing the expression (3), we use the method of principal components. Taking into account expression (10), it is necessary to calculate the factor loadings for the selected principal components and choose the largest from them in absolute value. The number of groups is defined as the number of principal components that explain a given level of variance of the initial indicators following expression (11).

We choose the level of explanation of the variance of the initial indicators as $\gamma = 0.80$. Under such conditions, it is necessary to choose the first four principal components. The values of the eigenvectors and the eigenvalues of the corresponding correlation matrices of the initial indicators are given in tables 5 and 6, and the values of the factor loadings – in tables 7 and 8.

Analysis of Tables 7, 8 allows us to formulate a conclusion, that we have the following distribution of initial indicators between subsets G_j:

Table 3
Indicator's values for data for first three quarters of 2019

Code	Values									
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
r-01	39228.9	20678.0	3774.4	5730.1	697.8	9.8	57.9	112.1	1248.9	2721.7
r-02	21736.3	11004.0	1322.5	8039.0	503.6	11.7	50.2	109.1	2978.8	2261.9
r-03	110916.2	25975.0	4163.1	13679.4	1971.8	7.7	59.5	112.9	858.1	3143.8
r-04	53475.4	17444.0	911.7	4294.1	804.4	13.7	50.9	108.9	105.4	2583.4
r-05	28004.0	15169.0	1301.6	3781.7	438.3	9.8	57.3	107.1	1121.4	2631.3
r-06	14401.8	3895.0	1008.4	3135.1	906.9	9.1	55.3	106.3	2852.6	3646.2
r-07	86201.6	22827.0	1456.1	5135.8	1369.6	9.5	57.8	111.7	351.4	2866.1
r-08	37051.0	5895.0	1522.7	4051.9	491.2	7.6	55.8	108.1	4151.6	1987.5
r-09	50780.4	17359.0	3627.6	17018.5	815.3	5.9	59.3	111.7	6897.8	2146.4
r-10	24742.5	28345.0	1361.0	4134.5	508.9	11.2	55.6	107.9	598.2	7529.0
r-11	7917.8	14675.0	175.7	898.1	57.2	14.2	58.4	108.8	59.5	5611.3
r-12	30915.6	8160.0	2744.3	6599.9	631.4	6.8	57.5	107.4	3842.8	1373.6
r-13	40158.9	24129.0	2075.6	7038.5	1451.0	9.7	58.9	112.4	598.3	4675.4
r-14	18915.3	11939.0	4653.9	5677.4	468.8	6.1	57.9	106.3	2844.4	5052.0
r-15	91151.2	22940.0	3673.1	10075.9	1121.6	11.1	56.1	109.7	1118.2	3515.0
r-16	26703.4	8958.0	1545.4	3649.4	277.9	8.6	57.9	111.0	2019.0	3480.3
r-17	33304.1	21871.0	878.7	4306.1	547.1	8.0	58.7	108.0	722.6	2697.3
r-18	14411.8	12093.0	1428.1	5437.1	308.9	10.4	53.4	109.9	2571.6	1119.1
r-19	51750.2	23955.0	3793.0	5187.6	360.9	5.0	62.2	108.3	984.1	1620.8
r-20	22025.6	24547.0	1092.2	5644.6	220.1	10.3	58.3	107.1	885.8	1692.7
r-21	25488.3	17527.0	1860.6	4677.0	356.6	8.7	56.6	109.7	1544.0	1870.5
r-22	45162.6	20718.0	1054.1	5544.1	483.4	8.5	58.7	109.6	944.5	3133.7
r-23	11193.4	6784.0	1330.5	2501.1	166.1	7.2	58.6	109.4	3952.4	1954.4
r-24	24933.9	20188.0	1212.6	5185.2	547.7	10.5	58.3	109.3	1150.5	1644.5

$G_1=\{X_1, X_3, X_4, X_5\}$;
$G_2=\{X_2, X_6, X_8\}$;
$G_3=\{X_7, X_9\}$;
$G_4=\{X_{10}\}$.

Note, that subset G_4 consists of one initial indicator X_{10}, so, partial composite index I_4 coincides with this indicator.

To calculate patrial composite indices I_j, $j = 1, 2, 3, 4$, we conduct a normalization procedure for initial data. In this case, we execute this step using formula (16), because this way allows keeping the proportions between the values of the indicator, which is important in the calculation of composite index's values.

We also take into account, that indicator X_6 is a disincentive, and other indicators are incentives. Weight coefficients we calculate, using formula (18). Values of composite indices I_j, $j = 1, 2, 3, 4$, have been calculated using linear convolution by expression (12). To calculate the comprehensive index, we also use weighted linear convolution like (19) with weight coefficients, obtained by the formula (20).

To correctly compare the results of calculations and identify changes in the levels of the

Table 4
Indicator's values for data for first three quarters of 2020

Code	Values									
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
r-01	36446.7	44134.0	4023.4	4345.4	679.2	10.4	56.9	109.0	720.5	3528.4
r-02	21419.8	25428.0	1376.7	7016.7	449.0	12.3	49.2	101.8	1806.3	2728.5
r-03	95582.3	58710.0	3737.0	10444.6	1755.7	8.2	58.4	106.5	235.0	5894.9
r-04	43994.1	43567.0	1300.5	3345.8	700.3	14.5	49.8	103.1	45.5	1188.8
r-05	26948.8	33675.0	1028.8	3118.3	397.7	10.5	55.8	108.7	438.6	5683.4
r-06	13637.5	7090.0	861.8	1956.5	769.9	10.2	54.3	104.3	1430.0	3680.1
r-07	78074.0	56801.0	1011.5	3907.7	1267.3	10.4	56.5	108.4	150.0	4121.6
r-08	30883.1	12247.0	1581.1	2212.1	400.2	8.1	54.4	107.4	1833.7	1031.1
r-09	49227.9	32672.0	4524.2	8784.3	762.2	6.6	58.2	105.1	3932.9	5024.2
r-10	26339.1	56737.0	958.0	3910.2	706.2	12.3	54.2	111.7	231.0	17025.4
r-11	6154.2	37587.0	169.3	620.4	45.8	15.2	57.3	112.5	51.0	299.9
r-12	30522.8	17182.0	3226.0	3772.5	646.3	7.4	56.2	106.4	1734.9	3386.5
r-13	40592.0	49388.0	1732.1	3632.5	1311.1	10.3	57.9	110.9	205.1	8912.5
r-14	20213.1	18536.0	6105.9	4248.0	400.4	6.9	57.0	109.4	1391.7	5460.3
r-15	80188.6	48838.0	3956.8	8287.4	1163.0	11.7	55.2	106.6	457.1	4618.8
r-16	28737.4	19118.0	1496.5	2010.7	297.3	9.1	56.7	109.2	1080.6	5255.5
r-17	30338.2	48048.0	1008.9	2983.4	596.6	9.1	57.2	110.1	282.9	2086.5
r-18	13649.8	26998.0	1472.1	3532.0	300.9	11.3	52.1	108.4	1543.0	1236.3
r-19	45981.1	61006.0	3608.1	3819.2	381.7	5.8	60.8	105.5	870.3	3537.0
r-20	22620.1	56237.0	763.4	2250.0	198.3	11.1	57.2	110.5	355.6	841.6
r-21	26941.7	39572.0	2763.3	4151.3	355.2	9.5	55.3	109.0	1069.1	3948.0
r-22	44996.7	43113.0	1112.1	3232.8	523.3	9.3	57.6	107.0	398.9	4440.9
r-23	10588.4	14647.0	1553.1	1448.2	126.7	8.6	57.2	107.7	1670.9	729.0
r-24	24360.0	43737.0	1687.1	3600.9	554.4	11.5	56.8	109.1	577.4	2961.3

comprehensive index for the relevant periods and the regions' structuring, the values for 2019 and 2020 will be combined into one sample. The normalization procedure is performed for the combined data.

Further calculations of both partial composite indices and comprehensive indices are executed for each period separately. The values of the selected eigenvector elements for calculating the weights by formula (18) are different, as well as the corresponding eigenvalues that will be used to calculate the weights of the generalized indicator by formula (20). So, for the data of 2019 and the data of 2020, we obtain different values of weight coefficients, which means that both composite and comprehensive indicators will be calculated according to different rules.

Therefore, for a more accurate comparison of the results, we propose to calculate corresponding weights as the average values of the appropriate components obtained for 2019 and 2020.

Weight coefficients for calculation of composite indices I_1-I_4 in accordance with distribution initial indicators to G_j have such values: $w_{11} = 0.30; w_{13} = 0.18; w_{14} = 0.27; w_{15} = 0.25; w_{22} = 0.25; w_{26} = 0.28; w_{29} = 0.47; w_{37} = 0.77; w_{38} = 0.23; w_{4,10} = 1.00$. Values of weight coefficients for calculation of comprehensive index are: $w_1^{COM} = 0.42$, $w_2^{COM} = 0.28$, $w_3^{COM} = 0.19$, $w_4^{COM} =$

Table 5
Most significant eigenvalues of correlation matrix and values of appropriate eigenvectors for data 2019

Eigenvalues			
λ_1	λ_2	λ_3	λ_4
3.64	2.42	1.36	0.87
Eigenvectors			
V_1	V_2	V_3	V_4
0.45	-0.16	0.13	-0.11
0.29	-0.39	-0.28	-0.14
0.38	0.23	-0.14	0.32
0.42	0.16	0.27	0.15
0.42	-0.17	0.23	0.14
-0.20	-0.49	0.35	0.02
0.22	0.12	-0.69	-0.20
0.12	-0.12	0.21	-0.26
0.01	0.57	0.25	0.23
-0.02	-0.34	-0.25	0.81

Table 6
Most significant eigenvalues of correlation matrix and values of appropriate eigenvectors for data 2020

Eigenvalues			
λ_1	λ_2	λ_3	λ_4
3.35	2.53	1.76	0.92
Eigenvectors			
V_1	V_2	V_3	V_4
0.49	-0.11	0.14	-0.26
0.28	-0.44	-0.05	-0.20
0.34	0.33	-0.19	0.06
0.46	0.14	0.24	0.09
0.45	-0.15	0.19	0.05
-0.22	-0.38	0.43	0.06
0.20	-0.04	-0.64	-0.27
-0.11	-0.36	-0.49	0.20
0.00	0.57	-0.02	0.21
0.22	-0.21	-0.10	0.85

0.11. The results of the calculations of comprehensive index values are presented in table 9.

A comparison of the calculation results of the comprehensive index shows that for most of Ukraine's regions there is a decrease in its values. In our opinion, this fact indicates a negative impact of the pandemic COVID-19 on economic development. At the same time, for some regions, in particular, Vinnytsia, Zhytomyr, Zaporizhia, Kirovohrad, Mykolaiv, Kharkiv,

Table 7
Values of factor loadings for data 2019

Initial indicators	Values of factor loadings			
	w_1	w_2	w_3	w_4
X_1	0.86	0.24	0.16	0.10
X_2	0.56	0.61	0.32	0.13
X_3	0.73	0.36	0.16	0.30
X_4	0.81	0.25	0.31	0.14
X_5	0.79	0.27	0.27	0.13
X_6	0.38	0.77	0.41	0.02
X_7	0.42	0.19	0.80	0.19
X_8	0.24	0.19	0.25	0.24
X_9	0.02	0.88	0.29	0.22
X_{10}	0.04	0.53	0.29	0.76

Table 8
Values of factor loadings for data 2020

Initial indicators	Values of factor loadings			
	w_1	w_2	w_3	w_4
X_1	0.89	0.17	0.19	0.25
X_2	0.52	0.70	0.06	0.20
X_3	0.62	0.52	0.25	0.06
X_4	0.84	0.22	0.31	0.09
X_5	0.83	0.23	0.25	0.05
X_6	0.40	0.61	0.57	0.05
X_7	0.36	0.07	0.85	0.26
X_8	0.21	0.58	0.66	0.19
X_9	0.01	0.90	0.03	0.20
X_{10}	0.40	0.33	0.14	0.81

Khmelnytskyi, and Chernihiv regions, there is an increase in the values of the indicator in 2020. This increase is especially noticeable for the Kirovohrad region. This can be explained by the fact that for a long time in this area was the best epidemiological situation in Ukraine. Also, the growth of industrial production, in particular pharmaceuticals one in Vinnytsia, Kirovohrad, Zaporizhia, and Kharkiv regions.

Let us consider the changes in the structure of Ukraine's regions in 2020 compared to 2019 in terms of the economic development comprehensive index. Given the relatively high density of values of the comprehensive index for different regions, the use of the approach to the grouping of regions, based on the analysis of the values of the delta, calculated by the formula (21), doesn't allow to determine their structure. Therefore, to solve this problem, we apply an approach based on the use of the Harrington desirability scale. For this purpose, we transform the values of the integrated indicator according to formulas (22) and (22). The distribution of regions by groups is executed according to the rules given in table 1. The results of the calculations are

Table 9
Comprehensive index values

Code	2019	2020	Code	2019	2020
r-01	0.45	0.46	r-13	0.48	0.49
r-02	0.40	0.38	r-14	0.48	0.46
r-03	0.67	0.65	r-15	0.55	0.55
r-04	0.37	0.37	r-16	0.38	0.37
r-05	0.37	0.38	r-17	0.39	0.39
r-06	0.39	0.34	r-18	0.36	0.34
r-07	0.49	0.50	r-19	0.48	0.50
r-08	0.44	0.36	r-20	0.36	0.35
r-09	0.68	0.59	r-21	0.38	0.41
r-10	0.40	0.48	r-22	0.41	0.42
r-11	0.28	0.26	r-23	0.39	0.32
r-12	0.47	0.43	r-24	0.38	0.39

listed in table 10.

The analysis of results obtained shows, that the first group with a relatively high level of economic development is quite large. Traditionally, this group includes Kyiv, Kharkiv, and Dnipropetrovsk regions, which in the "pre-pandemic" period had a fairly high level of economic development. These regions have a fairly high production potential, they account for a significant share of foreign investment in 2020 and therefore the pandemic has not had such a destructive impact on the economic development of these regions. Zaporizhia and Poltava regions also have significant potentials and were distributed to this group. The lowest level of economic development is in the Luhansk region, and in 2020 the situation has not changed.

It should be noted that for many regions there have been no changes in the level of economic development, although there has been a decrease in the value of the corresponding comprehensive index. For those regions where changes are taking place, they are usually associated with a decline in economic development. The only exception is the Kirovohrad region.

The most significant decrease in the level took place in Zakarpattia, Ternopil, and Chernivtsi regions. These are the regions that were the first to suffer from the pandemic and were in the "red" zone for a long time, which negatively affected all indicators of economic development and led to a significant reduction in the corresponding comprehensive index values.

Thus, the results of the research demonstrate the fundamental possibility of applying the proposed approach to the study of economic development of regions by constructing an integrated indicator. The analysis of the structure of the regions showed the real impact of the pandemic on the development of almost all regions, which led to the corresponding structural changes.

5. Conclusions

The study of economic development trends both in the economic system of the country as a whole and at the level of individual regions remains the focus of the most significant research. The results of such studies are especially important in periods of global challenges, one of which

Table 10
Identifying the structure of regions for data 2019 and 2020

Code	$d = H(Z)$		Level of development	
	2019	2020	2019	2020
r-01	0.72	0.75	above average	above average
r-02	0.47	0.35	average	below average
r-03	0.99	0.99	relatively high	relatively high
r-04	0.29	0.28	below average	below average
r-05	0.27	0.34	below average	below average
r-06	0.44	0.15	average	relatively low
r-07	0.86	0.88	relatively high	relatively high
r-08	0.68	0.25	above average	below average
r-09	0.99	0.97	relatively high	relatively high
r-10	0.46	0.84	average	relatively high
r-11	0.00	0.00	relatively low	relatively low
r-12	0.81	0.66	relatively high	above average
r-13	0.84	0.86	relatively high	relatively high
r-14	0.84	0.77	relatively high	above average
r-15	0.95	0.95	relatively high	relatively high
r-16	0.37	0.31	average	below average
r-17	0.42	0.43	average	average
r-18	0.24	0.13	below average	relatively low
r-19	0.83	0.88	relatively high	relatively high
r-20	0.22	0.18	below average	relatively low
r-21	0.37	0.56	average	average
r-22	0.56	0.58	average	average
r-23	0.39	0.06	average	relatively low
r-24	0.34	0.42	below average	average

at this stage of the world community development was the COVID-19 pandemic. Solving the problems of assessing the level of development of regions, their structuring, identifying gaps and breaks in the development of individual territorial units is complicated by the significant multidimensionality of their description. The use of analytical methods of information processing based on economic and mathematical models allows us to present it in a concentrated form without significant losses, which contributes to the adoption of sound management decisions and the development of strategic plans for regional development. Therefore, models that allow for a significant reduction in baseline and identify latent characteristics of the studied phenomena are important for studies. In particular, such approaches include models based on the comprehensive index assessment technology.

The approaches offered in the article allow estimating the level of economic development of regions by block convolution of the set of initial indicators into a single complex measure – an economic development comprehensive index. Thus, the toolkit which allows to carry out a grouping of initial indicators to take into account the weights of components at the construction of such indicators, and also the weights of partial composite indices at their convolution into the economic development comprehensive index is offered. The article proposes some approaches to grouping regions by the level of economic development. An approach based on the transformation of the comprehensive index values with the projection of the result on the desirability scale is chosen for practical implementation. This way allows to rank regions, determine their structure by this characteristic and assess structural changes over time.

According to the research outcomes, it can be concluded that the COVID-19 pandemic has a destructive impact on the economic development of the vast majority of Ukraine's regions, which was reflected both in changes in the values of the comprehensive index and in the regions' structure.

The direction of further research is the development of analytical tools to take into account indicators of non-metric origins in the assessment procedures for the identifying level of economic development.

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The use of genetic algorithms for multicriteria optimization of the oil and gas enterprises financial stability

Marta V. Shkvaryliuk, Liliana T. Horal, Inesa M. Khvostina, Natalia I. Yashcheritsyna and Vira I. Shiyko

Ivano-Frankivsk National Technical University of Oil and Gas, 15 Karpatska Str., Ivano-Frankivsk, 76019, Ukraine

Abstract

The article considers the problem of optimizing the financial condition of oil and gas companies. The offered methods of optimization of a financial condition by scientists from different countries are investigated. It is determined that the financial condition of the enterprise depends on the effectiveness of the risk management system of enterprises. It is proved that the enterprises of the oil and gas complex need to develop a system for risk management to ensure the appropriate financial condition. The financial condition is estimated according to the system of certain financial indicators, the integrated indicator of financial condition assessment is constructed using the method of taxonomy. According to the results of the calculation of the integrated indicator, it is concluded that this indicator does not have a stable trend. On the basis of the conducted researches it is offered to carry out optimization of an integral indicator of a financial condition with use of genetic algorithm in the Matlab environment. Based on the obtained results, recommendations of the management of the researched enterprises on increase of management efficiency are given.

Keywords

genetic algorithms, multicriteria optimization, oil and gas enterprises, financial stability

1. Introduction

The loss of financial stability of any enterprise in a turbulent environment, which is exacerbated by negative external factors is a reality today. This fact leads to many negative consequences, one of which is bankruptcy and liquidation of the enterprise. Due to the fact that the reduction of financial stability of enterprises that provide Ukraine's economic and energy security has become a reality, there is an urgent need to optimize their financial stability. Examining various scientific sources related to the solution of this problem, we can conclude that there are many

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marta.shkvaryliuk@gmail.com (M. V. Shkvaryliuk); liliana.goral@gmail.com (L. T. Horal); inesa.hvostina@gmail.com (I. M. Khvostina); yashcheritsyna@gmail.com (N. I. Yashcheritsyna); vnkShiyko@gmail.com (V. I. Shiyko)
https://nung.edu.ua/heading/marta-shkvarilyuk-iem (M. V. Shkvaryliuk); https://nung.edu.ua/en/node/790 (L. T. Horal); https://nung.edu.ua/en/node/1019 (I. M. Khvostina); https://nung.edu.ua/en/node/1021 (N. I. Yashcheritsyna); https://nung.edu.ua/person/shiyko-vira-igorivna (V. I. Shiyko)
0000-0001-6066-5619 (L. T. Horal); 0000-0001-5915-749X (I. M. Khvostina); 0000-0002-2926-5550 (N. I. Yashcheritsyna); 0000-0002-2822-0641 (V. I. Shiyko)

ways to optimize financial stability. However, we propose to use the genetic algorithm method in the process of the studied enterprises financial stability optimizing , which by optimizing the financial stability will generate optimal values of the enterprise internal factors. Which in the future will allow the company's management to make optimal management decisions and reduce the risk of its loss..

