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Predictors of oil shocks. Econophysical approach in environmental science

A O Bielinskyi[1], I Khvostina[2], A Mamanazarov[3], A Matviychuk[4], S Semerikov[1], O Serdyuk[5], V Solovieva[6], V N Soloviev[1]

[1]Kryvyi Rih State Pedagogical University, 54 Gagarina Ave, Kryvyi Rih 50086, Ukraine
[2]Ivano-Frankivsk National Technical University of oil and gas, 15 Karpatska St., 76019, Ukraine
[3]Tashkent Branch of State University named after M. V. Lomonosov
[4]Kyiv National Economic University named after Vadym Hetman, 54/1 Prospect Peremogy, 03057, Ukraine
[5]Bohdan Khmelnitsky National University of Cherkasy,81 Shevchenko Boulevard, 18000, Ukraine
[6]State University of Economics and Technology, 16 Medychna St., Kryvyi Rih, 50000, Ukraine

E-mail: krivogame@gmail.com

Abstract. The instability of the price dynamics of the energy market from a theoretical point of view indicates the inadequacy of the dominant paradigm of the quantitative description of pricing processes, and from a practical point of view, it leads to abnormal shocks and crashes. A striking example is the COVID-stimulated spring drop of spot prices for crude oil by 305% to $36.73 a barrel. The theory of complex systems with the latest complex networking achievements using pragmatically verified econophysical approaches and models can become the basis of modern environmental science. In this case, it is possible to introduce certain measures of complexity, the change in the dynamics of which makes it possible to identify and prevent characteristic types of critical phenomena. In this paper, the possibility of using some econophysical approaches for quantitative assessment of complexity measures: (1) informational (Lempel-Ziv measure, various types of entropies (Shannon, Approximate, Permutation, Recurrence), (2) fractal and multifractal (Multifractal Detrended Fluctuation Analysis), (3) recurrent (Recurrence Plot and Recurrence Quantification Analysis), (4) Lévy's stable distribution properties, (5) network (Visual Graph and Recurrence based) and (6) quantum (Heisenberg uncertainty principle) is investigated. Each of them detects patterns that are general for crisis states. We conclude that these measures make it possible to establish that the socially responsive exhibits characteristic patterns of complexity and the proposed measures of complexity allow us to build indicators-precursors of critical and crisis phenomena. Proposed quantitative measures of complexity classified and adapted for the crude oil market. Their behavior in the face of known market shocks and crashes has been analyzed. It has been shown that most of these measures behave characteristically in the periods preceding the critical event. Therefore, it is possible to build indicators-precursors of crisis phenomena in the crude oil market.

1. Introduction

For achieving solutions in sustainability development, we need long-term potential actions. Recent advances in this aspect have contributed to successful solutions to real problems, thus improving the sustainability of energy, the environment, and the quality of life. Therefore, the focus of attention is on modern theoretical approaches and tools to problems in sustainable energy, with modeling, analysis, and control problems, communication systems, process control, environmental systems, intelligent manufacturing systems, transportation systems, structural systems, and so forth [32, 85, 132, 170]

The world and worldwide energy industry have experienced a significant impact during the global coronavirus pandemic. Although it has reminded the importance of energy, even before, the expert energy community was discussing the novel era for global energy [224].

In the last decade, there has been a growing body of literature addressing the utilization of complex network methods for the characterization of dynamical systems based on time series. While both nonlinear time series analysis and complex network theory are widely considered to be established fields of complex systems sciences with strong links to nonlinear dynamics and statistical physics, the thorough combination of both approaches has become an active field of nonlinear time series analysis, which has allowed addressing fundamental questions regarding the structural organization of nonlinear dynamics as well as the successful treatment of a variety of applications from a broad range of disciplines [181, 242].

Financial and spot markets, as a complex system, have been attracting the attention of many scientists like engineers, mathematicians, physicists, and others for the last two decades. Such vast interest transformed into a branch of statistical mechanics – econophysics [6, 131]. The integration of different methods from physics, economics, mathematics, engineering, and computer science has led to interdisciplinary area of science where knowledge, methodologies, and tools of different fields are applied for modeling, explaining, and forecasting economic and social phenomena.

Econophysics, based on a rich arsenal of research on critical phenomena [197], very successfully copes with the description of similar events in economics and finance. These are crises and crashes that are constantly shaking the world economy. The introduced measures of complexity should, to one degree or another, respond to such phenomena.

The key idea here is the hypothesis that the complexity of the system before the crashes and the actual periods of crashes must change [20, 21, 23, 44, 49, 183, 184, 186-191, 193]. This should signal the corresponding degree of complexity if they are able to quantify certain patterns of a complex system. Introduced measures are dynamic and could be applied along with a diagnosable time series whose abnormal changes could be detected and prevented.

In addition to a few methodological similarities, there are also many important differences such as emphasis by econophysicists on statistical mechanics rather than mechanical models, reservations towards rational agent theory, and rejection by of many standard assumptions of mainstream economics, etc.

It cannot be said that econophysical ideas were not considered in the theory and practice of sustainable development. The paper [165] outlines the “ecological econophysics” that could be scientifically more suitable than mainstream economics. The authors of the work [59] applied econophysical methodologies (in particular multifractal analysis) to S&P Global Clean Energy Index, New York Stock Exchange (NYSE), and the price of the crude oil. Empirical results show that the clean energy index is much more dependent on crude oil market rather than NYSE. Better understanding of clean energy market structure and usage of more reliable instruments for its analysis will lead to bigger investments to this sector and, perhaps, make energy production more efficient. Econophysical analysis of other indices and indicators of energy sustainability is also a fruitful direction [15, 33, 169, 200, 225]. However, for the current period, these problems urgently need quantitative assessments of the energy market indicators using the tools of the theory of complex systems and econophysics. This work is intended to fill this gap to a certain extent.