2. Background

Important contribution to the study of financial sustainability was made by Drobyazko et al. [1], Azarenkova et al. [2], Mokeev et al. [3].

Drobyazko et al. [1] propose an economic and mathematical model of the assessing financial stability process by calculating the integral indicator of the service sector financial stability. To study the stability and controllability of the assessing financial stability process, the types of control maps for each of the coefficients were determined. Proposed apparatus of neural networks makes it possible not only to determine the most profitable activity of an enterprise but also to assess the financial condition of each of its research objects.

The main results of the Azarenkova et al. [2] study are following: the theoretical and essential characteristics of enterprise financial sustainability has been determined; the financial status of PJSC "Turboatom" has been analyzed; the taxonomic index of financial sustainability has been calculated and the forecast of its significance has been made, the approaches to increase enterprise financial sustainability have been proposed.

Mykoliuk et al. [4] provides practical advice for enterprises to achieve the highest possible level of energy and financial security.

Mokeev et al. [3] proposed eigenstate method to analyse the basic indicators of the enterprise as it allows to construct an economical stability model of such enterprise, describe the methodology for analyzing the economic stability of an enterprise on the basis of eigenstate method, provides formulas for calculating the complex indicator of economic stability. The efficiency of the methodology is demonstrated with evidence from the economic stability analysis of the trading company.

Ma et al. [5] analyze the relationship between enterprise management and financial performance, analyze the mean and heterogeneity of the enterprise management team characteristics, mathematically models its relationship, constructs fractional differential equations, and tests it through empirical research. The influence of the enterprise management age characteristics, international experience, education level, team size and government background on the financial performance of the company.

A positive aspect of modern research on solving the problem of optimizing financial stability is also the fact that in many works the problem is proposed to be solved by building an effective system of risk management in enterprises. In particular, this issue is considered by Sprčić et al. [6], Cohen et al. [7], Drobyazko et al. [1], Shkvaryliuk et al. [8], Kostetska et al. [9].

Research results of Sprčić et al. [6] have revealed low levels of ERM development in listed Croatian companies. Managers are focused on financial and operative risk management, while strategic and other risks have been neglected. Regression analysis has indicated somewhat unexpected but important conclusion – the explored risk management rationales have weak predictive power in explaining corporate risk management decisions in Croatian companies. The level or risk management system development is dependent only on the size of the company and value of the growth options [6].

Cohen et al. [7] distinguish three major findings from our study. First, importantly, all three types of participants see a strong link between ERM and the financial reporting process. Second, despite recognition of the broad nature of ERM, the predominant experiences of the actual roles played by triad members center on agency theory, while resource dependence may be relatively underemphasized by all triad members. Finally, CFOs and AC members indicate that auditors may be especially underutilizing ERM in the audit process, suggesting an “expectations gap” [7].

Ocheretin et al. [10] proposes an approach to modeling the business climate of the country, which is based on the financial and economic indicators, and makes it possible to assess the development trends of the studied indicator. The proposed approach is based on the taxonomy method.

The analysis of sustainability and security of enterprises was carried out using a wide range of classical and advanced modeling methods, in particular, by Matviychuk et al. [11].

In our previous studies [12] it was possible to achieve an increase in the efficiency of modeling financial risk through the formation of an ensemble of models.

Soloviev et al. [13] demonstrated the possibility of studying complex socio-economic systems as part of a network paradigm of complexity.

After conducting research on scientific papers that solve the problem of the economic entities financial stability loss risk reduction, it can be concluded that the optimization of financial condition through the use of a genetic algorithm has not been carried out. Therefore, this issue is relevant and needs research.

The analysis of methodological approaches to assessing the financial stability of economic entities shows the lack of a single methodological basis for assessment. Moreover, differences are manifested both in the components of financial stability, the system of indicators, and in the method of their consideration in the analysis of financial stability. All this necessitates the development of fundamentally new approaches and tools for assessing the financial stability of enterprises [14].

3. Methodology

Consider the dynamics of average indicators of liquidity, solvency, profitability and business activity of Ukraine’s oil and gas industry, table 1.

According to table 1, it was established that the current ratio does not exceed the regulatory value of 100% in contrast to 2018, where the value of the indicator is 103.0%. Current ratio characterizes the ability of enterprises to repay their current liabilities for up to 1 year through current assets. The liquidity indicator shows that the analyzed enterprises do not have enough resources that can be used to repay short-term creditors’ claims, the change in the studied indicator ranges from 84.7% to 98.7%. In 2018, the overall increase over four years increased by 14.96% and in 2019 there is a decrease of 5.47% compared to the base value.

The value of the Cash ratio did not reach the normative value of 20%. During 2015-2019

Table 1
Dynamics of financial indicators' average values of Ukrainian oil and gas industry

Indicators	Years				
	2015	2016	2017	2018	2019
Current Ratio	89,6	93,3	98,7	10 3	84,7
Equity-to-assets	18,1	16,9	18,4	3	1
ROA	-0,1	0,1	0,8	3,4	1,6
WCP	-0,1	0,1	1,5	6,9	3,8
CashRatio	1	1	1,9	1,3	1
NPM	0	0,3	1,8	4,5	4,4
ROTA	0	0,5	1,4	4,7	3,1
Tot. Ass Turn.	0,8	0,7	0,8	1	0,7
Rec. Turn.	3,4	2,9	4,1	4,5	3,5

Table 2
Growth rates (basic) of financial indicators' average values of Ukrainian oil and gas industry, %

Indicators	Years				
	2015	2016	2017	2018	2019
Current Ratio	1,00	1,04	1,10	1,15	0,94
Equity-to-assets	1,00	0,93	0,10	1,86	1,67
ROA	1,00	1,00	8,00	34,00	16,00
WCP	1,00	1,00	15,00	69,00	38,00
CashRatio	1,00	1,00	1,90	1,30	1,00
NPM	1,00	1,00	6,00	15,00	14,67
ROTA	1,00	1,00	2,80	9,40	6,20
Tot. As. Turn.	1,00	0,88	1,00	1,25	0,88
Rec. Turn	1,00	0,85	1,21	1,32	1,03

the value of the indicator ranges from 1.0% to 1.9%, i.e., money and their equivalents are not enough to meet the current liabilities of economic entities. The Equity-to-Assets – the solvency indicator characterizes the share of equity of enterprises in the total amount of funds invested in its activities. The value of the Equity-to-Assets does not exceed the normative by 50% and fluctuates during the study period in the range of 16.9-33.8%, so there is every reason to believe that enterprises are not solvent.

According to table 2, return on assets (ROA) determines the return of 1 hryvnia assets of economic entities, its value increases rapidly from 0.1% in 2016 to 3.4% in 2018, then decreases to 1.6%. The dynamics of working capital profitability is similar, the value of the indicator increases from 0.1% in 2016 to 6.9% in 2018, decreases to 3.8% in 2019. The profitability indicator is the net margin (NPM), which reflects the ratio of net profit to the total revenue of the enterprise whose value increased from 0.3% in 2016 to 4.5% in 2018 and decreased to 4.4% in 2019. The ratio of operating profit to assets of oil and gas companies (ROTA) characterizes the return on total assets, the value which also increased from 0.5% in 2016 to 4.7% in 2018 and decreased to 3.1% in 2019.

The state of business activity in the context allows you to determine the productivity of assets of enterprises. The value of the indicator in the industry ranges from 0.7-1 for the study period. Working capital turnover – an indicator of business activity, which shows the efficiency of enterprises working capital usage in terms of revenue generated by them. According to the values of the indicator, the efficiency of its generation for the period under study is 0. The turnover of receivables shows how many times during the year receivables are repaid. The ability of entities to repay receivables during the period under review changes abruptly. The highest value is observed in 2018 and is 4.5 and the lowest value is 2.9 in 2016, due to the crisis of payments in the country.

Given the results of the calculated average values of Ukraine's oil and gas industry financial indicators and their dynamics, it is possible to draw conclusions about the instability of their trends, which necessitates the calculation and modeling of an integrated indicator of financial condition. The taxonomy method was used for its construction, the results of its calculation and forecasting are shown in figure 1.

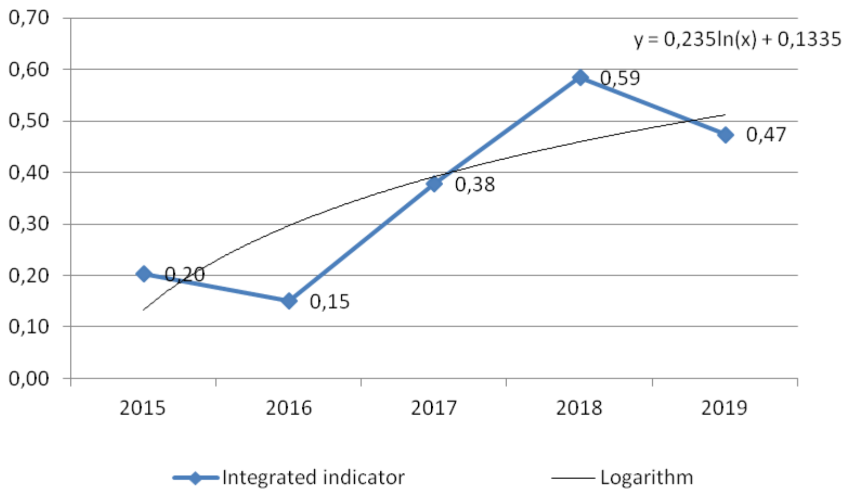

Figure 1: Dynamics of the integrated indicator of Ukrainian oil and gas enterprises' financial condition.

According to figure 1, it is determined that the value of the integrated indicator of Ukraine's oil and gas companies financial condition increases from 2016 to 2018, then decreases to 0.47 in 2019. The decrease in the integrated indicator shows a certain signaling ability to reduce financial stability of oil and gas companies increase in the general level of financial risks.

Variation of indicators acquires a difference due to a significant difference between the maximum and minimum values of the sample. It can be noted that the taxonomy index in the industry was unstable during the analyzed periods: the closer this indicator is to 1, the lower the level of risk. As you can see, the level of risk of the financial condition is quite high for the analyzed period. According to the results of the forecast, the negative trend of the taxonomic indicator can be stated, so if we do not change the conditions of operation and development of oil and gas companies in the future, negative trends will further worsen the financial condition of companies in Ukraine's oil and gas industry. There is a need to develop effective management solutions to provide enterprises with a vector of positive development, this problem is of national importance and needs proper attention.

To begin with, it is necessary to find the optimal values of input parameters that form a stable financial condition of the enterprise, provide a sufficient level of the studied industry financial condition integrated indicator at a level above average, 0.6-1.0. Thus, we have the first restrictions on the function of the financial condition integrated indicator.

The problem of optimization problems has been in the field of view of domestic economics representatives for a long time. To determine the extreme values classical methods of higher mathematics are widely used, consider one of the most interesting and modern options – genetic algorithms. The most popular from a scientific point of view is, the use of an algorithm for finding optimization solutions using mathematical modulation of genetic processes. In his works, he shows the possibilities and patterns of heredity and variability in genetics in the transfer to the problem of determining the optimization values. Ideas and methods of genetics play an important role in genetic engineering and are applied to economic problems. The mechanism of heredity means the role of genes as elementary carriers of hereditary information. Scientists showed the work of the so-called "genetic" operators of ascent, mutation, mathematical implementation of single-point and multi-point crossover, the search for the most adapted individual.

In mathematics, the problem of stability arises when a physical object is perturbed in phase space, that is, when external forces take it out of equilibrium. As a result, the object can: move away from equilibrium; be in a slight deviation from it; return to equilibrium, withstanding adverse fluctuations.

In fact, the behavior of an object in an disturbed state determines the stability or instability of its undisturbed equilibrium state. Thus, the equilibrium state of an object can be considered stable when, after perturbation, it enters a state close to equilibrium or when it returns to it.

To study the phenomenon of stability in more detail, it is necessary to use the concept of "area of stability". It is often necessary to determine the effect of changing certain parameters on stability. To do this, build the stability area of the object in the space of changing parameters. The area of stability is determined by the set of values of the parameters of the object for which it is stable. Going beyond the parameter limit limits the object from steady to unstable. When the limit of stability is exceeded, the level of risk increases significantly. It is clear that the transition from the zone of stability to the unstable position is determined not by the boundary line, but by some area that can be called transitional.

Drawing analogies between economic and mechanical equilibrium, we should pay attention to the differences between static and dynamic equilibria. At static equilibrium the motion of an object ceases, whereas at dynamic equilibrium the physical body continues to move, but at the same time certain total characteristics of the system remain unchanged. An example is the flow of water in the riverbed: the height of the water and the speed of the flow can be constant, and its parameters, such as inflow and outflow, can change. In other words, static equilibrium implies the ability of the system in it, after minor deviations to return to the previous state, and dynamic equilibrium can be interpreted as the ability of a mechanical system in motion under the influence of certain forces, not to deviate from a given trajectory at insignificant accidental stresses or deviations.

Speaking of the enterprise financial stability, it is advisable to draw analogies with the dynamic stability, because the functioning of the oil and gas sector, its functions, the implementation of the whole complex of active and passive operations is nothing but a dynamic process. Thus, the financial stability of enterprises – one of the key dynamic characteristics of its activities, which largely reveals its viability. In the future, we will consider that the financial stability of the enterprise is a dynamic category, which is the ability to return to equilibrium after leaving it as a result of a certain impact. Sustainability in economic systems, despite some similarities with technical ones, is a much more complex concept. In view of this, analogies can be made for economic systems only conditionally. Due to the fact that a universal approach to assessing financial stability as a single scalar indicator has not yet been developed, we propose to use the tools of multi-criteria optimization, which can be used to implement the concept of economic equilibrium.

We believe that a financially stable enterprise must achieve a certain equilibrium – the optimal ratio between risk, return, liquidity and other key financial performance indicators, on which depends its financial stability. As target functions we will take the key financial indicators of the oil and gas company: current liquidity, autonomy ratio, net margin and receivables turnover, which according to the correlation-regression analysis have the greatest impact on the integrated indicator of financial stability. To achieve a certain equilibrium of the bank should optimize all these criteria, taking into account its real financial condition. In addition, the task of constructing Pareto-effective financial indicators that optimize the integrated indicator of financial stability is proposed.

To find solutions to the multicriteria optimization problem, we use the method of genetic algorithm, which has proven itself well for solving this class of problems. A genetic algorithm is a heuristic search algorithm that is applied to optimization and modeling problems by random selection based on the use of mechanisms resembling evolutionary processes in nature. They are a kind of evolutionary methods of calculation. Genetic algorithm – a method of optimization based on the concepts of natural selection and genetics. In this approach, the variables that characterize the solution are represented as genes on the chromosome. The genetic algorithm, operating on a finite number of solutions (population), generates new solutions in the form of various combinations of parts of the solutions of this population. Operators such as selection, recombination and mutation are used for this purpose.

In a genetic algorithm, a chromosome is a numerical vector that corresponds to a variable. Each of the chromosome vector positions is called a gene.

The genetic algorithm is actually a kind of random search and is based on approaches that resemble the mechanism of natural selection. In a genetic algorithm, some random set of initial data, called a population, is first formed. Each element of the population is called a chromosome and represents some solution of the problem in the first approximation, i.e. satisfies the system of constraints of the problem. Chromosomes evolve during iterations called generations (or generations). During each iteration, the chromosome is evaluated using some degree of compliance (fitness function), which is also called compliance function. A mutation is an operation that implements random changes in different chromosomes.

The simplest mutation is to randomly alter one or more genes. In a genetic algorithm, a mutation plays an important role in restoring genes dropped from a population during a selection operation so that they can be used in new populations. In addition, it allows the formation of genes that were not present in the original population. The intensity of mutations is determined by the mutation rate, which is the proportion of genes that are mutated in this iteration. Too small a value of this factor means that many genes that could be useful will never be considered. At the same time, too large a value of the coefficient will lead to large random perturbations. Descendants will no longer be like their parents and the algorithm will lose the ability to learn while maintaining hereditary traits.

We used the Matlab Optimization Toolbox to find Pareto-effective sets of unit coefficients. The standard adaptive feasible mutation function was chosen as the mutation operator, which is used for constrained tasks and allows you to randomly generate directions based on the most recent successful or unsuccessful generations. To perform the crossover operation, the Scattered method was used, which involves creating a random binary vector and selecting genes from the first parent chromosome for which the corresponding value is 1, or from the second parent chromosome when the value is 0 when combining these genes to form a new offspring.

A multicriteria problem is often understood not as a verbal description of the problem, but as its model, namely: a multicriteria problem is a mathematical model of making an optimal decision based on several criteria. These criteria may reflect assessments of the different qualities of the object or process about which the decision is made. Formally, the multicriteria problem as a model is given in the form:

$$F(x) \to \max \text{ for all } x \in \mathbb{D}, \tag{1}$$

where $\mathbb{D}$ is the set of valid solutions; $F(x)$ is a vector function of the argument x (integral indicator of financial condition), which can be represented as follows:

$$F(x) = [f_1(x), f_1(x) \ldots, \ f_k], \tag{2}$$

where $f_1(x), f_2(x) ... f_k(x)$ – scalar functions of the vector argument x each of which is a mathematical expression of one optimaliti criterion.

Since this model uses a vector objective function, it is often called the problem of vector optimization. Obviously, problem (1) does not belong to the class of mathematical programming problems, because the models of this class of problems always contain only one objective function of the vector argument.

Here we consider a complex vector criterion, which can be used to achieve the maximum effect, without necessarily reaching the extreme in all functions. The existence of a solution that literally maximizes all target functions is a rare exception. The problem of vector optimization in the general case does not have a clear mathematical solution. To obtain a solution, it is necessary to use additional subjective information of a specialist in this subject area, which is commonly referred to as a decision maker. This means that when solving the problem by different specialists with the involvement of different sources of information, most likely different answers will be received. Problems of vector optimization are currently considered in the framework of decision theory [10], the main feature of which is the presence of uncertainty. This uncertainty cannot be ruled out by various modeling techniques and objective calculations. In multicriteria problems, the uncertainty is that it is not known which criterion to prefer and to what extent. To eliminate this uncertainty, it is necessary, firstly: to formulate a special principle of optimality, and secondly: to involve additional subjective information of the decision-maker based on his experience and intuition.

Therefore, in accordance with the above information, we formulate the objective functions and conditions of optimization. The function $G(x)$ is defined as an integrated indicator of financial condition calculated by the taxonomy method and takes into account the levels of unit indicators that were previously selected to be included in the integrated indicator of financial condition $Y(x)$:

$$Y(x) = -0,17432 - 0,00076x_1 + 0,00095x_2 + 0,055039x_3 + 0,125108x_4 \quad (3)$$

and Finscore ($G(x)$):

$$G(x) = 3,639721 + 0,009527x_1 - 0,03471x_2 + 0,13336x_3 - 0,16618x_4. \quad (4)$$

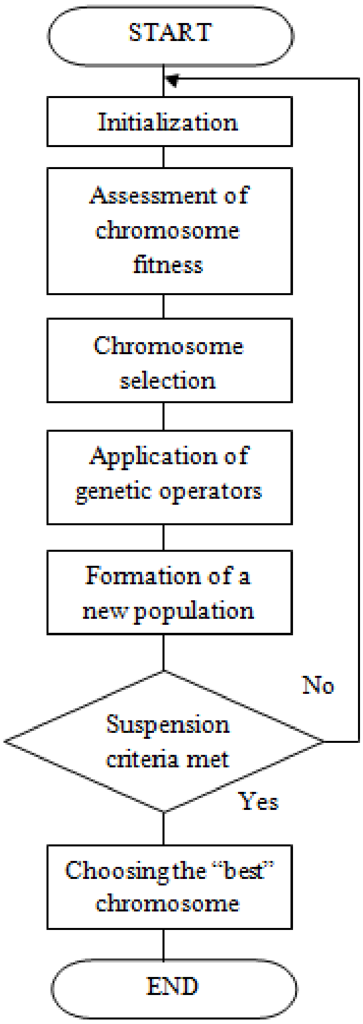

Figure 2: Conditional block diagram of the genetic algorithm.

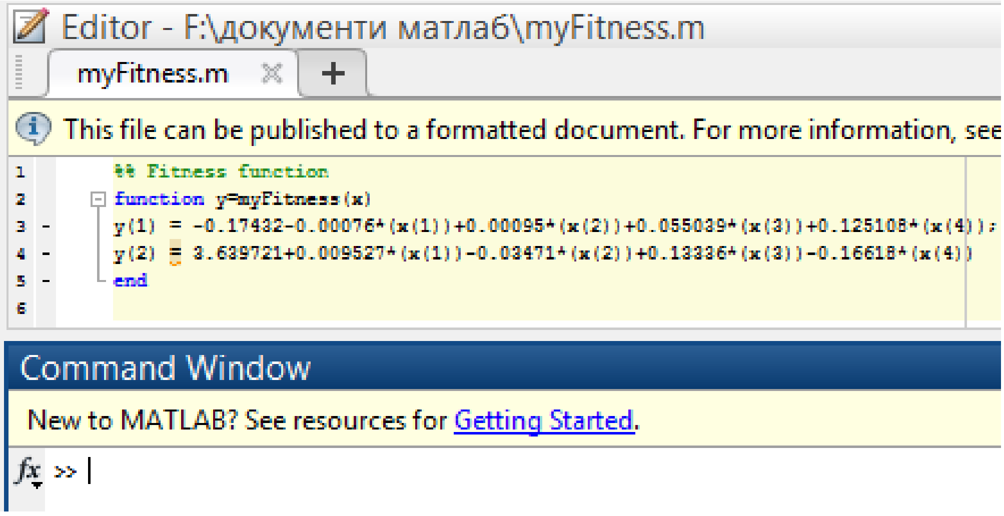

Figure 3: M-file in Matlab.

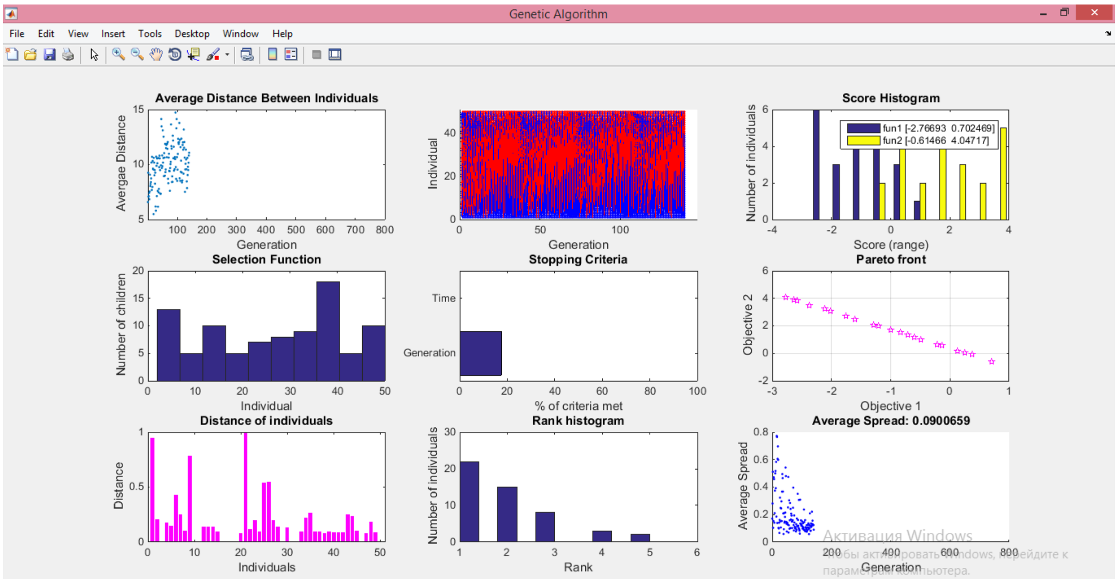

Figure 4: Program results.

Therefore, taking into account the constructed relationship between the studied enterprise financial condition $Y(x)$ and Finscore $G(x)$ integrated indicator and its unit indicators, it is necessary to optimize the complex indicator of financial condition, maximize it by optimizing independent variables.

First, let's set the problem by describing the stages of optimization. Figure 2 shows the stages of the optimization process.

Therefore, we need to optimize, namely to maximize the financial condition, it should be as close as possible to 1, but there are some limitations, which will be described below. Next, we will implement the financial condition integrated indicator function optimization in MatLab with pre-imposed restrictions on independent variables. First, we form an m-file in which we introduce the optimized function, in the economic-mathematical model of the integrated indicator of the financial condition of the object under study. The M-file with the optimized function with the given restrictions is shown in figure 3.

To implement a multi-criteria task, use the built-in Optimization ToolBox. The obtained optimization results are shown in figure 4.

4. Conclusions

With regard to the optimization of financial stability, this means that we have obtained many key indicators that shape the financial condition of the enterprise and it provides an opportunity to form an effective management strategy aimed primarily at achieving optimal values of indicators that together and determine the economic essence of the financial stability of the enterprise.

The analysis of existing approaches to assessing the financial condition of enterprises shows the lack of a unified methodological basis for this issue. The key problem is the lack of a single indicator that would accumulate all aspects of financial condition. The article proposes a method of assessing and optimizing the financial condition based on the concept of maximizing the financial stability of enterprises Finscore. The formulated problem of multicriteria optimization of indicators CurrentRatio, Equity-to-Assets, NPM and ReceivablesTurnover allows to obtain Pareto-optimal combinations, which achieve financial stability as a maximized value (optimal ratio between key indicators of the financial condition of the enterprise).

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Fuzzy modelling of the country's migration attractiveness

Hanna B. Danylchuk[1], Liubov O. Kibalnyk[1], Oksana A. Kovtun[2], Oleg I. Pursky[3] and Zenon Stachowiak[4]

[1]*The Bohdan Khmelnytsky National University of Cherkasy, 81 Shevchenko Blvd., Cherkasy, 18031, Ukraine*

[2]*University of Educational Management, 52A Sichovykh Striltsiv Str., Kyiv, 04053, Ukraine*

[3]*Kyiv National University of Trade and Economics, 19 Kyoto Str., Kyiv, 02156, Ukraine*

[4]*Cardinal Stefan Wyszyński University, 5 Dewajtis Str., 01-815 Warsaw, Poland*

Abstract

The article deals with the analysis the current state of migration in the context of globalization and identifies the most important corridors for the labour movement. The main donor countries of migrants are developing countries, with low socio-economic indicators, difficult environmental conditions and high levels of poverty. According to forecasts, the most migratory flows will take place in the countries of North America and in Europe, which is due to rising trends in unemployment in the countries of the "third world" and the demand for cheap labour, changes in the structure of the economies of developed countries, changes in labour market demand. The main world regional corridors in 1990–2019 have been identified through statistical analysis. And their growing and declining trends. The need to use economic and mathematical modelling techniques to analyse and determine the migration attractiveness of recipient countries in an uncertain environment has been substantiated. It has been shown that fuzzy logic tools are the most effective in this case. Based on the results of the simulation using the Mamdani method, the world's attractiveness rating for migration is calculated, which with a "high" thermo leads such countries as Italy, France, United Arab Emirates. The findings suggest that migrants are attracted by countries with the lowest inflation rates, high and average GDP per capita and average or low taxation levels.

Keywords

labour movement, fuzzy logic, country's migration attractiveness

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abdanilchuk@gmail.com (H. B. Danylchuk); liubovkibalnyk@gmail.com (L. O. Kibalnyk); kovtun.oa71@gmail.com (O. A. Kovtun); Pursky_O@ukr.net (O. I. Pursky); zenstach@wp.pl (Z. Stachowiak)

https://scholar.google.com.ua/citations?user=bfPE5scAAAAJ (H. B. Danylchuk); https://scholar.google.com.ua/citations?user=tPw46YYAAAAJ (L. O. Kibalnyk); http://umo.edu.ua/institutes/imp/struktura-institutu/kaf-upravl-proekt/sklad/kovtun-oksana-anatolijivna (O. A. Kovtun); https://knute.edu.ua/blog/read/?pid=39582&uk (O. I. Pursky); https://bazawiedzy.uksw.edu.pl/info/author/UKSW-71567/Zenon+Stachowiak?tab=main&lang=en&cid=92972 (Z. Stachowiak)

0000-0002-9909-2165 (H. B. Danylchuk); 0000-0001-7659-5627 (L. O. Kibalnyk); 0000-0002-0159-730X (O. A. Kovtun); 0000-0002-1230-0305 (O. I. Pursky); 0000-0001-8842-7743 (Z. Stachowiak)

1. Introduction

Since the 19th century, international labour migration has been the subject of scientific research in various fields. Since the late 1980s and early 1990s, population migration as a social and economic phenomenon has become particularly relevant and global in the context of the development of integration processes which have led to a significant labour movement. Today, migration is characterized by the permanence of territorial flows and the diversity of expression forms. Changes in migration flows are mainly reflected in population growth, territorial movements and the emergence of new types of migration movements. Migration processes have a decisive impact on the socio-economic development of countries and regions as a whole.

By Massey [1] the period of post-industrial migration (since the mid-1960s) has been associated with its emergence as a global phenomenon cause of the number and diversity of donor and recipient countries dramatically increasing. Developing countries, third world countries, have intercepted the baton of dominant donors of migrants. The main flows of migrants were from industrialized countries to post-industrial ones.

The second wave of globalization (from about 1950-s to 1980-s) led to the migration movement from less developed countries (Afghanistan, Pakistan, India, Vietnam, Morocco, Egypt, Turkey, etc.) to more developed West and East countries (Europe, USA, Canada, Japan, etc.). Most migrants were low-skilled workers, employed under temporary employment programmes. In the early 1980s such Asian countries as Korea, Taiwan, Hong Kong, Singapore, Malaysia and Thailand experienced migration.

During the second and the beginning of the third wave of globalization, along with the traditional countries of immigration like the United States, Canada, and Australia, some European countries (Germany, France, etc.) began to accept migrants actively. During that period, the number of migrants in the world rose sharply. From 1970-s to 1980-s there were 82 million people migrants and, in 1980, there were 99 million people. Over the next 10 years, the number of international migrants increased by 56 million people [2].

Recently, many scholars believe that the end of the twentieth and the beginning of the twenty-first century is a phase of unprecedented international migration. However, other researchers, notably Miller and Castles [3] refers to this assumption as a “myth”, as the number of migrants in the world is growing, but their share in the world population does not exceed 2-3 % in the history of migration. More than a century ago, during the period of mass migration to the New World, the share of migrants in the world did not exceed 3 % of the world’s population, so Hatton and Williamson [4] refer to the opinion of Miller and Castles [3] about false migration. It should be noted, however, that the scale of international migration in recent decades has never been greater in the history of human development.

In our view, international migration should be seen as a widespread socio-economic process with many causes and consequences, playing a significant role in the economic, inter-ethnic and demographic changes in the development of communities and society as a whole. It have a projection in the social, political and cultural life.

Considering the relationship between socio-economic and migration processes, it is necessary to take into account the theory of factors that are the causes of “repulsion” or “attraction”. The theory of factors of “attraction – repulsion” was developed by Ravenstein [5], who formulated the “Laws of migration” on the basis of census data in England and Wales at 19th century.

Ravenstein [5] concluded that migration can be explained by factors of "attraction – repulsion" and unfavourable conditions in one region (strict legislation, excessive taxes, etc.), "displace" people from their residence place as well as favorable conditions created in other regions, which "attract" them.

Ravenstein [5] divided all the factors of migration into internal (push factors) and external (pull factors), which can be divided into five groups, namely: economic, social, cultural, political and environmental. Among the economic factors, it is worth mentioning, first of all, the level of wages, the quality of life, the level of unemployment, the stability of economic development and the type of tax system.

The migration laws formulated by Ravenstein [5] were as follows: the main reason for migration is better conditions in another locality than in the one where the person lives; the extent of migration decreases with increasing distance; migration often occurs in several stages; population migration is two-way; people's mobility is determined by their personal characteristics (gender, age, social class, etc.).

Also, the density and volume of migration flows are influenced by the structural changes that occur in social and economic systems related to the cyclicality of the economy, the technological transition and the mismatch of requests (needs) of the National Labour Market Structure for Highly Qualified Professionals [6].

Consequently, the impact of migration processes on the social and economic development of certain regions, countries and their associations is beyond doubt, and scientists and public administrators need to have tools, which would make it possible to develop effective employment policies, to identify possible losses, imbalances and profits for the State from migration flows, to form social packages of support for forced migrants, to improve migration policies etc.

In view of the above, the issues of modelling and forecasting migration flows in the face of globalization challenges are of particular relevance. Considerable attention is paid to the outlined scientific problem in the research of foreign and domestic scientists. Thus, in the work of Novik [7] using the simulation method, namely system dynamics, a system of causal relationships has been constructed, and the main factors influencing the decision of individuals to travel abroad have been identified. In the proposed model two feedback loops are presented, one of them reinforcing (reinforcing loop) and the other – counteracting (counteracting loop). It is the counteracting loop that allows the system after numerous iterations, occurring continuously dynamically in each period, to reach a balance.

Econometric methods and models, namely regression analysis methods, are the most commonly used in studies on migration processes at the regional and national levels. This method is the basis for forecasting and for studying the impact of various objective factors on the dynamics and volumes of migration flows. In scientific researches of Averkyna and Kudrei [8], Ovchinnikova [9] in constructing regression models, it has been proved that the main factors influencing migration processes are the index of average wages, GDP per capita, labour demand, employment and unemployment, Provision of social benefits, employers' need for workers to replace vacant jobs.

Ocheretin et al. [10] proposed approach for modeling the business climate of countries using the taxonomy method. The resulting model can be used to increase the choice of influencing factors when modeling the attractiveness of countries for migrants.

Babenko et al. [11] investigate the possibilities of applying machine learning methods to modeling macroeconomic indicators which have impact on migration processes.

Also Sathler et al. [12] applied the method of statistical spatial modelling, taking into account heterogeneous influences on migration flows, visualized the spatial distribution of net migration, indexes of the internal and external flows among the municipalities of the Brazilian Amazon. The authors have shown that the selected variables demonstrate spatial relationships, and spatial regression models provide more accurate estimates of the indexes by including autoregression with spatial lag.

Aksonova and Derykhovska [13], Porat and Benguigui [14] use a cluster analysis to study migration flows, the results of which confirm that the development of individual regions (countries) is uneven and asymmetrical in terms of the main indicators of labour migration, as well as being the basis for the search for cluster convergence directions.

Akbari [15] applies web-based analysis techniques to research international migration flows when the network is seen as a set of nodes (countries) and arcs with some directions. The migration process is viewed as a socio-spatial network that has a set of nodes located in a geographical space and interconnected ribs of arcs with a certain length. This method allowed the author to identify certain patterns of international migration (asymmetric and reciprocal) and migration clusters.

Through the dynamic multi-factor model based on the assumptions of the theory of positional games, Tarasyev and Jabbar [16] carried out a prediction of the migration behaviour of the individual depending on the economic situation. The impact of migration on the socio-economic development of the recipient country through the Cobb-Douglas production function is also assessed.

Using the agent modelling methodology, Kniveton et al. [17] proved that it is a reliable method of modelling autonomous human behaviour in migration decisions, taking into account not only socio-economic influences but also environmental ones.

It is also suggested that agent models should be applied in situations where human behaviour needs to be investigated in conflict situations affecting migration decisions – internally displaced persons, refugees, undocumented migrants [18]. This concept makes it possible to construct adequate models of the movement of forced migrants and to forecast their likely places of residence, which will enable the public authorities of the State to deal effectively and in a timely manner with the problems of housing and social guarantees, employment and so on.

Using the structural vector autoregression and the estimated model of the dynamic stochastic general equilibrium (DSGE) of a small open economy, Smith and Thoenissen [19] conclude that migration shocks have a significant impact on the volatility of GDP per capita, to replace investment income per capita as well as investment housing and housing.

In summary, contemporary globalization processes have led to the emergence of a new pattern of historical migratory movements and flows, which has led to the emergence of new mega trends. Therefore, traditional methods of analysis and modelling do not always allow for an adequate assessment and forecasting of these processes, hence the need to apply fuzzy logic methodology, that would allow ranking countries according to their migration attractiveness in an uncertain environment [20].

2. Research methods

As noted above, it is useful to use a fuzzy inference engine in the form of a fuzzy set, which corresponds to the current values of the input variables, using fuzzy knowledge base and fuzzy operations, to assess migration processes. Fuzzy sets theory is used specifically to solve problems in which the input data is unreliable and poorly formalised. Currently, fuzzy logic is used in the construction of neural networks, genetic algorithms, and the design of fuzzy systems. Fuzzy logic provides effective means of representation of uncertainty and inaccuracies of the real world, and the presence of mathematical means of representation of uncertainty of input information makes it possible to construct models corresponding to realities [21].

The foundations of the fuzzy sets theory of and fuzzy logic were laid in the 1960s by Zadeh [22]. Thanks to this research a new scientific branch has appeared, which received the name "fuzzy logic". His work laid the foundations for the approximate human reasoning modeling and gave impetus to the development of a new mathematical theory. L. Zadeh introduced the term "fuzzy set", suggesting that the ownership function can accept any value within [0; 1] not just the values of 0 or 1. Also, a series of operations on fuzzy sets has been defined and a generalization of known methods of logical inference has been proposed by introducing the notion of a linguistic variable.

A fuzzy inference engine used in expert and knowledge-based management systems, established subject matter experts or learning neural networks. In turn, the training set of networks is based on experimental data as a set of fuzzy predicate rules of the form:

Rule 1: if $x \in A_1$, than $y \in B_1$,

Rule 2: if $x \in A_2$, than $y \in B_2$,

…

Rule N: if $x \in A_n$, than $y \in B_n$,

where x – input variable, y – output variable, A and B – membership functions defined accordingly x and y.

Expert $A \to B$ knowledge, reflecting the unclear causal relationship between input and output, is called fuzzy connections R:

$$R = A \to B, \tag{1}$$

where "→" is called fuzzy implication.

The relation R can be seen as a fuzzy subset of a direct product $X{\times}Y$ of a complete set of assumptions X and inferences Y. Thus, the process of obtaining a fuzzy output B' with observation A' and knowledge A B represented as follows:

$$B' = A' \cdot R = A' \cdot (A \to B), \tag{2}$$

where "·" – convolution operation [23].

Fuzzy inference algorithms differ in the type of rules, logical operations, and dephasation methods. The most common modifications to the fuzzy inference algorithm are the Mamdani and Sugeno algorithms. The main difference between the two is the way the values of the output variable in the rules are specified, and the knowledge base. In Mamdani-type systems, the values of the input variables are given by fuzzy terms, in Sugeno-type systems it is as a linear combination of the input variables. For tasks where identification is more important, it is useful to use Sugeno algorithm, and for tasks where explanation and justification are more important, Mamdani algorithm will have the advantage.

Mamdani [24] algorithm was one of the first to be used in fuzzy output systems. Formally, Mamdani algorithm can be defined as follows.