It should be noted that modern machine learning technologies are promising in ensuring the energy sustainability of complex systems. Machine learning (ML), a subset of artificial intelligence, refers to methods that have the ability to "learn" from experience, enabling them to carry out designated tasks. Examples of machine learning tasks are detection, recognition, diagnosis, optimization, and prediction. Machine learning can also often be used in different areas of complex systems research involving the identification of the basic system structure (e.g., network nodes and links) and study of the dynamic behavior of nonlinear systems (e.g., determining Lyapunov exponents, prediction of future evolution, and inferring causality of interactions). Conversely, machine learning procedures, such as "reservoir computing" and "long short-term memory", are often dynamical in nature, and the understanding of when, how, and why they can function so well can potentially be addressed using tools from dynamical systems theory [204].

The calculation and analysis of econophysical measures of complexity will be carried out using the example of a time series of daily spot prices for crude oil which is considered to be the most volatile in the commodity market [45].

On the other hand, the oil market is a complex system and the theoretical approaches developed by the theory of complex systems and, in particular, econophysical, are obviously applicable to it.

The paper is planned in this way. Section 2, we present our classification of WTI crude oil market shocks and crashes for the period from January 2, 1986, to September 21, 2020. In Section 3 the technique and results of recurrence analysis are described. In Section 4, we describe the information measures of complexity. In Section 5, we describe the multifractal analysis methodology and its results for the oil market. Section 6 demonstrates how one of the complexity indicators based on nonlinear dynamics methods are defined and worked. Section 7 presents the theory and empirical results on network measures of complexity and their robustness for crude oil price time series. Section 8 defines the quantum complexity measure. Section 9 contains conclusions and some recommendations for further research.

2. Data and classification of oil shocks and crashes

A large number of academic literature has provided evidence that there is a relationship between oil and macroeconomic variables. As a rule, the results suggest that oil prices have a significant impact on the world economy and, as an energy active, it also has strategic power in terms of international trade. Thus, we advanced into action and set the task (1) to make an appropriate classification of such events that are predictable and not predictable and (2) to construct such indicators that will identify in advance shocks and crashes in order to allow investors and ordinary users to work in this market.

Further, the corresponding study will present that oil price is regime-switching. Such switching reveals in high risk (completely random) and low risk (deterministic) environments. Such events with a high risk are completely unpredictable, their appearance is unexpected and there no patterns that would indicate their appearance. Some of those events are much more predictable, less efficient, and exhibit corresponding complexity patterns that can serve as indicators of further falling. All the mentioned events will be classified and presented on the table.

The data we use here for our analysis are the daily closing prices of the West Texas Intermediate (WTI) crude oil over a period of time from January 2, 1986, to September 21, 2020 [45]. During this period, the oil market experienced periods of varying degrees of volatility. The main ones are presented in table 1.

We will call price jumps not exceeding 30% as shocks, all the rest - crashes. Let us analyze how econophysical measures of complexity "react" to oil shocks and crashes. Further estimations will be applied for the initial time series and its normalized returns which can be calculated as

$$g(t) \cong [G(t) - \langle G \rangle] / \sigma. \tag{2.1}$$

where $G(t)$ presents log-returns, and σ is the standard deviation of G .

We predefine the most noticeable of such abnormal phenomena that are the most volatile, influential, and reasonable. Their loss in price is noticeable, comparing to other events.

Table 1. List of the crude oil market major shocks and crashes since December 1987 till April 2020

The Date of Beginning	Oil High Price, $	The Date of Ending	Oil Low Price, $	Falling, %
23.11.1987	19,31	21.12.1987	15,12	21,70
20.04.1989	24,62	08.05.1989	19,41	21,16
14.05.1990	19,73	25.05.1990	16,12	18,30
07.08.1990	29,60	09.08.1990	25,69	13,21
16.01.1991	32,25	18.01.1991	20,05	37,83
21.03.1994	15,37	28.03.1994	14,15	7,94
25.08.1995	19,91	05.10.1995	16,86	15,32
22.02.1996	22,14	04.03.1996	19,24	13,10
26.03.1998	16,92	14.04.1998	15,18	10,28
21.04.1998	15,57	15.06.1998	11,69	24,92
25.01.2000	30,28	27.01.2000	27,22	10,11
20.09.2000	37,22	28.09.2000	30,26	18,70
12.03.2003	37,87	28.04.2003	25,25	33,32
18.03.2005	56,80	23.03.2005	49,43	12,98
13.03.2008	110,21	01.04.2008	100,92	8,43
21.05.2008	132,99	04.06.2008	122,30	8,04
22.09.2008	122,61	23.12.2008	30,28	75,30
29.04.2011	113,39	27.06.2011	90,65	20,05
01.05.2012	106,17	21.06.2012	77,91	26,62
20.06.2014	107,95	29.01.2015	44,12	59,13
16.09.2019	63,10	03.10.2019	52,41	16,94
03.03.2020	47,27	30.03.2020	14,10	70,17
17.04.2020	18,31	20.04.2020	-36,98	301,97

The calculations of indicators for them will be carried out within the framework of the algorithm of a rolling (sliding, moving) window. According to the procedure, we emphasize the frame of predefined length in which the calculation of the corresponding measure is obtained. Then it is shifted along the time by a predefined value, and the procedure is repeated until the entire series is exhausted. Comparing the calculated measure of complexity and the actual time series of crude oil, we can analyze changes of complexity in the system. Our measures can be called indicators or precursors if they behave in a specific way for all periods of crashes and shocks, for example, decrease or increase during the pre-crash or pre-shock periods.

3. Recurrence Analysis

In 1890 the mathematical foundations of recurrence were introduced by Henri Poincaré, resulting in the *Poincaré recurrence theorem* [164]. This theorem states that certain systems will return to their arbitrarily close, or exactly the same initial states after a sufficiently long but finite time. Such property in the case of deterministic behavior of the system allows us to make conclusions regarding its future development.

Analysis of the behavior of energy commodities has been a topic of great interest for a long time. Such tools as the correlation dimension, the Lyapunov exponents, the BSD statistics, the Kolmogorov-Sinai (KS) entropy, etc. were used to identify either such prices are presented to be chaotic or not. Some research papers proclaim strong evidence of chaos. Others conclude that, for example, crude oil future prices exhibit non-linear and complex behavior, but at the same time stochastic process [145]. Matilla-García, considering three futures series (natural gas, unleading gasoline, and crude oil), find that the presence of chaos could not be rejected for natural gas and crude oil [140]. Barkoulas et al. [11] employing correlation dimension, the Lyapunov exponent, and, namely, recurrence plot, found that crude oil cannot be characterized as a deterministic system, and it does not exhibit the recurrence properties typical of deterministic nonlinear structures. Mastroeni et al. [138] give some insights into the chaotic paradigm and deterministic processes of energy commodity prices. Their results obtained with different statistical tests for chaos and, moreover, with the help of recurrence analysis reveal a prevalent deterministic feature. Also, recurrence regimes switch between laminar states and periods of chaotic behavior. Such different results that were presented years ago and now allow us to assert that chaotic and stochastic characteristics coexist in the energy commodity prices.

Hua X et al. [80] examine the information between carbon and energy markets using a multilayer recurrence network. From their method, they obtain an information linkage coefficient to measure the linkage relationship between layers that represent different markets. Such an approach represents changes in mutual information between energy and carbon prices in different stages. It can be used to quantitatively study the formation mechanisms of some markets and even handle a signaling role before the distortion of market efficiency.

In our analysis, we would like to employ characteristic tools of classical recurrence analysis and quantification analysis for handling crisis states that lose their deterministic and recurrent properties while they occur.

3.1. Time Delay Method

Identification and prediction of the abnormal phenomena of the system is a defiant problem in many disciplines, such as economics, meteorology, and seismology where usually the information about the system's properties comes from time series of some univariate experimental data. The basic idea of recurrence analysis is based on the extraction of the information about the temporal evolution of trajectories which lie on compact attractor in phase space.

Usually, not all relevant variables can be captured from our observations. Often, only a single variable may be observed. *Thakens'* theorem [201] ensures that the observational states of the system can be expressed through a d - dimensional vector or matrix, where each of its components reflects the properties of the whole system.

For an approximate reconstruction of the original dynamics of the observed system, we project the time series onto a Reconstructed Phase Space [53, 91, 154] with the commonly used time delay method [91] which relied on the *embedding dimension* and *time delay*.

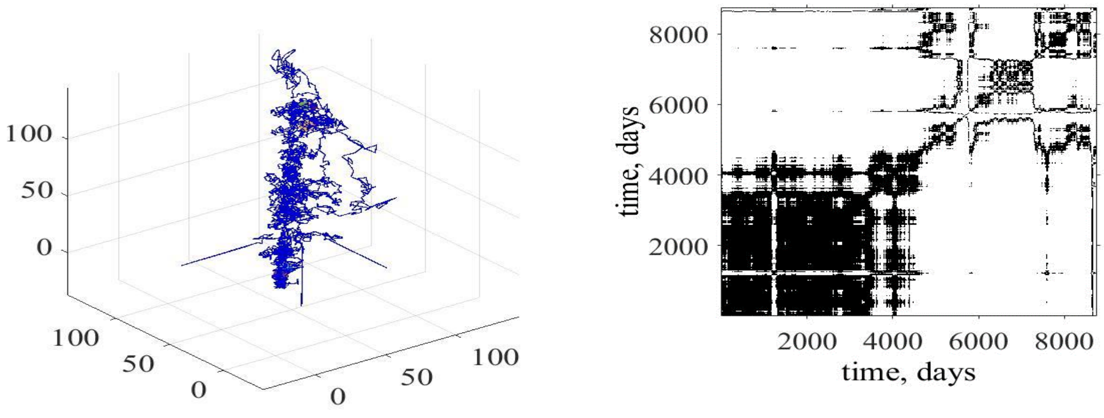
The embedding dimension the dimensionality of the reconstructed system (corresponds to the number of relevant variables that may differ from one system to another. The time delay parameter specifies the temporal components of the vector components. As an example, in recurrence analysis, Webber Jr and Zbilut [220] recommend setting the embedding dimension between 10 and 20. Regarding the analysis of financial systems, values between 1 and 20 for the embedding dimension are considered to be reasonable as well as the time delay.