Let the knowledge base contain only two fuzzy rules of the kind:

Rule 1: if $x \in A_1$ and $y \in B_1$, than $z \in C_1$,

Rule 2: if $x \in A_2$ and $y \in B_2$, than $z \in C_2$,

where x, y – names of the input variables, z – name of the output variable, A_1, A_2, B_1, B_2, C_1, C_2 – some fuzzy sets, assigned by membership functions $\mu_{A_1}(x)$, $\mu_{A_2}(x)$, $\mu_{B_1}(y)$, $\mu_{B_2}(y)$, $\mu_{C_1}(z)$, $\mu_{C_2}(z)$, the precise value of z_0 should be determined on the basis of the information given and the clear values x_0, y_0.

The operation of implication of fuzzy sets consists of the following four steps:

1. *Fuzziness*: The degrees $\mu_{A_1}(x_0)$, $\mu_{A_2}(x_0)$, $\mu_{B_1}(y_0)$, $\mu_{B_2}(y_0)$ of each premise of each rule. The ownership functions defined on the input variables apply to their actual values to determine the degree of truth of each premise of each rule.
2. *Fuzzy inference*: there are "cut" level for the assumptions of each of the rules (using min operation). The calculated meaning of truth for the assumptions of each rule applies to the conclusions of each rule. This results force one fuzzy subset that will be assigned to each output variable for each rule.

$$\alpha_1 = \mu_{A_1}(x_0) \cap \mu_{B_1}(y_0), \alpha_2 = \mu_{A_2}(x_0) \cap \mu_{B_2}(y_0), \tag{3}$$

where "∩" is the operation of the logical minimum (*min*).

Then there are "cut" membership functions:

$$\mu'_{C_1}(z) = (\alpha_1 \cap \ \mu_{C_1}(z)), \mu'_{C_2}(z) = (\alpha_2 \cap \ \mu_{C_2}(z)). \tag{4}$$

3. *Composition*: using the maximum transaction (*max*, designated as "∪") to find the found cut functions. Result is the resulting fuzzy subset for the output variable with the membership function:

$$\mu_{\epsilon}(x) = \mu_C(z) = \mu'_{C_1}(z) \cup \mu'_{C_2}(z) \tag{5}$$

4. *Clarity* (to find z_0) by centroid method:

$$y = z_0 = \frac{\alpha_1 z_1^* + \alpha_2 z_2^*}{\alpha_1 + \alpha_2} \tag{6}$$

where "ω" – function domain of $\mu_{\Sigma}(x)$ [23].

3. Results and discussions

The United Nations estimates the number of international migrants in 2019. It has reached 272 million people. Consider the structure and trends of global labour migration processes from 1990 to 2019 (table 1), using the geographical topic [25]. In 2019, more than half of all international migrants lived in North America (82.3 million people) and Europe (59 million people). North Africa and West Asia ranks third with the largest number of international migrants (49 million people), Sub-Saharan Africa (24 million people), Central and South Asia (20 million people), and East and South-East Asia (18 million people). Latin America and the Caribbean (12 million people) and Oceania (9 million people) recorded low numbers of international migrants. Through the indexes of the average absolute increase we generated the forecast for 2025 and 2030 (table 1).

Table 1
Labour migration by geography, 1990-2019, mil. people and forecast for 2025, 2030 [25]

Years	Countries							
	North Africa and West Asia	South Africa	Central and South Asia	East and Southeast Asia	Oceania	Latin America and the Caribbean	Europe	North America
1990	4.73	7.16	6.84	26.17	13.29	17.61	27.61	49.61
1995	5.02	6.69	8.34	21.25	14.28	18.91	33.34	53.49
2000	5.36	6.57	10.51	20.47	13.15	20.32	40.35	56.86
2005	6.02	7.22	12.95	18.96	14.22	23.28	45.36	63.59
2010	7.13	8.26	15.75	19.58	15.86	32.56	50.97	70.68
2015	8.07	9.44	17.87	19.44	21.34	42.05	55.63	75.01
2020	8.93	11.67	18.30	19.63	23.57	48.59	58.65	82.30
2025 (Forecast)	9.63	12.42	20.21	18.54	25.28	53.75	63.82	87.75
2030 (Forecast)	10.33	13.17	22.12	17.45	26.99	58.91	68.99	93.20

There is an upward trend in international labour migration in North America, Europe, Latin America, Oceania, Central and South Asia and South and North Africa, and a downward trend only in East and South-East Asia.

The 10 largest regional migration corridors in 2019. Presented in table 2, five of which represent nearly half of the world's migration flows (124 million people).

Table 2
Largest regional labour migration corridors in the world in 2019, mil. people [25]

№	Contributing countries groups	Recipient countries groups	Number of migrants
1	Europe	Europe	41.86
2	Latin America and the Caribbean	South America	26.58
3	North Africa and West Asia	North Africa and West Asia	18.93
4	Central and South Asia	North Africa and West Asia	18.52
5	South Africa	South Africa	18.31
6	East and Southeast Asia	East and Southeast Asia	14.32
7	North Africa and West Asia	Europe	13.05
8	Central and South Asia	Europe	11.20
9	East and Southeast Asia	North America	10.24
10	Latin America and the Caribbean	Latin America and the Caribbean	8.24

The "Europe to Europe" direction, which has 4.19 million people international migrants, is the largest regional migration corridor in the world. A large proportion of these migrants have moved between European Union countries. In 2010–2019, it increased by more than 5 million people international migrants, compared to 2000–2010, with an average annual increase of more than 0.5 million people.

The direction "Latin America and the Caribbean to North America" is the second largest regional migration corridor in 2019 (26.6 million people people). During the period 1990–2000, in this direction the number of migrants increased by 0.9 million people per year, but the growth slowed between 2000–2010 and 2010–2019 (0.5 and 0.3 million people per year, respectively).

The next three largest regional migration directions were almost the same in 2019 (18-19 million people international migrants). The number of international migrants inside the corridor "North Africa and West Asia" increased by 7.3 million people in 2010–2019, while the corridor "Central and South Asia to North Africa and West Asia" increased by 5.4 million people.

While international migration is a global phenomenon, only 20 countries received two thirds of all international migrants in 2019. Almost half of all international migrants live in 10 countries only. The largest number of migrants is in the United States of America, with 51 million people migrants, or about 19% of the world's total number of migrants admitted in 2019. The most attractive countries for migration in 2019 were Germany (13.1 million people) and Saudi Arabia (13.1 million people), the Russian Federation (12 million people) and the United Kingdom (10 million people). Of the 20 most attractive countries for migration, seven are in Europe, four – in North Africa and Western Asia, three – in Central and South Asia, two – in East and South-East Asia and North America, and one each in Oceania and Sub-Saharan Africa.

Between 1990 and 2019, the United States recorded the largest absolute increase in international migrants (27.4 million people). The countries with the largest increases were Saudi Arabia (8.1 million people per year), the United Arab Emirates (7.3 million people per year), Germany (7.2 million people per year) and the United Kingdom (5.9 million people per year).

In 2019 one third of all international migrants come from only 10 countries. In 2019 India has become the leading country of the international migrants origin (17.5 million people). The second largest migrants contributor was Mexico (11.8 million people), followed by China (10.7 million people), the Russian Federation (10.5 million people) and the Syrian Arab Republic (8.2 million people).

Thus, based on data on the volume and intensity of migration processes, it can be concluded that different regions and countries have different attractions for migrants.

The level of the migration attractiveness of countries was determined through the Mamdani fuzzy inference.

The fuzzy inference simulation process was conducted in the Matlab environment using the Fuzzy Logic Toolbox. To construct the Mamdani fuzzy inference system, four input and one output (linguistic) variables in the fuzzy inference system were specified: GDP (gross domestic product per capita), IR (inflation rate), UR (unemployment rate), PIT (personal income tax) and EMA (Evaluation of migration attractiveness) (figure 1). These variables were selected from correlation and regression analysis as relevant to the migration attractiveness of countries.

The phasing of the introduced linguistic variables and the definition of their terms have been carried out. The parameters of the membership functions for these term sets are shown in table 3 and the structure of the functions are shown in figure 2.

Based on the rules, a decision-making mechanism is created, predicts the value of the perfor-

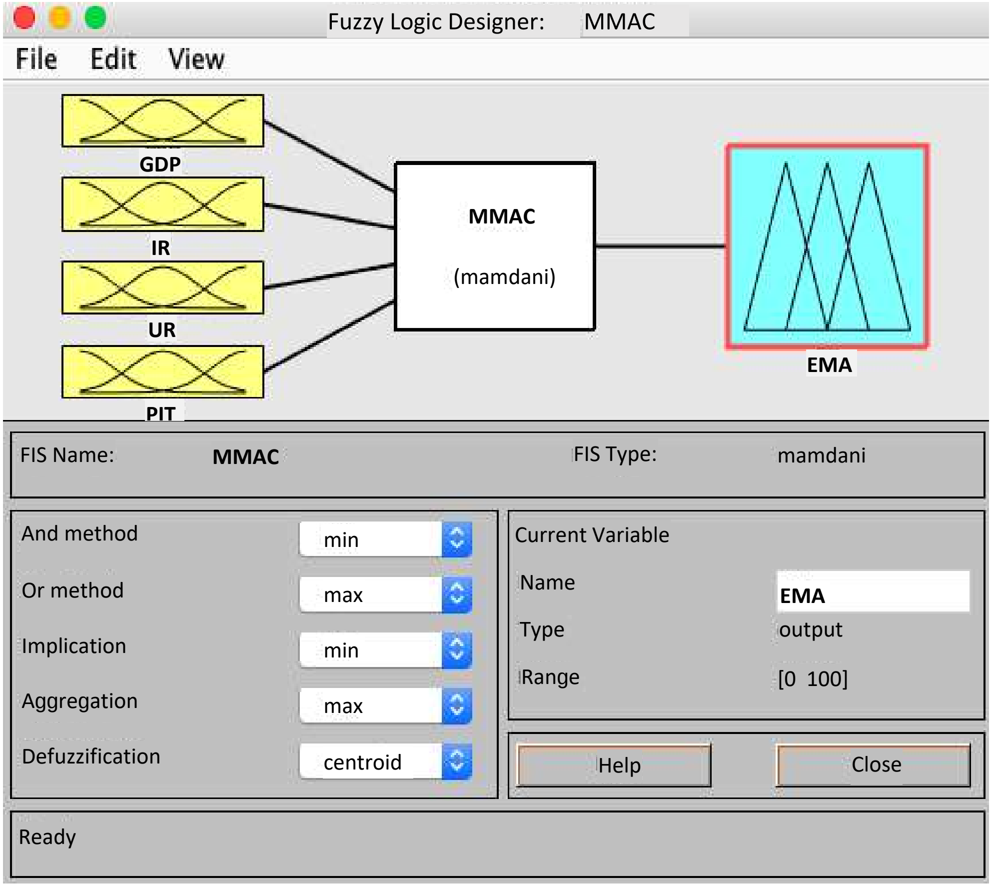

Figure 1: Structure of the model of the evaluation the labour migration attractiveness of countries in 2019 through the Fuzzy Logic Toolbox.

mance variable. The peculiarity of this model is its flexibility, it can be filled with other rules, its content and quantity adjusted.

On the basis of the input variables statistics an evaluating of migration attractiveness was calculated. Figure 3 illustrates Mamdani fuzzy inference with the example of France. The results of the evaluation of migration attractiveness of selected countries are presented in table4.

The analysis of the recipient countries rating allows to draw conclusions that none of the studied countries has received the score «ultrahigh». Italy, France and the United Arab Emirates are among the top three most attractive countries with a «high» term. The middle-level attraction cluster is Saudi Arabia, the UK, Canada, the US, and Australia. Russia has the lowest rating. The results are largely consistent with the statistical analysis, but allow for a clearer definition of a country's ranking. It should be noted that the proposed fuzzy model for the evaluating the migration attractiveness of recipient countries can be further developed and refined by introducing additional input variables.

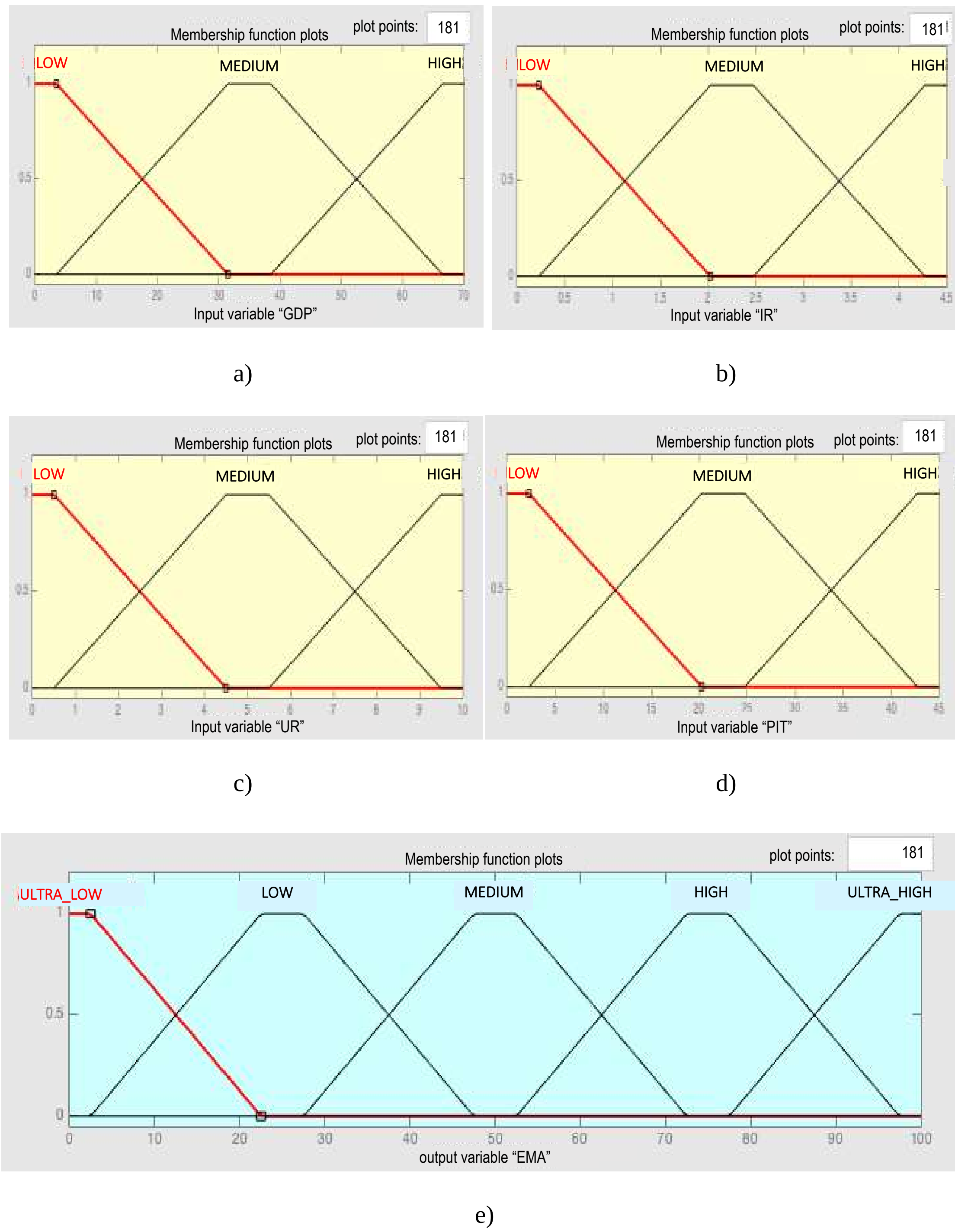

Figure 2: Membership function graphs for input (a, b, c, d) and output (e) linguistic variables.

Table 3
Attributes of the attribution model of the evaluation the labour migration attractiveness of countries in 2019

Linguistic variable	Evaluating interval	Term	Evaluating rate
		low	0 – 23.3
GDP (thousand $)	[0; 70]	medium	23.3 – 46.6
		high	46.6 – 70
		low	0 – 1.5
IR (%)	[0; 4.5]	medium	1.5 – 3
		high	4 – 4.5
		low	0 – 3.3
UR (%)	[0; 10]	medium	3.3 – 6.6
		high	6.6 – 10
		low	0 – 15
PIT (%)	[0; 45]	medium	15 – 30
		high	30 – 45
		ultralow	0 – 20
		low	20 – 40
EMA	[0; 100]	medium	40 – 60
		high	60 – 80
		ultrahigh	80 – 100

Table 4
Rating of recipient countries' attractiveness to international labour migration, 2019 [25, 26, 27, 28, 29]

Country	Input variables				Output variable	Rank
	GDP, thousand $	IR, %	UR, %	PIT, %	EMA	
Italy	44.20	0.6	9.8	43	68.9	1
France	49.44	1.1	8.3	45	62.8	2
United Arab Emirates	69.90	1.9	2.4	0	60.3	3
Saudi Arabia	48.91	2.1	5.9	0	58.3	4
Great Britain	48.71	1.7	4.1	45	56.3	5
Canada	51.34	1.9	5.4	33	52.9	6
USA	65.12	1.8	3.9	37	52.5	7
Germany	56.05	1.4	3	45	51.1	8
Australia	53.32	1.6	5.3	45	50	9
Russia	29.18	4.5	4.4	13	46.6	10

4. Conclusion

Thus, using the Mamdani fuzzy inference method, a system has been developed that allows to make a decision on the selection of the optimal country for labour migration based on the analysis of the most popular recipient countries in 2019. The parameters of the Mamdani model are interpreted quite easily, and the use of fuzzy logic makes it possible to model economic problems effectively in order to analyse the economic indicators of the migration attractiveness of countries. As a result of the implementation of this model, recipient countries are ranked

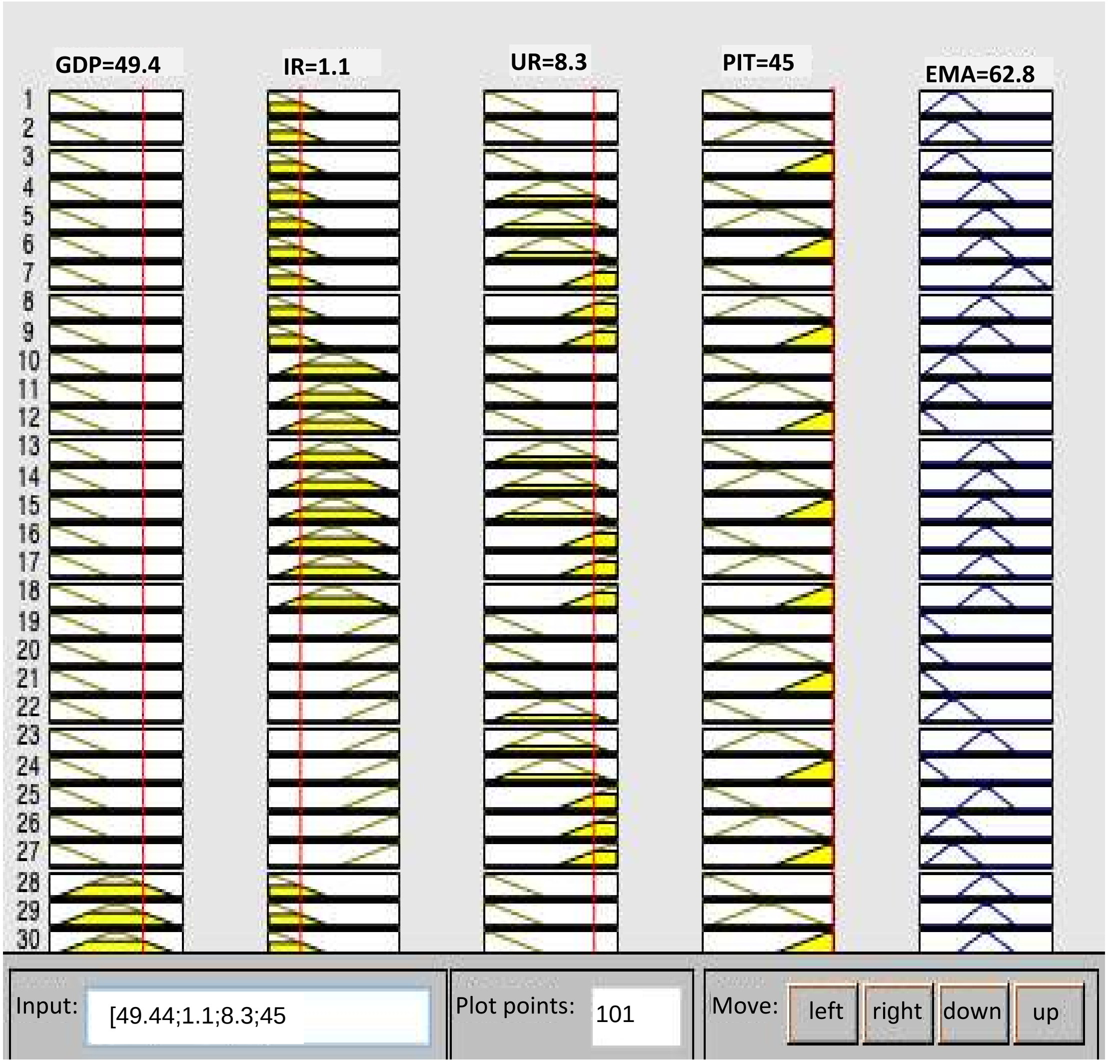

Figure 3: Implementation of Mamdani fuzzy inference using Fuzzy Logic Toolbox of Matlab package.

as being the most attractive for migrant employees. Such countries as Italy, France and the United Arab Emirates, which have medium and high levels of per capita GDP, low inflation, and a "loyal" level of tax, were in the first three positions.