3.2. Recurrence Plot

Recurrence plot (RP) have been introduced to study dynamics and recurrence states of complex systems. When we create RP, at first, from recorded time series we reconstruct phase-space trajectory. Then, according to Eckmann et al. [52], in terms of d_E - dimensional space, we consider a trajectory $\vec{X}(i)$ on the reconstructed trajectory. The recurrence plot is an array of dots in a $N \times N$ matrix, where dot is placed at (i, j) whenever $\vec{X}(j)$ is sufficiently close to $\vec{X}(i)$, and both axes are time axes which mathematically can be expressed as

$$R_{ij} = \Theta(\varepsilon - \|\vec{X}(i) - \vec{X}(j)\|), \quad \text{for } i, j = 1, \ldots, N \tag{3.1}$$

where $\| \, \|$ is a norm (representing the spatial distance between the states at times i and j); ε is a predefined recurrence threshold, and Θ is the Heaviside function (ensuring a binary R). As a result, the matrix captures a total of N^2 binary similarity values. A synthetic example is presented in figure 3.1.



(a) (b)

Figure 3.1. Phase portrait (a) and corresponding RP (b) of the crude oil price.

Maximum distance seems to be a suitable choice to determine the pairwise similarity between two vectors. It is often used as it is independent of the phase space dimension, easy to calculate, and allows some analytical expression [58, 205, 206].

Also, as it can be seen from equation (3.1), the similarity between vectors is determined by a threshold ε. The choice of $\varepsilon > 0$ ensures that all vectors that lie within this radius are similar to each other, and that dissimilarity up to a certain error is permitted [163].

The fixed radius for recurrent states is the commonly used condition, which leads to equally sized ε-neighborhoods. The shape in which neighborhoods lie is determined by the distance metric. Applying the fixed threshold with the distance metric, we define recurrence matrices that are symmetric along the middle diagonal. The self-similarity of the multi-dimensional vectors reflects in the middle diagonal which is commonly referred to as *line of identity* (LOI). In contrast, it is not guaranteed that a recurrence matrix is symmetric, if the condition of fixed number of nearest neighbors is applied. For specific purposes LOI that responds to trivial recurrence states might be excluded from the RP [221].

The visualization of trajectories and hidden patterns of the systems is the "destiny" of RP [135, 222]. The dots within RP, representing the time evolution of the trajectories, exhibit characteristic large-scale (*homogeneous*, *periodic*, *drift*, and *disrupted* [66, 136, 205]) and small-scale (*isolated recurrence points, diagonal, vertical*, and *horizontal lines*) patterns.

3.3. Recurrence Quantification Analysis

For a qualitative description of the system, the graphic representation of the system suits perfectly. However, because of subjective intuition and further interpretation of such large and small-scale patterns, additional metrics in term of quantitative analysis, which are based on previously mentioned large and small-scale patterns, were introduced by Webber and Zbilut and called *recurrence quantification analysis* (RQA). Later, it has been extended and intensively used by Marwan et al. [136, 219, 233].

Usually, first acquaintance with classical RQA starts with recurrence point density, or, as it is known, recurrence rate (RR)

$$RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j}.$$

It enumerates the probability that any state of the system will recur. It is the simplest measure that is computed by taking the number of the nearest points forming short, spanning row and columns of the recurrent plot. It summarizes them and divides by the number of possible points in the recurrence matrix of size N^2. Higher RR indicates higher proportion of similar values across time or because time series presents very little change in its dynamics. RP can help distinguish such changes in qualitative terms, but other measures from quantitative perspectives might give additional information about forces driven change in the system.

The remaining measure relies on the frequency distribution of line structures in the RP. First, we consider the distribution of the line length of diagonal structures in the RP

$$P(l) = \sum_{i,j=1}^{N} \left\{ (1 - R_{i-1,\, j-1}) \cdot (1 - R_{i+l,\, j+l}) \cdot \prod_{k=0}^{l-1} R_{i+k,\, j+k} \right\}.$$

The proportion of recurrence points that form line segments of minimal length μ parallel to the matrix diagonal is the measure of determinism (*DET*)

$$DET^{(u)} = \frac{\sum_{l=\mu}^{N} l \cdot P(l)}{\sum_{i,j=1}^{N} R_{i,j}} = \frac{\sum_{l=\mu}^{N} l \cdot P(l)}{\sum_{l=1}^{N} l \cdot P(l)}.$$

Systems that exhibit deterministic dynamics are mainly characterized by diagonal lines. Long diagonal lines indicate periodic signals, but short diagonal lines stand for chaotic behavior. Regarding the quantitative analysis, typically, only the lines with minimal length $\mu = 2$ are considered. If $\mu = 1$ then *DET* and *RR* are identical. For some systems, *DET* becomes more reliable if $\mu > 2$. Here, μ serves as a filter, excluding the shorter lines. However, it should be noted that too large μ may spoil the histogram $P(l)$ and thus the reliability of *DET*.

The results of calculations of window dynamics of the considered recurrence measures are presented in figure 3.2. Measures *RR* and *DET* are calculated for the entire time series of the oil price for window length of 250 days and a step of 5 days.

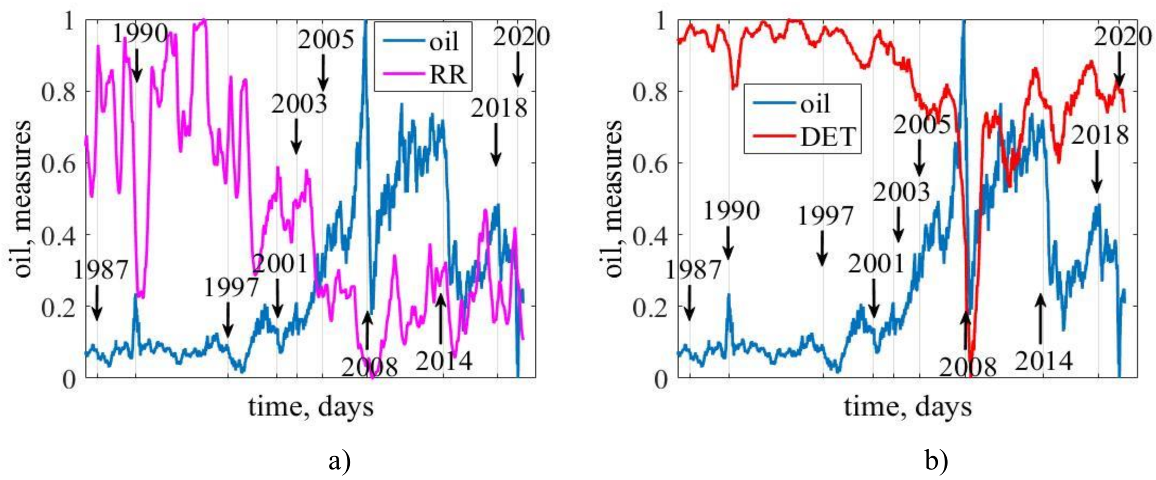


Figure 3.2. Dynamics of RR (a) and DET (b) for the oil price time series.

It is evident that the two recurrent measures during abnormal periods decrease long before the actual anomaly. The complex system becomes less recurrent and deterministic which is logical in the periods of approaching critical phenomena. Consequently, RR and DET can be used as precursors of the critical and crash phenomena.

A comparative analysis of the measures under consideration revealed an obvious advantage of the recursive measure. In addition to the smoothness of the measure itself, it can be calculated for windows of small sizes, which leads to inaccurate or incorrect results for other methods.

4. Informational measures of complexity

Complexity is a multifaceted concept, related to the degree of organization of systems. Patterns of complex organization and behavior are identified in all kinds of systems in nature and technology. Essential for the characterization of complexity is its quantification, the introduction of complexity measures, or descriptors [92].

Complexity is seemed to be very subjective and undefined thing. We may speak about signs of complexity considering:

- the number of components involved into the system;
- the size of irregularities in the system: the lack of repetitiveness, a large number of unorganized interconnections lead to huge complexity;
- the amount of information (number of bits) of the structure; the amount of space needed for object reconstruction, accurate interpretation;
- the algorithmic complexity of a certain task: minimum time needed to carry out specific task on a computer;
- the communication complexity of a task: the number of bits that have to be transmitted in order to solve specific task [28, 116, 208].

Historically, the first attempt to quantify complexity was based on Shannon's information theory [176] and Kolmogorov complexity [95].

4.1. Lempel-Ziv complexity

The complexity of the information generated from ergodic sources can be calculated with Lempel-Ziv complexity (LZC) that links both the concept of complexity (in Kolmogorov-Chaitin sence) and entropy rate [24, 243]. With the LZC we can measure the capacity of an ergodic dynamical process to generate new patterns. As we performed measurements and obtained a time series of some discrete values, we can apply LZC measure that requires low computational costs and gives the entropy rate of the measureable phenomena [55, 91].

Historically, S Da Silva et al. [40, 42, 68, 69] were the first who applied LZC for financial systems, considering the deviation of actual time series for a random as a measure of actual market efficiency in absolute [41, 42, 68, 69] or relative [40] terms. Applying this approach to high-frequency tick-by-tick return data from 43 companies listened on Bovespa, authors Da Silva and Giglio detect decreasing efficiency rate for the majority of the stocks after the financial crisis of 2008 [42]. In [110], authors have examined the algorithmic (Kolmogorov) complexity for tracking a transition between regular and random patterns in financial systems.

Some fruitful results were obtained with the LCZ and visibility graph-based analysis [227]. Here, they perform comparative analysis of stock market indices and their shuffled versions where their complexity is measures. Correspondingly, is was confirmed that proposed model seemed to be reasonable and financial indices were presented to be less complex (chaotic) comparing to their shuffled versions.

In this paper [178], multivariate complexity measures, multiscale coarse-graining procedure were combined to study self-reproducing chaotic systems which complexity is determined by different initial states for its multistability.

Meanwhile, multiscale types of permutation entropy and LZC are employed to study the complexity of the mentioned systems. As results present, multiscale measures make complexity analysis better, but need to be careful, as for multivariate time series such multiscale approach on the example of permutation entropy presented to be unnecessary. Nevertheless, both measures have potential application value in real applications.

A brief analysis of the problem indicates that the complexity of the crude oil market during a crisis have not been studied with the use of LZC. In this section, we use the Lempel-Ziv complexity measure and its multiscale version to study this market. In previous papers [23, 41, 69, 76, 183, 191] we tested LZC measure for the cryptocurrency market. Current work is dedicated to the crude oil market.

Varying dynamics of the corresponding measures should signal about approaching shock or crash phenomena [23, 191].

4.1.1. The Concept of Kolmogorov and Lempel-Ziv complexity. The idea of Kolmogorov complexity (algorithmic complexity) [95] is to measure the amount of information in the finite objects using the theory of algorithms. Speaking about finite size objects, we often consider binary strings and their compressed structures. Therefore, approximately, the Kolmogorov complexity of some string x is the amount of information in that string. In other words, a string of repeated patterns where all bits are equal to zero, has a very low complexity (presents little information), while completely chaotic string becomes hardly compressed and, thus, has a lot of information. In this case, if the complexity of our string is equal to k, we expect here k bits of information. Information about both random and regular parts of a string is included to the Kolmogorov complexity.