The model could be improved to reflect global trends in 2020 and the impact of the pandemic on global migration flows.

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Computational method determining integral risk indicators of regional socio-economic development

Oleg I. Pursky[1], Tetiana V. Dubovyk[1], Iryna O. Buchatska[1], Iryna S. Lutsenko[2] and Hanna B. Danylchuk[3]

[1]*Kyiv National University of Trade and Economics, 19 Kioto Str., Kyiv, 02156, Ukraine*

[2]*National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", 37 Peremohy Ave., Kyiv, 03056, Ukraine*

[3]*The Bohdan Khmelnytsky National University of Cherkasy, 81 Shevchenko Blvd., Cherkasy, 18031, Ukraine*

Abstract

In this study, we present the computational method for risk assessment of the socio-economic development of regions. An attempt has been made to develop a method for the determination of integral risk indicators of socio-economic development based on the joint use of the methods of factor analysis and expert evaluation. This approach has increased the reliability of the calculations and made it possible to analyze the influence of socio-economic indicators on the risk level of socio-economic development. The integral risk indicator shows the effect of the inconsistency in the level of factor provision on the socio-economic development of the *j*-th region (district) in comparison with the general situation in the country (regions). The closer the value of integral risk indicator is to 1, the higher the level of risk in this region. Using Kyiv region districts as an example, the process of risk assessment for regional socio-economic development has been considered. The results obtained in this investigation demonstrate that the presented computational method solves the problem of formalization of risk assessment for the socio-economic development of regions.

Keywords

computational method, risk assessment, socio-economic development of regions

1. Introduction

The market today is functioning in a turbulent environment facing continuous change because of hyper-competition, changing demands of customers, regulatory changes, and technological advancement [1]. Modern world economic conditions, economic globalization, acceleration of market development processes, information technologies, socio-political factors require public

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Pursky_O@ukr.net (O. I. Pursky); tatiana_dubovik@i.ua (T. V. Dubovyk); i.buchatska@knute.edu.ua (I. O. Buchatska); lutsenkois0802@gmail.com (I. S. Lutsenko); abdanilchuk@gmail.com (H. B. Danylchuk)

https://knute.edu.ua/blog/read/?pid=39582&uk (O. I. Pursky); https://knute.edu.ua/blog/read/?pid=41510 (T. V. Dubovyk); https://knute.edu.ua/blog/read/?pid=43144&uk (I. O. Buchatska); https://kafedra.management.fmm.kpi.ua/test/?p=1260 (I. S. Lutsenko); https://scholar.google.com.ua/citations?user=bfPE5scAAAAJ (H. B. Danylchuk)

0000-0002-1230-0305 (O. I. Pursky); 0000-0001-9223-4629 (T. V. Dubovyk); 0000-0003-2413-7370 (I. O. Buchatska); 0000-0001-5134-811X (I. S. Lutsenko); 0000-0002-9909-2165 (H. B. Danylchuk)

administration new approaches to the formation of socio-economic strategies, development of adequate methodological solutions, and tools in the field of governance, especially it concerns socio-economic development management of regions [2, 3]. Using modern information technologies and new electronic communication channels significantly reduce costs related to organization and support social activity and business, and the new possibilities allow re-designing socio-economic development strategy at any moment [4]. In connection with the globalization and the processes of post-industrial economy development, the effect of unpredictability appears in the change of socio-economic systems state due to the increasing influence of economic crises, suddenly emerging threats and risks [5, 6, 7, 8, 9]. One of the urgent problems in the risk analysis of socio-economic systems is the construction of adequate methods. This is due to the multidimensionality of socio-economic systems, the stochasticity of their behavior, as well as the complex interaction between the elements of the systems [6, 8].

In socio-economic studies, to improve the reliability of the procedure for assess-ment of socio-economic development using mechanism for determining the integral indicators based on factor analysis, taking into account consider the knowledge and experience of experts [10]. Accordingly, the aim of this study is to develop a reliable computational method for risk assessment of regional socio-economic development on the basis of the joint use of the methods of factor analysis and expert evaluation [11]. This article poses and solves the problem of formalization of risk assessment for regional socio-economic development with the Kyiv region districts as an example.

2. Computational method of risk assessment

This section presents a method for determining the integral risk indicator of regional socio-economic development. The method is based on a model for determination of the socio-economic development integral indicator which is described in detail in [10]. In this model, methods of factor analysis and expert evaluations are used to determine integral indicators. To reducing the dimension of the feature space (socio-economic indicators), one of the methods of factor analysis is used [12, 13, 14], the principal component analysis (PCA) [15, 16]. Based on the reduced set of independent factors, a single integrated indicator is obtained, which combine all these factors in the best way [16, 17]. The main disadvantage of factor analysis methods is the reliability of the conclusions, in particular, in this model of determining the integral indicators [10], the weight of the factor is determined by the dispersion of initial indicators, which is not always reliable in socio-economic studies, since in this case the importance of indicators for the socio-economic system is not taken into account [10, 17, 18]. Therefore, within the framework of this model, in order to increase the reliability of the algorithm for determining the integral indicators based on factor analysis, expert evaluation procedures have been introduced in the mechanism of determining the weight of the factors [10, 19, 20]. In this case, the generalized weight of factors that takes into account both the weight of the factor, determined on the basis of expert evaluations, and the weight of the factor determined statistically, can be obtained as the weighted average of these two evaluations [10, 21]:

$$w_i = (\overline{q_i} + \overline{v_i}) \Bigg/ \sum_{i=1}^{n} (\overline{q_i} + \overline{v_i}), \tag{1}$$

where $\overline{q_i} = q_i \Big/ \sum_i^n q_i, \overline{v_i} = v_i \Big/ \sum_i^n v_i$ are expert and statistical (factor analysis) weighted coefficients of the factor, respectively. Thus, the complex indicator of socio-economic development for j-th region is calculated as the sum of factors with the corresponding weighted average weight coefficients w_i [10]:

$$I_j = \sum_{i=1}^{n} w_i F_{ij} \ (j = 1, 2, ..., n), \tag{2}$$

where n is a number of factors; F_{ij} is the value of the i-th factor for the j-th object (region). Taking into account the proposed complex indicators of socio-economic development (2), the value of the integral risk indicator R in the region can be calculated using the following formula [22, 23]:

$$R_j = 1 - \frac{\left(I_j / I_{avg}\right)}{m}, \tag{3}$$

where R_j is the integral risk assessment for socio-economic development of j-th region; I_{avg} is the numerical value of the complex indicator I in average in study regions; m is the number of factors. In this formula (3), the risk assessment is calculated taking into account the average value of the complex indicator I_{avg} for all regions and the number of factors. In this case, the risk R_j varies in the range from 0 to 1. The integral indicator R_j shows the effect of the inconsistency in the level of factor provision on the socio-economic development of the j-th region (district) in comparison with the general situation in the country (regions). The closer the value of R_j is to 1, the higher the level of risk in this region.

Figure 1 illustrates a general scheme of the developed computational method for determining the regional socio-economic development integral risk indicator. This method makes it possible to develop a procedure for automated data processing of socio-economic research and includes the following stages:

1 stage. The entering of values of indicators of socio-economic development and expert evaluations in the form of the matrix of indicators and the matrix of expert evaluations of indicators, followed by its normalization to a single scale of measurements.

2 stage. The calculation of the pairwise correlations matrix and determination of its eigenvalues and eigenvectors.

3 stage. Obtaining a matrix of factors by multiplying the normalized matrix of indicators and the matrix of eigenvectors, and normalization of factors and calculation of their variance.

4 stage. Determining the number of N factors included in the integral risk indicator (figure 1) on the basis of eigenvalues of the matrix of pairwise correlations of indicators and the given boundary value L dispersion of normalized indicators or in other words – a sampling of the minimum number of factors with maximal eigenvalues λ_i is made, the sum values of which are not less than nL [10].

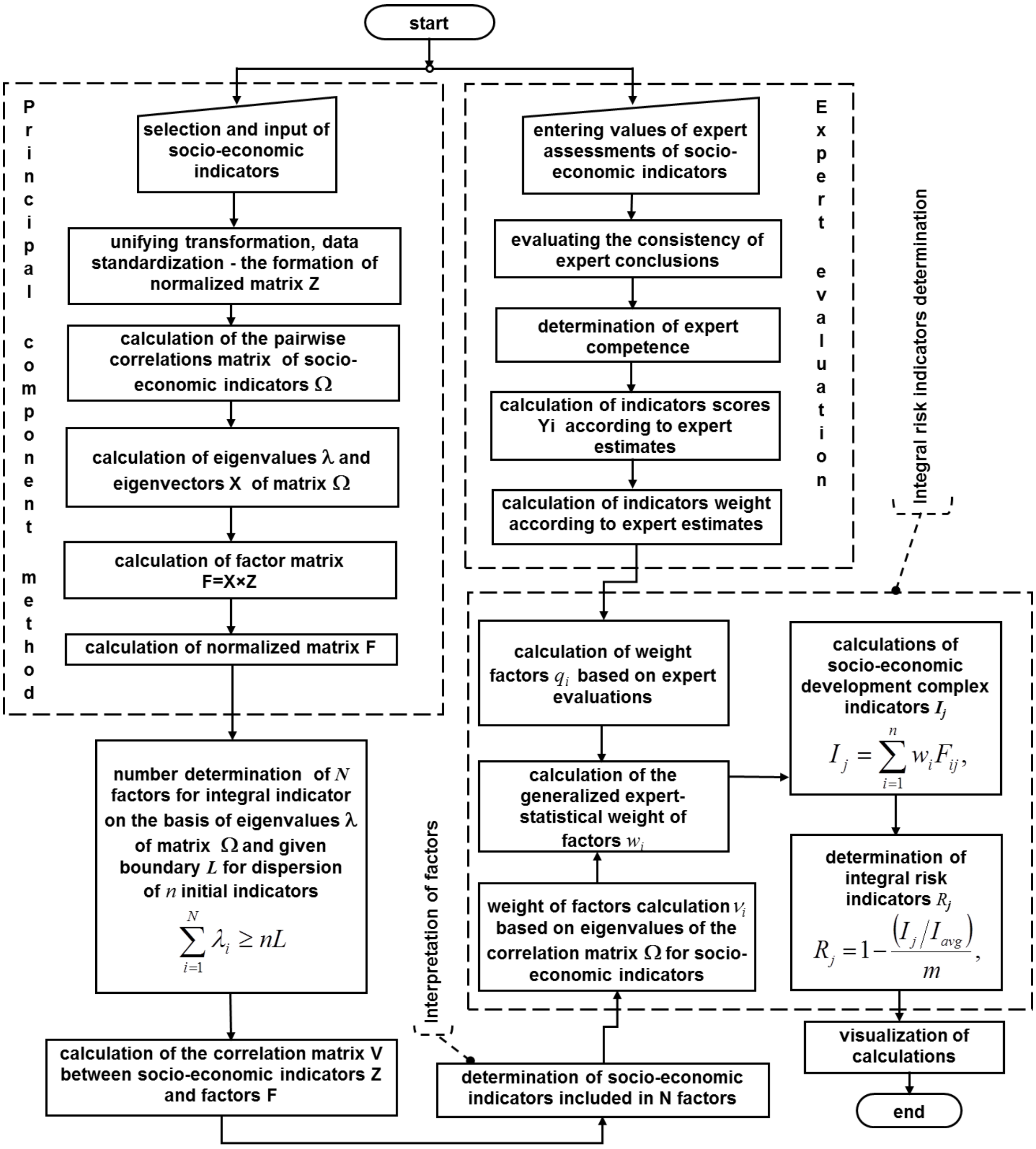

Figure 1: General scheme of the computational method for risk assessment of regional socio-economic development.

5 stage. Determining the relative contribution $\%(F_i)$ of each of the N factors in the description of the total dispersion of all n indicators as the ratio of the eigenvalue λ_i of the factor F_i to the total dispersion of the features, which is also equal to n:

$$\sum_{i=1}^{N} \lambda_i \geq nL, \%(F_i) = \lambda_i \Big/ \sum \lambda_i = \lambda_i / n \qquad (4)$$

6 stage. The determination of the experts competence and calculation of Kendall's coefficient of concordance for evaluation of the consistency of their conclusions.

7 stage. The determination of weighting coefficients of factors (1) included in the integrated indicators (2).

8 stage. The calculation of integral risk indicators of regional socio-economic development (3) and the visualization of the results of data processing.

3. Risk assessment for regional socio-economic development of the Kyiv region districts

In this section, we consider the process of risk assessment for regional socio-economic development in accordance with the developed computation method for integral risk indicators (figure 1) on the example of the Kyiv region districts. Listed in a single scale of measurements and normalized values of socio-economic indicators of districts are presented in table 1.

Table 1
Normalized values of socio-economic indicators

Districts	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11
D1	-0,209	-0,054	-0,094	-0,129	0,077	-0,270	-0,063	-0,157	-0,171	0,065	-0,310
D2	-0,069	-0,075	-0,461	0,186	-0,005	0,294	-0,049	-0,027	-0,062	-0,240	-0,301
D3	0,044	-0,050	-0,088	0,300	0,253	-0,108	-0,068	-0,252	-0,298	-0,465	-0,236
D4	-0,077	0,016	0,005	0,174	0,223	-0,005	0,005	-0,142	-0,221	-0,287	-0,328
D5	0,468	-0,026	0,027	0,199	0,004	0,046	-0,031	0,111	0,058	-0,220	0,129
D6	0,036	0,013	-0,130	-0,129	0,255	-0,228	0,011	0,018	-0,053	0,097	-0,063
D7	-0,109	-0,071	-0,312	0,174	-0,173	-0,330	-0,001	-0,084	-0,044	-0,167	-0,312
D8	0,048	0,219	0,127	0,161	0,026	0,251	0,038	-0,211	0,021	-0,172	0,004
D9	-0,035	-0,086	0,131	-0,192	0,360	-0,142	-0,066	-0,043	-0,072	0,110	-0,090
D10	-0,170	-0,064	0,110	0,123	-0,219	0,054	-0,069	-0,219	-0,139	0,076	-0,175
D11	-0,125	-0,061	-0,131	0,035	-0,259	0,037	-0,064	-0,243	-0,014	0,066	0,087
D12	0,475	-0,067	-0,030	0,110	-0,058	-0,288	-0,049	0,071	0,020	-0,071	0,209
D13	-0,193	0,928	0,466	-0,205	-0,129	0,089	0,948	0,594	0,671	0,200	0,250
D14	-0,128	-0,052	-0,102	0,186	0,331	0,037	-0,037	0,014	0,027	-0,147	-0,071
D15	0,085	-0,041	-0,006	-0,381	-0,219	0,183	-0,061	0,110	0,229	0,306	0,181
D16	-0,170	0,073	0,110	-0,230	0,067	-0,176	0,162	0,198	0,256	0,254	0,313
D17	-0,003	-0,066	-0,292	-0,230	-0,166	-0,091	-0,069	0,250	0,129	0,156	0,089
D18	-0,189	-0,086	0,338	0,060	-0,285	0,106	-0,069	-0,222	-0,288	0,159	0,206
D19	0,043	-0,077	0,226	0,161	0,151	-0,228	-0,068	-0,088	-0,190	-0,002	0,101
D20	-0,166	-0,066	-0,169	-0,167	0,270	-0,031	-0,066	0,108	0,076	0,096	-0,118
D21	0,274	-0,080	0,189	-0,066	-0,126	0,114	-0,069	0,034	-0,057	0,097	-0,081
D22	0,090	-0,067	0,043	0,174	0,084	-0,168	-0,069	-0,097	-0,166	-0,132	-0,045
D23	0,389	-0,057	0,165	0,211	-0,044	0,285	-0,069	0,056	-0,035	-0,268	-0,003
D24	-0,182	-0,082	-0,088	-0,419	-0,067	0,465	-0,063	-0,154	0,188	0,231	0,275
D25	-0,125	-0,020	-0,036	-0,104	-0,352	0,106	-0,062	0,375	0,136	0,259	0,291

According to the National State Statistics Service of Ukraine [24], one of the main indicators that characterize the level regional socio-economic development are (table 1):

(P1) Number of cars per 1000 people;
(P2) Services rendered per unit of population, UAH;
(P3) Natural increase (reduction) of the population;
(P4) Registered unemployment rate;
(P5) Average monthly salary, UAH;
(P6) Provision of housing by the population, m^2 per person;
(P7) The ratio of m^2 of built housing to the population;
(P8) Preschool establishments per unit of population;
(P9) General educational institutions per unit of population;
(P10) Number of crimes per 1000 people;
(P11) Emissions of pollutants.

It should be noted that the list of indicators, depending on the goals and objectives of the risk assessing, may change, thereby changing its emphasis. Thus, for the Kyiv region we have a matrix of initial socio-economic indicators in the size of 25 × 11 (25 districts of the Kyiv region: Baryshivsky (D1), Bilotserkivsky (D2), Boguslavsky (D3), Boryspilsky (D4), Borodyansky (D5), Brovarsky (D6), Vasylkivsky (D7), Vyshgorod (D8), Volodarsky (D9), Zgurivsky (D10), Ivankivsky (D11), Kagarlytsky (D12), Kyiv-Sviatoshynsky (13), Makarivsky (D14), Myronivsky (D15), Obukhovsky (D16), Perejaslav-Khmelnytsky (D17), Polissya (D18), Rokytnyansky (D19), Skvyrsky (D20), Stavyshchensky (D21), Tarashchansky (D22), Tetiivsky (D23), Fastivsky (D24), Yahotynsky (D25)).

On the basis of the normalized matrix of socio-economic indicators (table 1), the pairwise correlations matrix of indicators is dimensioned 11 × 11. For the pairwise correlations matrix of indicators, we determine eigenvalues λ (table 2) and eigenvectors $\mathbf{X}$. The matrix of factors F is obtained by multiplying the normalized matrix of socio-economic indicators (table 1) into the matrix of the eigenvectors of the pairwise correlations matrix. The obtained factors are normalized. The normalized factor matrix is used to calculate the matrix of correlations between factors and indicators of socio-economic development that is required for the interpretation of factors.