Example, considering a 2-part description p and d, both of which describe regular and random sequences of a shortest program for x, the amount of meaningful information is presented to be of a size p. Taking into account x and y strings, where x is some regular string and y is a randomly generated string, their shortest programs are presented to be (p_x, d_x) and (p_y, d_y). The Kolmogorov complexity of y is larger than for x, while $KS(x) \approx |p_x|$ and $KS(y) \approx |p_y|$. Thus, it should be clear that most of information content in x is its regular part, and in y its irregular part.

As Kolmogorov complexity was considered to be non-computable, Lempel and Ziv [110] suggested a method for computing the complexity of finite size objects. The idea is very similar to LZ76 algorithm. Consider a sequence of characters S composed from alphabet $\sum$. Afterwards, we can present the sequence synthesis scheme as a concatenation of non-repeatable substrings:

$$H(S) = S(1,\ i_1)S(i_1+1,\ i_2)\ldots S(i_{k-1}+1,\ i_k)\ldots S(i_{m-1}+1,\ N),$$

where $S(i_{k-1}+1,\ i_k)$ is the substring of S generated at the k^{th} step, and any factor $S(i_{k-1}+1,\ i_k-1)$ that is a substring of the string $S(1,\ i_k-1)$ is included to an exhaustive history $H(S)$ of the sequence S. The number of factorized sequences in $H(S)$ is the corresponding LZC. The algorithmic complexity for a random sequence is calculated by expression $LZC_r = N/\log(N)$. Then, the normalized LZC is defined as

$$LZC = \frac{LZC}{LZC_r}.$$

For further calculations, the crude oil price subseries of the fixed length is obtained and logarithmic returns are calculated. Then, they are transformed into the series of two states (bits). However, except two state system, we can specify *n*-state system. In the case of three states, unlike the binary coding system, a certain threshold σ is settled, and the states g are coded as follows [42, 68, 69]:

$$g = \begin{cases} 0 & if\ g < -\sigma, \\ 1 & if\ -b \le g \le b, \\ 2 & if\ g > b. \end{cases}$$

As the financial markets exhibit scale-invariant properties that manifest themselves through the power laws of the distribution, the classical LZC procedure is unacceptable and often leads to erroneous conclusions. Therefore, the multiscale procedure should help to overcome such difficulties.

According to the procedure, first of all, we allocate non-overlapping segments of the scale factor (length) τ and each of those is averaged. Then, we switch to the next scale and repeat the same "coarse-graining", but with larger length of non-intersecting windows [38]. When $\tau = 1$, the "granular" series is exactly similar to the original one. For $\tau > 1$, the coarse-grained series follows the expression

$$y_j^{\tau} = \frac{1}{\tau} \sum_{i=(j-1)\tau+1}^{j\tau} g(i), \quad \text{for } 1 \le j \le N / \tau, \tag{4.1}$$

with corresponding scale factor τ. The length of each "granular" row depends on the length of the window and is even N / τ. The illustration of calculations for one of scales is presented in the figure 4.1.

Figure 4.2 presents the results of calculations of multi-scaling LZC measure for scale factor $\tau = 6$. The calculations were performed for a rolling window of 250 days and an increment of 5 days. The data in figure 4.2 indicate that the LZC measure is noticeably reduced in the case of averaged over the scales from 1 to 6 (m_6) for all crashes and critical events in the immediate vicinity of the crisis point.

As the results of calculations showed, the length of sliding window of 250 days turned out to be optimal for the separation of crises and fixing the LZC measure as an indicator. It is decreasing before the actual crisis point, signaling about decreasing complexity during such events.

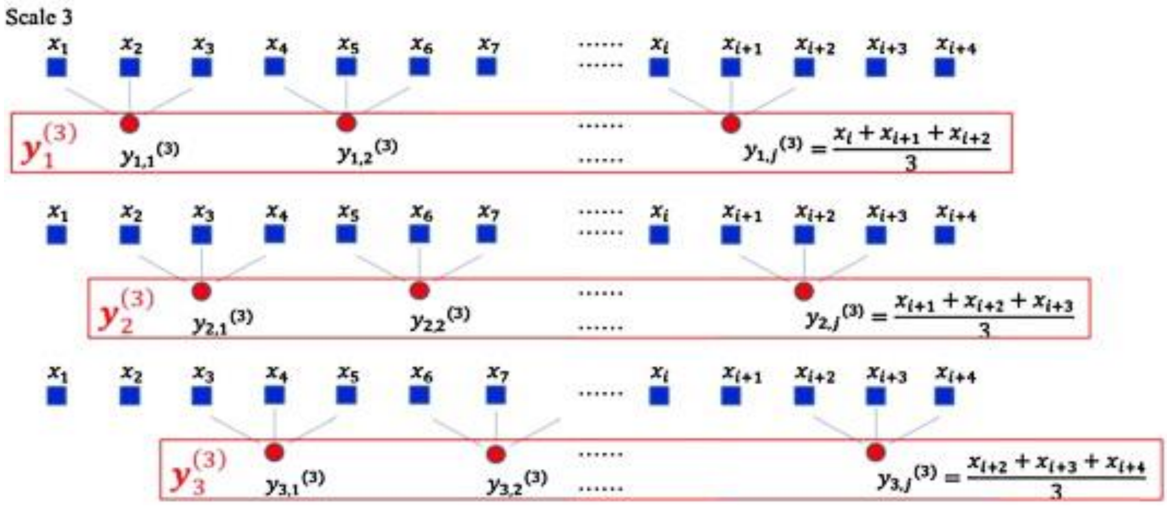


Figure 4.1. Illustration of the coarse-graining procedure for $\tau = 3$ [180].

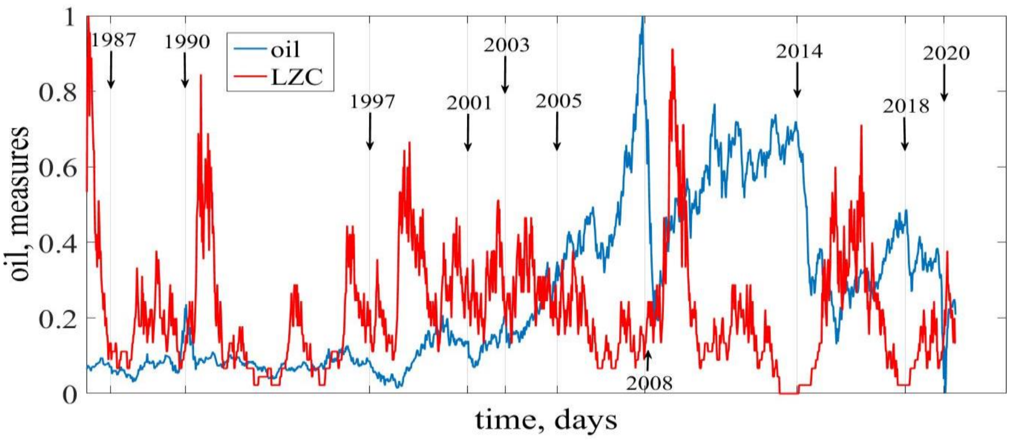


Figure 4.2. Comparative dynamics of the oil price and multi-scaling LZC.

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4.2. Entropy as a measure of complexity

Nowadays, the most important quantity that allows us to parameterize complexity in deterministic or random processes is entropy. Originally, it was introduced by R Clausius [34], in the context of classical thermodynamics, where according to his definition, entropy tends to increase within an isolated system, forming the generalized second law of thermodynamics. Then, the definition of entropy was extended by Boltzmann and Gibbs [27, 67], linking it to molecular disorder and chaos to make it suitable for statistical mechanics, where they combined the notion of entropy and probability.

After the fundamental paper of Shannon [177] in the context of information theory, where entropy denoted the average amount of information contained in the message, its notion was significantly redefined. After this, it has been evolved along with different ways and successful enough used for the research of economic systems [20, 23, 89, 182-184, 190, 192, 210].

A huge amount of different methods, as an example, from the theory of complexity, the purpose of which is to quantify the degree of complexity of systems obtained from various sources of nature, can be applied in our study. Such applications have been studied intensively for an economic behavior system.

In this work [134] it is presented that the complexity of the price varies on different time horizons. The empirical results show that the highest market efficiency (randomness) presented to be on small time scales up to one or two weeks. For higher time scales, longer than one quarter, the multiscale entropy pattern shows changeable entropy levels. During such events as U.S. recessions in the recent 25Â years, multiscale entropy level is presented to be decreased indicating about their drastically reduced level of complexity. Possible extreme events, in terms of the relative market efficiency, are analyzed, supposing that due to them short-term but not the long-term market complexity was affected. Such novel analysis of oil market can reveal its completely new sides.

Zou Y et al. [229] propose wavelet entropy-based approach for forecasting model. According to their work, they use wavelet entropy algorithm to determine optimal wavelet families and decomposition scale to make higher the forecasting performance. In terms of conventional performance evaluation criteria for the forecasting accuracy the proposed algorithm outperforms traditional models.

Combining the Symbolic Time Series Analysis with the Shannon entropy, on the example of WTI and Brent crude oil indices, Mensi W et al. [2, 144] approve that efficiency degree of both indices varies over time and these variations can be detected with corresponding approach. Researchers conclude that Shannon entropy can have practical implication for forecasting, portfolio management, and hedging crude oil market risks. In periods of indicating low market efficiency, investor can start buying/selling when the price is below/above the indicator's value.

The presence of patterns within a series is a key criterion in evaluating randomness, so it is appropriate to establish such methods that will be based on the different patterns and their repetition [48]. In this regard, Pincus described the methodology *Approximate entropy* (ApEn) [157] to gain more detail analysis of relatively short and noisy time series, particularly, of clinical and psychological. Its development was motivated by the length constraints of biological data. Since then it has been used in different fields such as psychology [158], psychiatry [231], and finance [18, 54, 107, 123, 156]. Duan and Stanley [51] showed that relying on respective changes in patterns of such measures as volatility, ApEn, and Hurst exponent, it is possible to effectively differentiate the real-world financial time series from random-walk processes. The empirical results prove that financial time series are predictable to some extent, and ApEn is appliable indicator of predictability degree in financial time series. Alfonso Delgado-Bonal [47] gives evidence of the usefulness of ApEn. The researcher quantifies the existence of patterns in evolving data series. In general, his results present that degree of predictability increases in times of crisis.

Efficiency of WTI crude oil market is also discussed here [101]. Kristoufek and Vosvrda discuss the contributions of the long-term memory, fractal dimension, and ApEn to the total inefficiency. According to their research, regularities in studied 25 commodities are not strongly pronounced. Kapica J [93] uses modified ApEn algorithm which is called as Sample entropy to verify the efficient market hypothesis for energy price movement. Results present that the behavior of the price is not random. The value of entropy for the brend type of crude oil differs from that which is for random time series with normal law distribution. It denotes that the price of crude oil is far predictable than for a purely random walk.