Table 2
Eigenvalues of the pairwise correlation matrix of indicators

λ	4,565	1,733	1,193	0,931	0,764	0,587	0,495	0,364	0,244	0,097	0,022
$\%(F_i)$	41,51%	15,76%	10,85%	8,46%	6,95%	5,34%	4,50%	3,31%	2,22%	0,89%	0,20%
$\sum \%$	41,51%	57,27%	68,11%	76,58%	83,53%	88,87%	93,37%	96,69%	98,91%	99,80%	100,00%

On the basis of the calculated eigenvalues of the pairwise correlations matrix (table 2) and the given threshold L of the dispersion for normalized socioeconomic indicators (table 1), the formula (4) determines the number of N factors in the integral risk indicator. In this case, the number of main components (factors) must be used, which exhaust at least 60-70% of the variance of the initial random variables. For example, at a given threshold of 0,6 from table 2 it is necessary to select N factors with maximal eigenvalues, the sum of values of which is not less than $0,6 \times 11$ = 6,6. The sum of the first three eigenvalues λ is 7,49 that is the integral index consists of the first three factors (N = 3) that explain approximately 68% (see formula 4) of the variance of the initial data (table 2). The calculate matrix of correlations between the normalized socio-economic indicators and the factors shows, which indicators are included in the given three factors (with the value of the variance of the indicators should not be less than the given limit value of 0,6). Table 3 shows the structure of factors: the coefficient of correlation between the indicators and factors in which they are included, statistical and expert weights coefficients and weighted average weight coefficient of factors. The first factor included the first four indicators: 1) the number of cars per 1000 people; 2) services rendered per unit of population; 3) natural increase (reduction) of the population; 4) the level of registered unemployment. The second factor included the eleventh indicator – emissions of pollutants. The third factor entered the seventh indicator – the ratio m^2 of the built housing to the population.

Table 3
Structure of factors and their weight coefficients

Factors	Socio-economic indicators	Correlation coefficient	Statistical weight coefficient	Experts weight coefficient	Weighted average weight coefficient
F1	P1	0,912	0,456	0,269	0,363
	P2	0,873			
	P3	0,741			
	P4	0,647			
F2	P11	0,794	0,293	0,086	0,191
F3	P7	0,663	0,251	0,644	0,446

By multiplying the obtained factors by the corresponding weighted average weight coefficients of factors, by formula (2) we obtain the values of integral risk indicators according to formula (3) that allow ranking the regions in terms of their risk assessment of socio-economic development (figure 2). In table 4 calculations results of integral risk indicators R_j of social and economic development of Kyiv region districts are presented. It is also worth noting that for a better understanding of the calculation procedures should be carefully study the model that presented in [10].

Table 4
The value of integral risk indicators of socio-economic development calculated for Kyiv region districts

Districts	R_j	Districts	R_j	Districts	R_j	Districts	R_j	Districts	R_j
D9	0,805	D22	0,744	D15	0,684	D19	0,669	D8	0,567
D16	0,803	D20	0,722	D25	0,677	D3	0,661	D14	0,553
D23	0,758	D17	0,711	D2	0,675	D24	0,636	D6	0,529
D10	0,755	D18	0,709	D1	0,671	D7	0,620	D13	0,502
D21	0,752	D12	0,704	D5	0,670	D11	0,608	D4	0,483

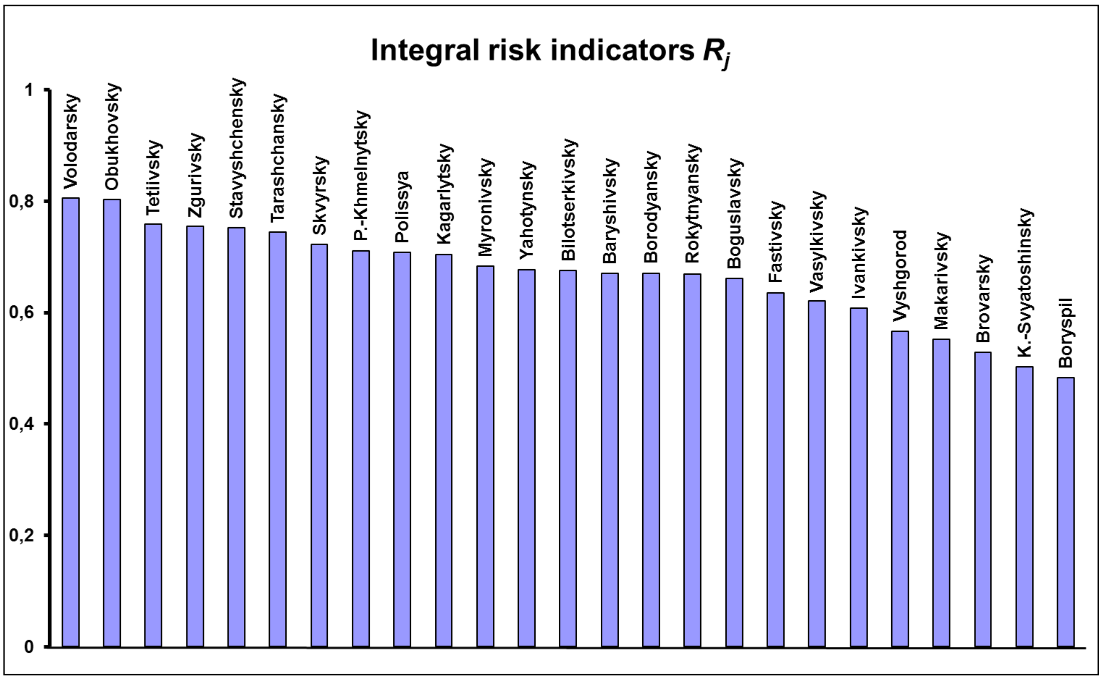

Figure 2: Ranking of regions in terms of their risk assessment of socio-economic development.

4. Conclusions

The obtained results demonstrate that the presented computational method solves the problem of formalization of risk assessment for the socio-economic development and can be used to analyze and predict the socio-economic situation in the region. In the framework of the presented method by changing the values of socio-economic indicators, it is possible to analyze and model the socio-economic situation in the region, which undoubtedly provides management with valuable information on possible risks and directions of effective strategies for socio-economic development. The management receives not only an adequate assessment of the risk level of socio-economic development of the region, but also the opportunity to determine the immediate causes and consequences that shape the current and future socio-economic situation in the region. The results of modeling the process of assessing the level of socio-economic development of Kyiv region showed that the main advantages of the method of determining the integrated risk indicators are the possibility of studying correlations between socio-economic indicators, between indicators and factors, interpretation of factors, determining negative and positive characteristics of socio-economic situations in context of research objects. The proposed method for risk assessment makes it possible to implement a unified approach to data analysis and to ensure the efficiency of constructing integral risk indicators.

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Modelling the logistics system of an enterprise producing two type of goods

Roman V. Ivanov[1], Yuriy V. Sherstennikov[1], Vasyl M. Porokhnya[2] and Tetyana V. Grynko[1]

[1]*Oles Honchar Dnipro National University, 72 Haharina Ave., Dnipro, 49000, Ukraine*
[2]*Classical Private University, 70B Zhukovskoho Str., Zaporizhzhia, 69002, Ukraine*

Abstract
The study is devoted to solving the scientific problem of optimizing the retail trade in the production and sale of two types of products, taking into account the change in potential demand for products. The economic and mathematical model of the production activity of the enterprise was developed taking into account logistics and market demand. The logistics scheme takes into account all the main links of the logistics system, as well as the connections between them. The considered scheme makes it possible to take into account the diversification of products manufactured by the enterprise. The mathematical model is designed for discrete time. A numerical optimization method has been developed for this mathematical model. The optimal solutions for several cases are found and investigated. The dynamics of the main characteristics of drugs was calculated for all considered cases. A comparative analysis of economic efficiency for the studied cases has been performed. The economic efficiency of retail network optimization is proved.

Keywords
model of the production activity, economic efficiency, retail network optimization

1. Introduction

Today in the scientific literature, much attention is paid to the modeling of logistics processes and production. The main purpose of article was to present the possibilities and examples of the use of Tecnomatix (Siemens) plant simulation to simulate logistics and manufacturing processes. This tool allows you to simulate discrete events and create digital models of logistics systems (for example, manufacturing), optimize the operation of factories, production lines, as well as individual logistics processes. A review of the execution of a Tecnomatix plant simulation for

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romanivanov1926@gmail.com (R. V. Ivanov); hm001@ukr.net (Y. V. Sherstennikov); vprhnp76@gmail.com (V. M. Porokhnya); greisy25@gmail.com (T. V. Grynko)
https://scholar.google.com.ua/citations?user=4vRuOxEAAAAJ (R. V. Ivanov); https://scholar.google.com.ua/citations?user=c1v9cL8AAAAJ (Y. V. Sherstennikov); http://virtuni.education.zp.ua/info_cpu/en/node/736 (V. M. Porokhnya); http://www.eup.dp.ua/grinko.html (T. V. Grynko)
0000-0003-2086-5004 (R. V. Ivanov); 0000-0003-4931-6455 (Y. V. Sherstennikov); 0000-0002-7882-4523 (T. V. Grynko)

simulating processes in manufacturing engineering and logistics was conducted and several selected simulations were presented.

Further research should elaborate on the few initial attempts to combine different modeling techniques with optimization [1]. Simulation is an appropriate technique to tackle unresolved issues where analytical computations fail [2].

Bucki and Suchánek [3] highlights the problems of mathematical modeling for a certain element of the logistics supply system, that is, the production system. The system of the production complex, consisting of a determining number of parallel subsystems, is modeled.

If a company is unable to adapt to changing market conditions, it cannot survive in today's market environment.

To survive in a highly competitive global economy, production systems must be able to adapt to new circumstances [4]. As it is clear from the above and other contexts, many different indicators must be taken into account for assessing the complexity of selected general process structures when designing a structure or optimizing production objectives [5].

The structure of production processes also depends on the production needs of certain products, which can show high variability. The organization of production processes is closely related to the process mappings and procedures responsible for production processes from individual components [6]. A key prerequisite for the effectiveness of the above and other production systems is the precise definition of the interaction between the links of the logistics system (LS) [6].

In addition, the requirements are growing in order to control possible destruction of production systems [7]. Rapid recovery of production in case of errors and other risks associated with the general I/O model, which is the production system, should be considered [8].

Production generally occurs as a series of individual actions that are performed manually, mechanically, or a combination of these. Optimization requires continuously processed orders in relation to individual projects. It can also be the sequential production of one type of product, in which we can easily identify a number of key performance indicators and manage and automate production [9, 10]. Today, the standard starting point for calculating and optimizing manufacturing systems is simulation. Computer simulations allow us to test various types of production quickly. Computer simulation makes it possible to check many consequences of changes in production, processes and selects the most efficient way to streamline logistics in the near future [11]. Simulation can be used both before calculating the design of the production system and in order to optimize the production system and in the design of production processes, respectively. In both cases, it is necessary to consider the simulation results as input and information for the design or redesign of a thoughtful system. In addition to defining the structure of production systems (in manufacturing and logistics systems, of a particularly cautious nature, see, for example, Popovics et al. [12], Greasley [13], Malik and Leduc [14]), simulations are useful for planning production and its sustainability and continuity [15, 16]. Specifically, simulations can help coordinate the needs of different departments and open management bottlenecks and improve resource allocation, allocation of production between production lines or factories, testing strategies, performance dimension, and so on [17, 18]. The analysis of the articles makes it possible to draw the following conclusions. Simulation modeling is an urgent problem in planning and optimizing an enterprise's LS. Most of the research is devoted to a detailed study of certain links of LS. Studies of the entire logistics system from the

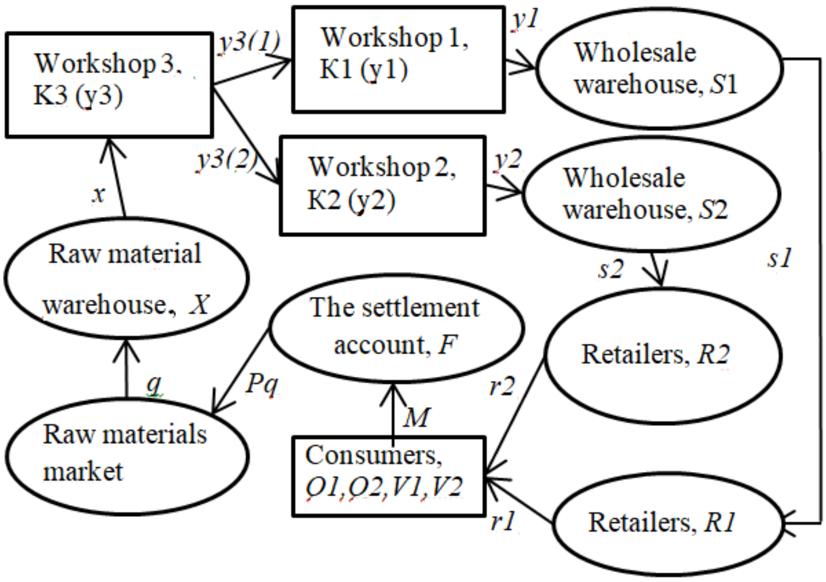

Figure 1: Scheme of the enterprise's logistics.

purchase of raw materials to the delivery of finished products to the final consumer have been insufficiently completed. The article aims to fill in part the existing research gap.

The aim of the study is to develop an economic and mathematical model of the LS of an enterprise producing two types of products [19]; using this model to develop a methodology for optimizing the work of such an important link in the LS as retail trade in the production and sale of two types of products.

To achieve the goal, the following tasks were set: draw up a system of equations for the proposed scheme of the enterprise's LS; on the basis of the obtained system of equations, develop a mathematical program for optimizing various modes of the LS operation; to compare the economic efficiency for different operating modes of the enterprise.

2. Results

We will consider the work of an enterprise that produces two types of products. Let the logistics system of an enterprise be represented by the diagram shown in figure 1.

Figure 1 shows that after the production link, each type of product is delivered to the end consumer via independent supply chains. In fact, wholesale warehouses for each type of product can be only certain areas, in some warehouse reserved for the first and second types of products. The same applies to retail chains. In part, these can be different outlets, to which either product 1 or product 2 is imported. It can also be supermarkets in which products 1 and 2 are sold in different (or even the same) departments. But in a mathematical description, it is convenient to represent these chains as independent. This presentation does not diminish the generality of the description.

In previous works [20, 21, 22] the production link was considered as an unstructured black box. Figure 1 represents the production process in two stages. Workshop 3 will carry out preliminary processing of raw materials and those operations that are common in the production of both types of products. Workshops 1 and 2 perform the final operations that are typical only for products of the first and second types, respectively. The presented scheme can also describe the joint work of a group of enterprises, some of them can be producers of products, while others can act as distributors of products.

Let us formulate a system of equations that describe the LS of the enterprise shown in figure 1. We use this notation. Each variable aj_i contains two indices. The j line index numbers the types of products ($j = 1,\ 2$); subscript i numbers time intervals (days) ($i = \overline{1, 730}$). We will consider a project with a planning horizon of two years.

1. A change in demand Qj_i for products on the market is an input effect for an enterprise whose task is to bring its output into line with demand.

$$rj_{i+1} = nj \cdot Rj_i \cdot (Qj_i - Vj_i) \tag{1}$$

   where rj_i is the rate of sales of the j product (pieces / unit of time) in the i-th; Rj_i – quantity j of goods in the retail trade network in the i-th period; Vj_i is the quantity j of the consumer's product (not yet consumed).
2. The quantity of goods in the retail network Rj_i is determined by the recurrent formula:

$$Rj_{i+1}=Rj_i+\left(sj_i-rj_i\right), \tag{2}$$

   where sj_i is the rate of deliveries (units per period) from a wholesale warehouse in a retail network.
3. The value Rj_i must be in the range $0 \leq Rj_i \leq Rmj$, where Rmj is the maximum possible quantity of the product in the retail system. The following formula for the rate of deliveries from the wholesale warehouse to the retail system corresponds to this requirement:

$$sj_{i+1} = \min\left[r_{ji} \cdot \left(1 + \frac{Rmj - Rj_i}{Rmj}\right), Rmj - Rj_i, Sj_i\right], \tag{3}$$

   where Sj_i is the stock of goods (quantity) in the j-th wholesale warehouse.
4. The rate of production yj_i is determined by the following formula:

$$yj_{i+1}=\frac{Yj_i}{tYj}, \tag{4}$$

   where Yj_i – the value of work in progress in the i-th period; tYj is a production parameter.
5. The value of work in progress is determined by the formula:

$$Yj_{i+1} = Yj_i + a3j \cdot y3_i - yj_i \tag{5}$$

6. The value (quantity) of stock of goods in the wholesale warehouse Sj_i is calculated by the formula:

$$Sj_{i+1}=Sj_i+yj_i-sj_i, \tag{6}$$

   where yj_i is the rate of flow that enters the warehouse from production.

7. The amount of raw materials purchased daily is determined by the following formula:

$$q_i = \begin{cases} q0, & \text{if } i < Tq, \\ kq \cdot q0, & \text{otherwise.} \end{cases} \tag{7}$$

Formula (7) allows you to describe the process of changing the volume of daily purchases of raw materials. Such a change may be necessary as a result of the project implementation during the initial period of time $[0; Tq]$.

8. The stock of raw materials in the raw material warehouse is calculated by the formula:

$$X_{i+1} = X_i + q_i - x_i. \tag{8}$$

9. The rate of supply of raw materials to the third workshop is calculated by the formula:

$$x_{i+1} = X_i / tX. \tag{9}$$

10. The amount of goods among consumers (not yet consumed):

$$Vj_{i+1} = Vj_i + rj_i - k1 \cdot Vj_i. \tag{10}$$

11. Such a formula has been adopted to determine the daily net profit of the enterprise, expressed in the corresponding monetary units (MU):

$$\begin{aligned} M_i = (1 - kp) \cdot [(1 - kad) \cdot (p1 \cdot r1_i + p2 \cdot r2_i) - \\ -p1 \cdot c1 \cdot y1_i - ks \cdot (S1_i + S2_i) - z \cdot (Rv1_i + Rv2_i) - Pq \cdot q_i], \end{aligned} \tag{11}$$

where pj is the unit price (MU per unit of production); cj – the share of the cost in the cost of production; z, ks – payment for storage of a unit of goods for one period in a retail network and in a wholesale warehouse, respectively (MU per unit of production); kp is the income tax rate; kad is the rate of value added tax; Pq – raw material unit cost (MU per unit of raw materials).

The system of equations (1) – (11) is a mathematical model of the logistic system of an enterprise that manufactures two types of products and operates in accordance with the scheme shown in figure 1. The system of equations (1) – (11) contains quantities of two types: variables with a subscript and constants. All calculations by model (1) – (11) will be performed with the following values of constants, which units of measure are described above

$$\begin{gathered} z = 0,01; ks = 0,04; c1 = 0,4; c2 = 0,45; \\ Pq = 2; Q1 = 500; Q2 = 500; tX = 10; \\ tS1 = 3; tY1 = 3; tY2 = 4; tY3 = 5; \\ kp = 0,25; k1 = 0,33; k2 = 0,33; kq = 0,5; \\ a31 = 0,3; a32 = 0,7; p1 = 9; p2 = 11; pk = 10 \\ Rm1 = 30; Rm2 = 58; tQ1 = 300; kQ1 = 0,8; \end{gathered} \tag{12}$$

$$n1 = 6,05 \cdot 10^{-5}; n2 = 7,56 \cdot 10^{-5}; kad = 0,06$$

Initial values:

$$\begin{array}{cccc} q_0 = 3,2 & x_0 = 0 & s1_0 = 0 & s2_0 = 0 \\ X_0 = 0 & Y1_0 = 0 & Y2_0 = 0 & M_0 = 0 \\ S1_0 = 0 & S2_0 = 0 & R1_0 = 0 & R2_0 = 0 \\ r1_0 = 0 & y1_0 = 0 & y2_0 = 0 & y3_0 = 0 \\ & V1_0 = 0 & V2_0 = 0 & \end{array} \tag{13}$$

The initial values (13) correspond to the case when a new project starts, so to speak, from scratch. Although the model allows you to describe an ongoing project.

Case 1.

We are considering an enterprise operating in accordance with the diagram in figure 1. We consider the demand for products unchanged throughout the entire life cycle T of the project. We are considering a project lasting two years (T = 730 days). The final products of the enterprise are produced only by the first and second shops. Parts of the production of the first and second shops make up 30% and 70% in the total amount of the final product, respectively. This means that in equation (5), the values of the parameters should be chosen as $a31$ = 0,3, $a32$ = 0,7. Figure 2 shows the ratio of work-in-process parameters for this case.

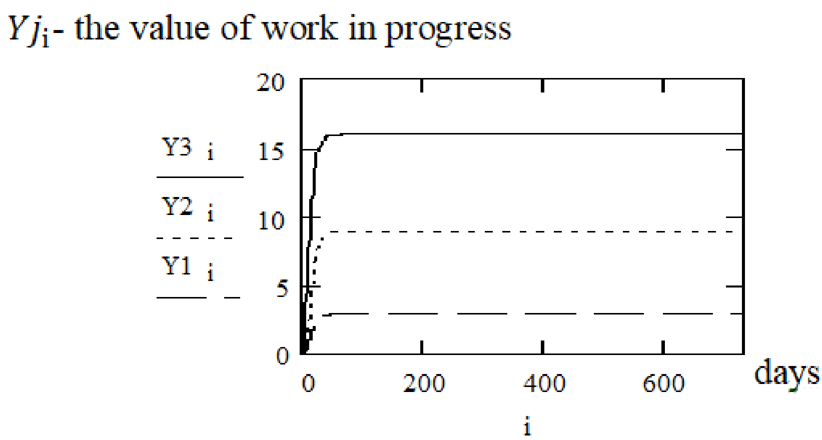

Figure 2: Dynamics of values of work in progress.

Figure 3 illustrates the dynamics of production capacities of each of the three shops, calculated in accordance with the system of equations (1)-(11).

Calculations have established that $y3_i = y1_i + y2_i$, as it should be.

Figure 4 shows that in the interval [0; 200] the rate of deliveries of the first product at retail $s1_i$ exceeds the rate of sales of the first product $r1_i$ and as a result the quantity of the first product in retail $R1_i$ in this time interval increases from 0 to $Rv1_i$, as can be seen from figure 5.

Then, by the 200th period, the rate of retail supply $s1_i$ becomes equal to the rate of sales of the first product $r1_i$ as a result of which the quantity of the first product $R1_i$ in retail stabilizes at the level $Rv1_i$ (figure 5).

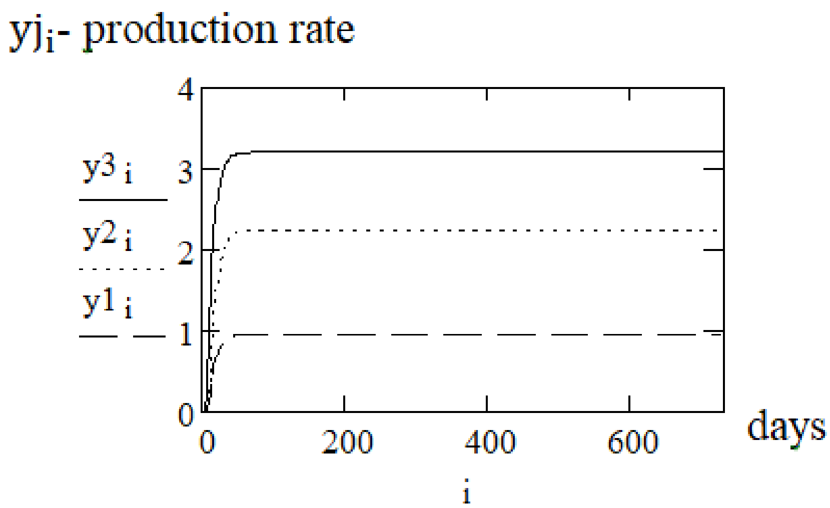

Figure 3: Dynamics of values of work in progress.

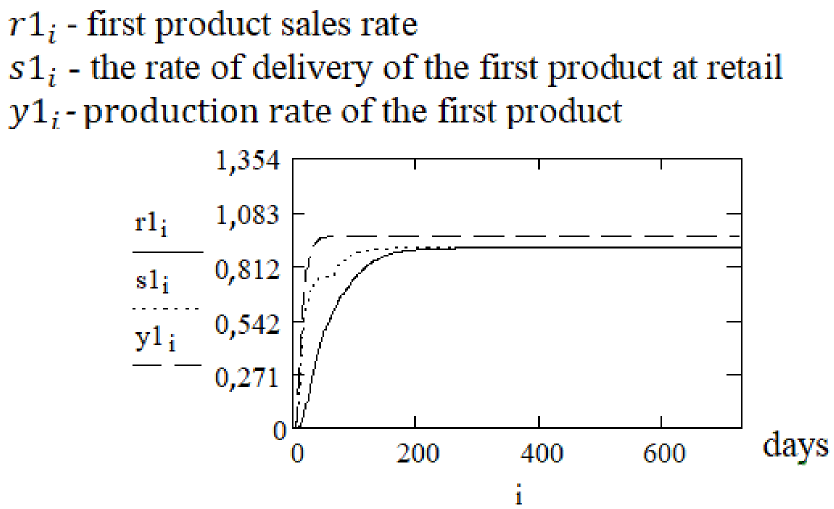

Figure 4: Dynamics of the main rates in LS for the first product.

Figure 4 shows that for all periods the rate of production $y1_i$ exceeds the rate of retail deliveries $s1_i$ and, as a consequence, the stock of goods in the wholesale warehouse $S1_i$ monotonically grows to a value of 51,3, which can be seen from figure 5.

Figure 6 shows the dynamics of the main indicators for the second product. The analogy between the behavior of the main indicators, which is visible from the comparison of figure 5 and figure 6 allows us to conclude that the dynamics of the main rates of both types of goods is similar.

This does not mean that you can arbitrarily set the rate of production of goods of each type. The rate of sales is the factor that determines the work of all parts of the LS.

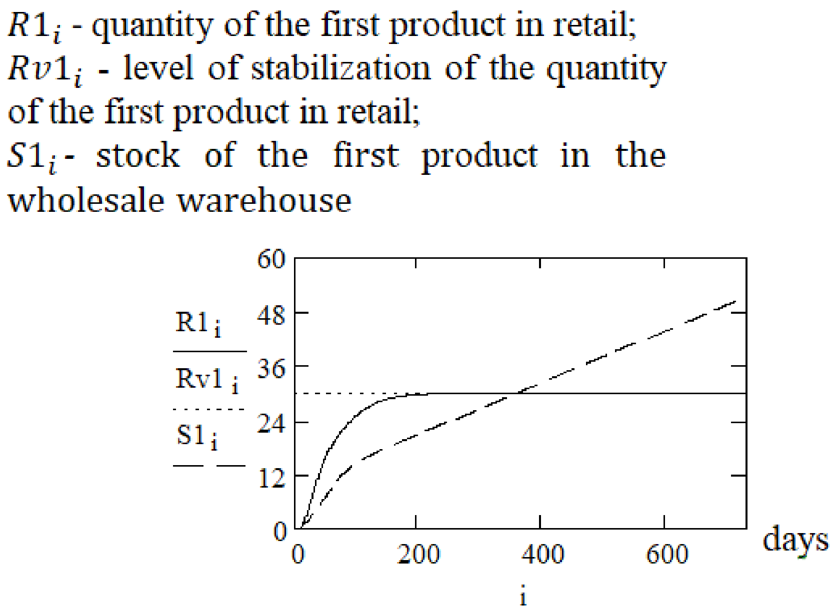

Figure 5: Dynamics of the main indicators in LS for the first product.

Figure 7 illustrates the dynamics of daily profit over the entire life cycle of a project. Figure 7, it can be seen that after reaching the maximum value in the 200th period, the daily profit begins to decrease monotonically. This decline means that the project life cycle will be limited in time.

The total profit over the lifetime of the project will be $\sum_{i=1}^{T} M_i$=1712 (MU).

Case 2.

Above, we considered the case of constant demand for both types of products. Now let us consider the case when, in the 300th period, the demand for products of the first type decreases abruptly by 20%, and for products of the second type remains unchanged. Figure 8 shows the dynamics of the main rates for the first type of goods for this case. The sales rate $r1_i$ and the retail supply rate $s1_i$ decrease by 19,6% in the 300th period.

The main rates for the second product remain unchanged. Figure 9 shows the dynamics of the quantity of goods in the wholesale warehouse $S1_i$ and in retail $R1_i$. The dynamics of the quantity of goods in retail $R1_i$ has not changed (figure 5). The dynamics of the quantity of goods in the wholesale warehouse $S1_i$ changes sharply after the 300th period. This abrupt change is explained in figure 8: there is a constant rate of deliveries y_i to the wholesale warehouse, while the rate of removal of goods $s1_i$ from the wholesale warehouse at the 300th period sharply decreases.

As seen from figure 10 the dynamics of the main indicators for the second product of goods remains unchanged (see figure 6).

Figure 11 illustrates the dynamics of daily profit throughout the entire life cycle of the project for the case under consideration. Figure 11 shows that with the 300th period, the daily profit decreases sharply. This decrease is due to a decrease in the rate of sales at a constant rate of

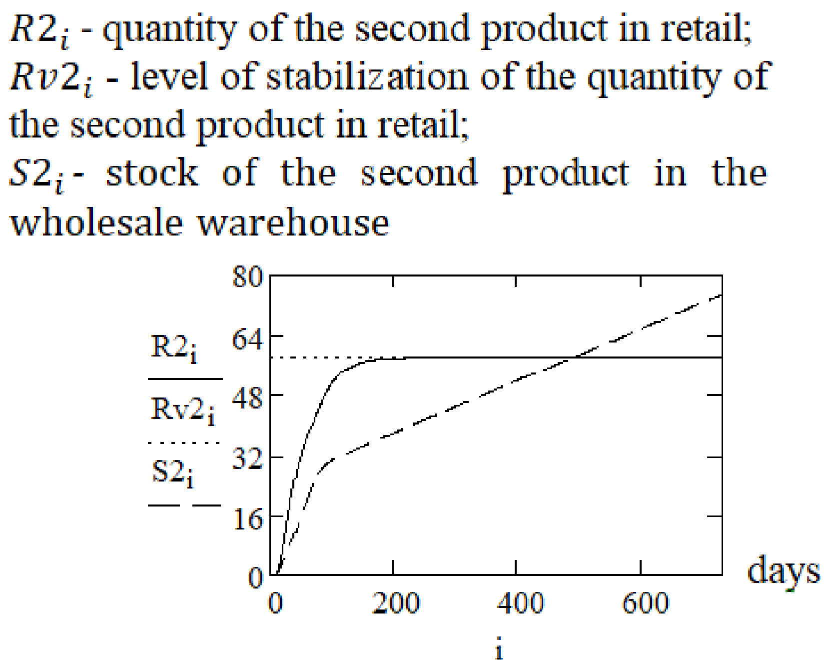

Figure 6: Dynamics of the main indicators in LS for the second product.

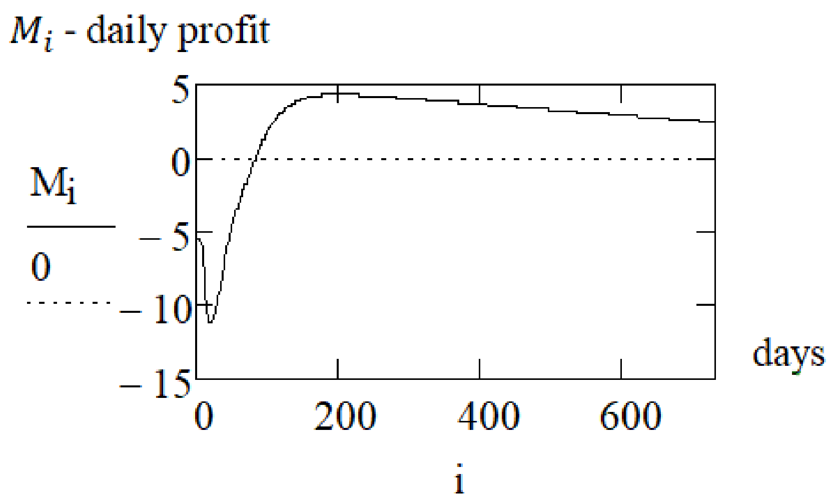

Figure 7: Dynamics of the daily profit of the enterprise.

production.

In this case, the total profit over the lifetime of the project will be $\sum_{i=1}^{T} M_i = 723,1$ (MU). This value is significantly less than case 1.

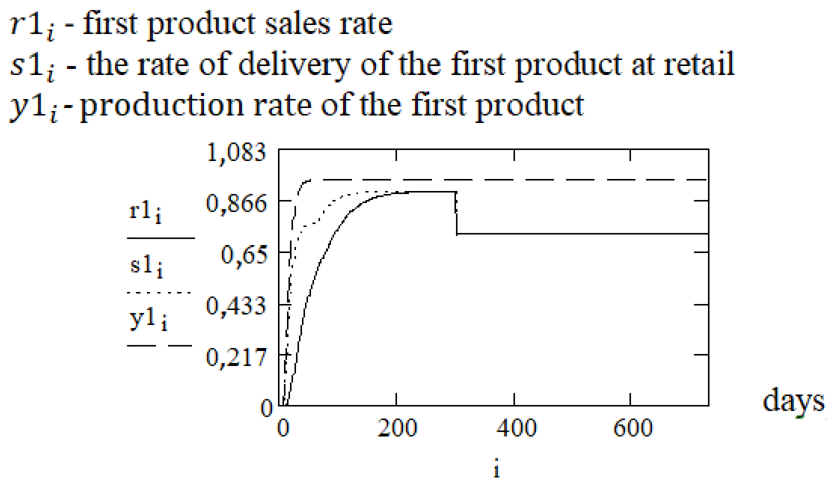

Figure 8: Dynamics of the main rates in LS for the first product in Case 2.

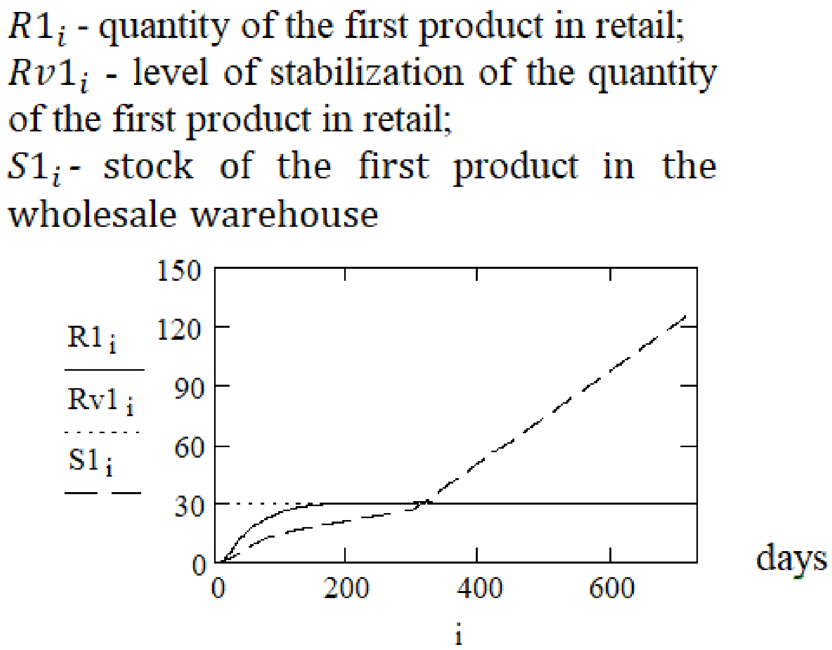

Figure 9: Dynamics of the main indicators in LS for the first product in Case 2.

Case 3.

Now consider the following situation. The demand for products of the first type decreases sharply as in the previous case, and for products of the second type remains unchanged. However, the company, foreseeing significant loss of profit, decides to increase the retail network for

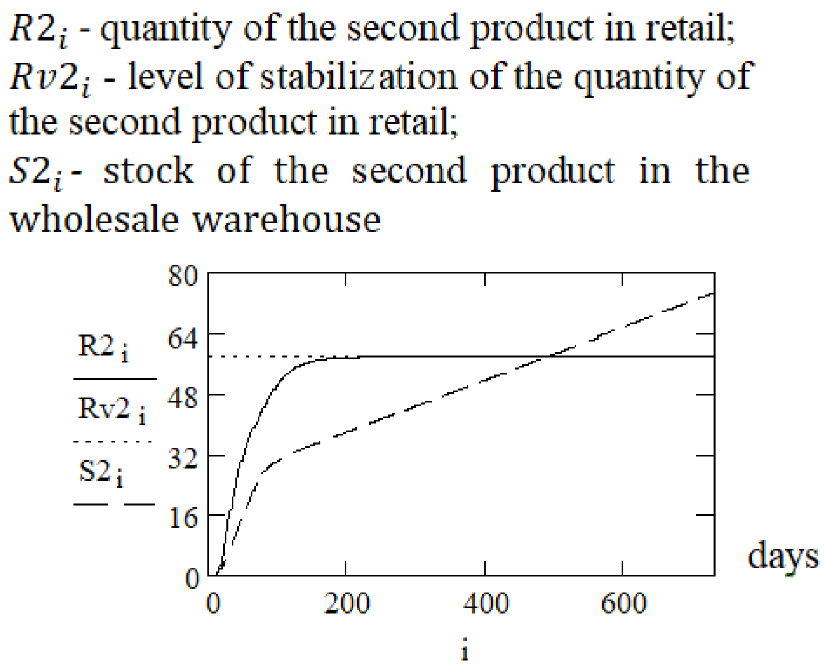

Figure 10: Dynamics of the main indicators in LS for the second product in Case 2.

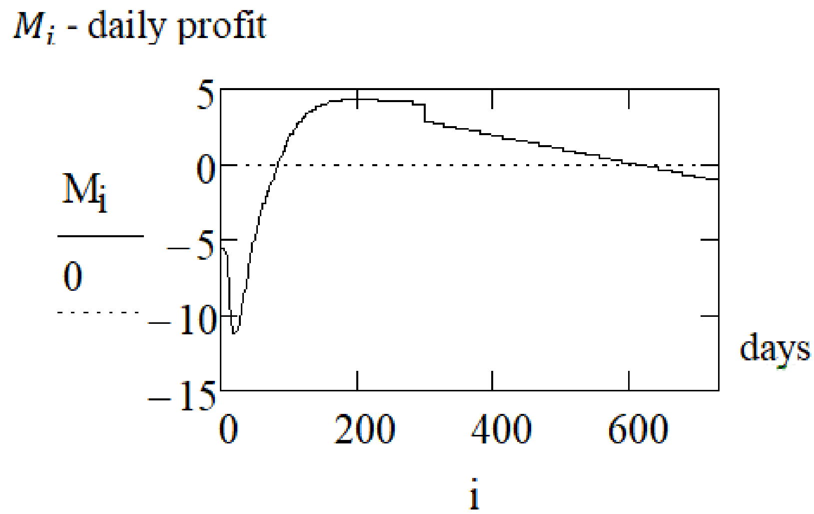

Figure 11: Dynamics of the daily profit of the enterprise in Case 2.

goods of the first type by 20% in the 300th period:

$$Rv1_i = \begin{cases} 30, & \text{if } i < 300 \\ 36, & \text{otherwise} \end{cases} \tag{14}$$

Figure 12 shows the result of calculating the main rates for the first product according to the model (1) – (11) taking into account (14).

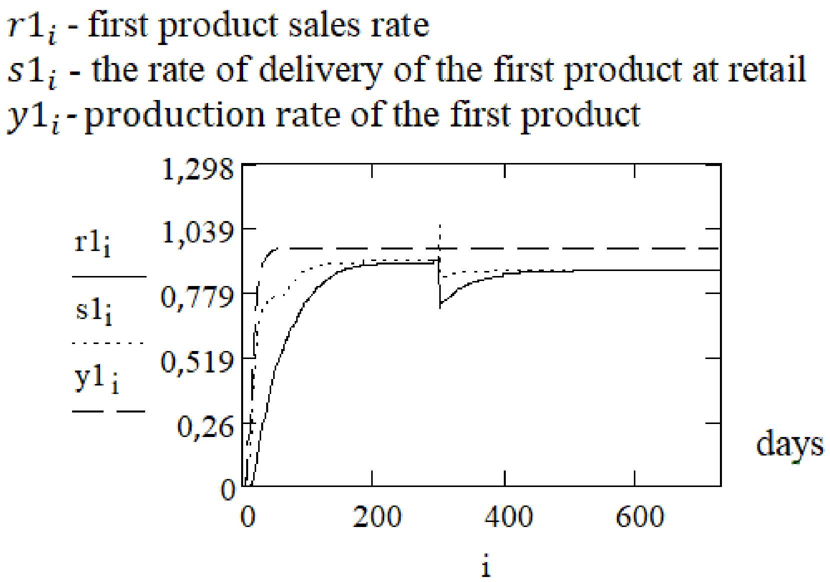

Figure 12: Dynamics of the main rates in LS for the first product in Case 3.

Figure 12 it can be seen that at the 300th period the sales rate $r1_i$ of the first product decreases sharply, but, unlike the previous case (see figure 8), after the 300th period it begins to gradually increase and at the 500th period it reaches the rate of deliveries in retail $s1_i$.

Figure 13 shows the dynamics of the levels of stocks of goods in the sweat warehouse $S1_i$ and in the retail trade $R1_i$.

The balance of goods in the wholesale warehouse at the end of the project in this case is 67,6, which is significantly less than the balance in the previous case – 128,5 (see figure 9).

This decrease is due to the increase in sales rates and is a favorable factor for increasing the total net profit, which in this case is $\sum_{i=1}^{T} M_i = 1442\ (MU)$, which is significantly more than case 2 (723,1 (MU)).

Comparison of economic results in the second and third cases shows the need to optimize drug parameters.

We will optimize the parameters of the retail trade, since it is the retail trade that directly affects the profit of the enterprise. We will assume that only the following retail transformation is available for both types of goods:

$$Rv1_i = \begin{cases} Rm1, & \text{if } i < tR1 \\ kR1 \cdot Rm1, & \text{otherwise} \end{cases}, \tag{15}$$

$$Rv2_i = \begin{cases} Rm2, & \text{if } i < tR2 \\ kR2 \cdot Rm2, & \text{otherwise} \end{cases}, \tag{16}$$

where $tR1 = tR2 = 300$.

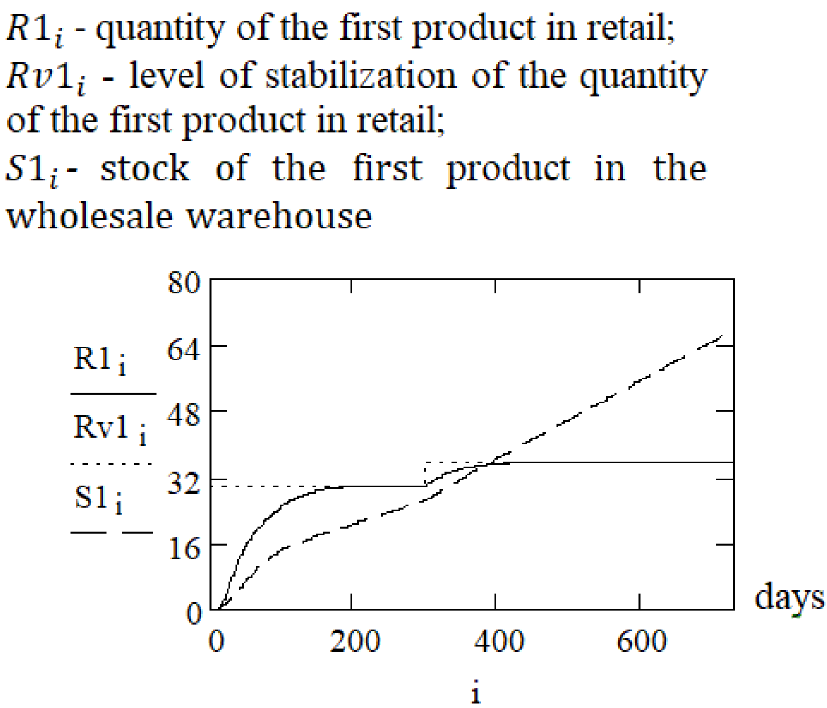

Figure 13: Dynamics of the main indicators in LS for the first product in Case 3.

Relations (15) and (16) mean that the company has the opportunity to choose the initial values of the retail capacity for each type of product and to perform the transformation of retail chains with a period of 300. Let us formulate an optimization problem for these conditions. Find the parameters $Rm1, kR1, Rm2, kR2$ at which the total net profit reaches its maximum:

$$F(Rm1, kR1, Rm2, kR2) = \sum_{i=1}^{T} M_i \to max. \tag{17}$$

The system of equations (1) - (11) serves as constraints for function (17). The solution to the optimization problem (17) system calculated Mathcad:

$$\begin{pmatrix} Rm1 \\ kR1 \\ Rm2 \\ kR2 \end{pmatrix} = \begin{pmatrix} 33,6 \\ 1,014 \\ 59,43 \\ 1,043 \end{pmatrix}, F = 2855\ (MU). \tag{18}$$

Figure 14 shows the dynamics of rates calculated for the optimal solution (18) for the goods of the first type. An analogy can be noted between figure 14 and figure 8.

The total amount of products produced during the lifetime of the project in both cases is the same

$$\sum y1_i = 685,4.$$

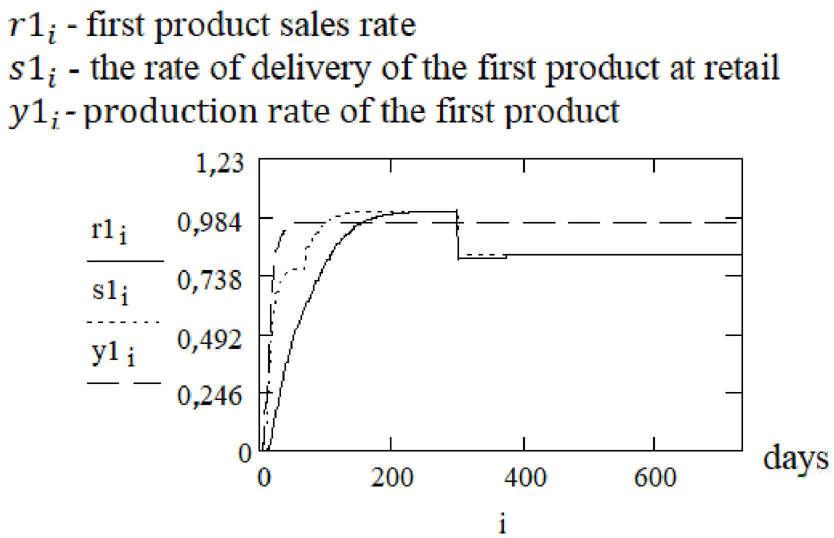

Figure 14: Dynamics of the main rates in LS for the optimal solution (18).

But the number of products sold for the optimal solution is $\sum r1_i = 587,5$, which exceeds the number of products sold for the case corresponding to figure 8: $\sum r1_i = 536,4$. The fact that the amount of products sold is exactly in this ratio is directly visible from the comparison of figure 8 and figure 14.

Such a ratio of the quantities of produced and sold products leads to the fact that unsold products remain in the wholesale warehouse by the end of the project. Its number is $S1_{730} = 128$ for the case corresponding to figure 8 (see figure 9) and $S1_{730} = 64$ for the optimal solution (18), which is shown in figure 15.

Figure 16 displays the dynamics of the indicators of the second product for the optimal solution (18).

Comparing figure 10 and figure 16 it can be seen that in the first case, by the end of the project, unsold products of the second type remain in the wholesale warehouse in the amount of $S2_{730} = 74,7$, while for the optimal solution (figure 16) this quantity is practically zero.

Figure 17 shows the dynamics of daily net income. Calculation of net profit for the period under review gives the result $\sum_{i=1}^{T} M_i = 2852\ (MU)$.

This value is significantly higher than the previous ones, which justifies the need to develop mathematical models for the functioning of LS. Since the mathematical models containing the main parameters of the LS allow you to formulate and solve the optimization problem with a minimum cost of funds and time.

Comparison of figure 7 and figure 17 shows that for the optimal solution (figure 17), the decrease in daily profit over time is much slower, which means that it is possible to create longer-term projects using the optimal parameters of LS.

Case 4.

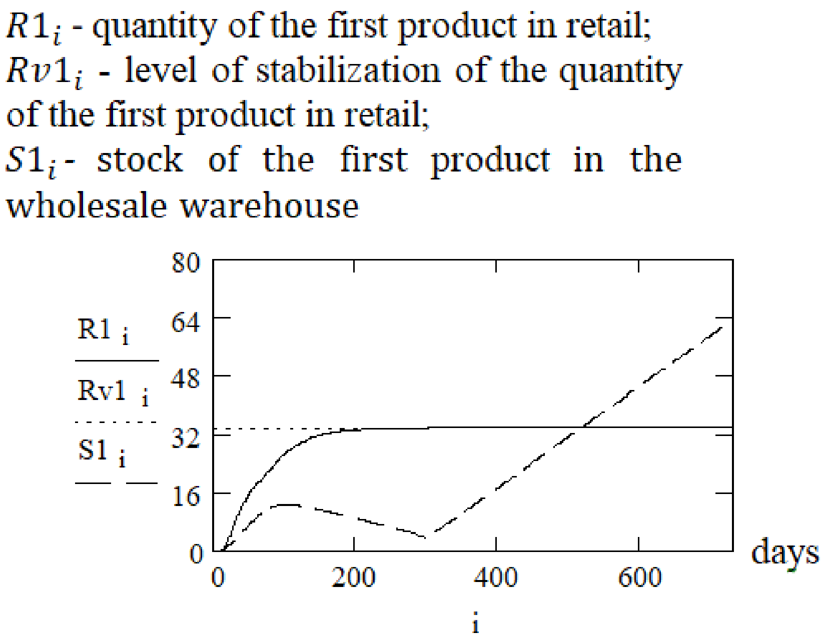

Figure 15: Dynamics of the main indicators in LS for the optimal solution (18).

We considered above the case when the transformation of retail chains for both types of goods occurred simultaneously at a fixed point in time - at the 300th period (see formulas (15), (16)).

Now, suppose that the company can choose the moments of transformation ($tR1, tR2$) of retail chains for each type of product independently.

The optimization problem in this case can be formulated as follows: find the parameters $Rm1, kR1, tR1, Rm2, kR2, tR2$ in formulas (15), (16) at which the total net profit reaches a maximum:

$$(Rm1, kR1, tR1,\ Rm2, kR2,\ tR2) = \sum_{i=1}^{T} M_i \rightarrow max \tag{19}$$

The system of equations (1) – (11) serves as constraints for function (19). The solution to the optimization problem (19) system calculated Mathcad:

$$\begin{pmatrix} Rm1 \\ kR1 \\ tR1 \\ Rm2 \\ kR2 \\ tR2 \end{pmatrix} = \begin{pmatrix} 33,47 \\ 1,115 \\ 292,5 \\ 58,7 \\ 1,049 \\ 200 \end{pmatrix}, F = 3272\ (MU). \tag{20}$$

Comparison of solutions (20) with (18) shows that the possibility of independent changes in retail chains for each type of product leads to an increase in economic efficiency by 14,6%.

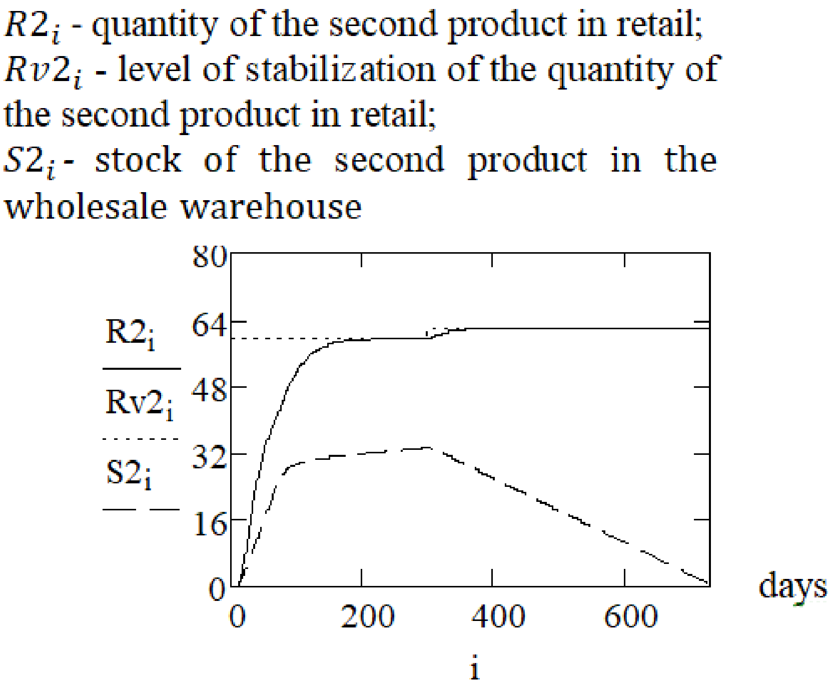

Figure 16: Dynamics of the main indicators for the optimal solution (18).

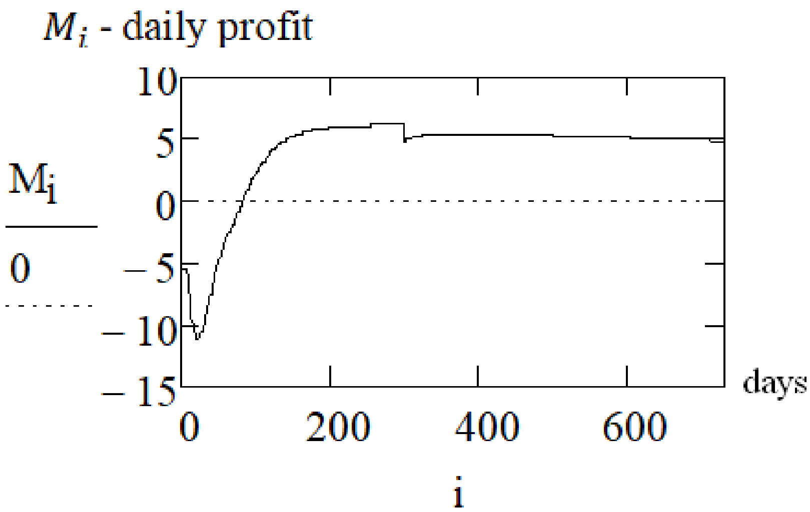

Figure 17: Dynamics of the daily profit of the enterprise for the optimal solution (18).

For a more complete comparison of solutions (20) and (18), the dynamics of the main levels of the drug is presented in figure 18 and figure 19 (compare with figure 15 and figure 16, respectively).

Figures 18 and 19 show that with the optimal solution (20), the capacities of wholesale storage

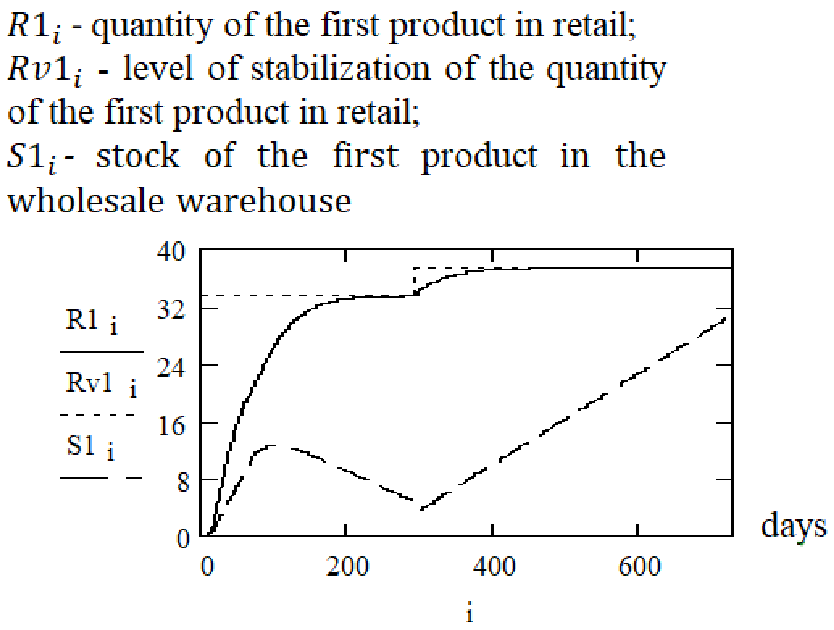

Figure 18: Dynamics of the main indicators in LS for the optimal solution (20).

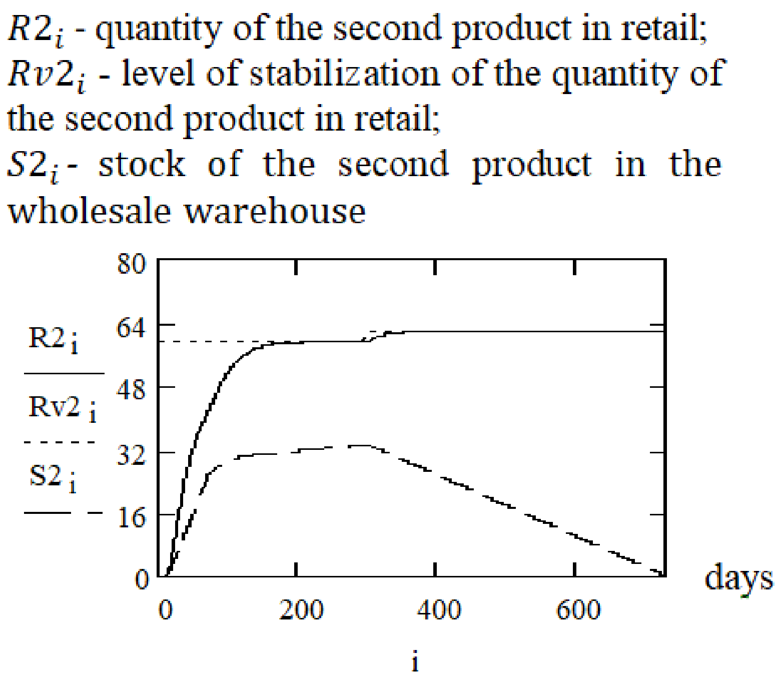

Figure 19: Dynamics of the main indicators for the optimal solution (20).

facilities can be very small. They may not even exceed the retail capacity for both types of goods.

3. Conclusions

1. The work considers a universal scheme of the enterprise's LS, which contains all the main production stages, starting from the procurement of raw materials and components and uploading the supply of finished products to the retail network. A feature of the author's approach is a mutually consistent description of the work of all links of the enterprise's LS, taking into account the demand for products throughout the entire planning horizon. The proposed LS scheme allows describing the diversification of production.
2. The system of mathematical equations for the proposed scheme of the enterprise logistics system is compiled. The system of equations is written in the form of finite differences. Time is considered as a discrete variable. Such a description is more consistent with the real situation at the enterprise, since management decisions at the enterprise are made at discrete moments in time. The system of equations contains the main characteristics of LS as well as the number of potential buyers, which makes it possible to take into account the market demand for products.
3. The proposed model was used to analyze four situations affecting retail chain management. The formulated optimization problems were solved numerically in the Mathcad system. It has been proven that managing a retail network in accordance with the optimal solution can give a significant economic effect.
4. The fact that the proposed system of mathematical equations contains such market parameters as the number of potential buyers and the number of goods in the hands of buyers allows us to include in the description the influence of an advertising campaign on the efficiency of selling goods. It also allows you to assess the impact of advertising on production diversification. These questions can be topics for further research.

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