Permutation entropy (PEn), according to the previous approach, is a complexity measure that is related to the original *Shannon entropy* (ShEn) that applied to the distribution of ordinal patterns in time series. Such a tool was proposed by Bandt and Pompe [9], which is characterized by its simplicity, computational speed that does not require some prior knowledge about the system, strongly describes nonlinear chaotic regimes. Also, it is characterized by its robustness to noise [3, 232] and invariance to nonlinear monotonous transformations [91]. PEn has become enormous tool for studying biomedical or climate time series. Here [8], Bandt studies day-to-day market data with Brownian motion. Turning rate and up-down balance parameters are considered and tested with respect to Brownian motion for finding changing points in crude oil price.

Bariviera et al. [10] analyze the information efficiency of the oil price per barrel for 32 years. During such long period, different economic events affected the global economy. It is presented that with the PEn and permutation statistical complexity it is possible to discriminate different degrees of information efficiency that changes regarding some geopolitical events which dovetail with the results of previously mentioned papers. According to the paper of Wei-Shing and Sheng-Yu [223], the oil price series is presented to be predictable. With calculated PEn, forbidden patterns, and statistical measure it is seen that the Brent oil price series has some seasonal behavior, but from 2010 crude oil market efficiency becomes apparent. In these papers [79, 228] not only the exact information and scale of the time series are considered, but also magnitude of the extracted information. Such approach is presented to be more robust, as it can, example, distinguish different markets, while reducing the standard deviation of all the markets.

The combination of entropy and symbolic dynamics turned out to be fruitful for analyzing the disorder for the time series of any nature without losing their temporal information.

4.2.1. Shannon entropy. The general approach can be described as follows. Formally, we represent the underlying dynamic state of the system in probability distribution form P and then the Shannon entropy S with an arbitrary base (i.e. 2, e, 10) is defined as

$$S[\mathrm{P}] = -\sum_{i=1}^{N} p_i \log p_i \ . \tag{4.2}$$

Here, in equation (4.2), p_i represents the probability that price i occurs in the sample's distribution of the oil price time series, and N is the total amount of data in our system. Dealing with continuous probability distributions with a density function $f(x)$, we can define the entropy as

$$H(f) = -\int_{-\infty}^{+\infty} f(x) \log f(x) dx. \tag{4.3}$$

According to the approach, the negative log increases with rarer events due to the information that is encoded in them (i.e., they surprise when they occur). Thus, when all p_i have the same value, i.e. where all values are equally probable, and $S[\mathrm{P}]$ reaches its minimum for more structured time series (events that are more certain). Equation (4.3) is obeyed to the same rules as equation (4.2). In figure 4.3 are the empirical results for Shannon entropy and the oil price time series.

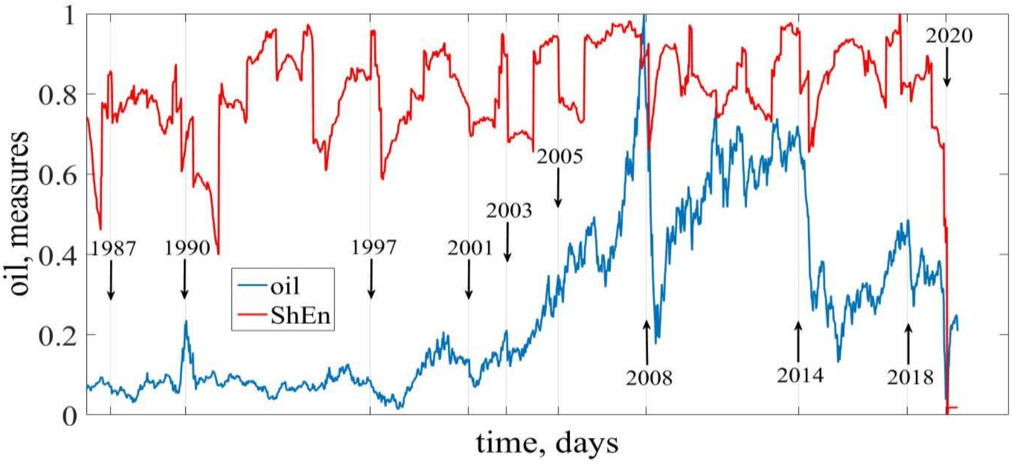


Figure 4.3. ShEn dynamics along with the entire time series of the oil price.

It can be seen from the figure that Shannon's entropy is rapidly increasing at the very moment of the crisis itself and is an excellent indicator of crisis phenomena.

4.2.2. Approximate entropy. ApEn determines the probability of similarity of a chosen pattern at the given time and the next pattern at the same time. Given the initial time series $\{x(i) \mid i = 1, \ldots, N\}$, a non-negative integer $d_E \le N$, and a filter r, which is positive real integer, we define embedded blocks

$$\vec{X}(i) = [x(i), x(i+1), \ldots, x(i+d_E-1)]$$

and

$$\vec{X}(j) = [x(j), x(j+1), \ldots, x(j+d_E-1)].$$

Then we find relative neighborhoods in phase space that are measures by the distance

$$d[\vec{X}(i), \vec{X}(j)] \equiv \max_{k=1,\ldots d_E}\{| x(i+k-1) - x(j+k-1) |\}$$

between all pairs of $\vec{X}(i)$ and $\vec{X}(j)$. For further estimations, we need to define the probability to find such pairs of patterns, the distance of which does not exceed the specified threshold r which can be calculated as

$$C_i^{d_E}(r) = \frac{1}{(N-d_E+1)} \sum_{j=1}^{N-d_E+1} \Theta\left(r - d\left[\vec{X}(i), \vec{X}(j)\right]\right),$$

where $\Theta(\cdot)$ is the Heaviside function which counts the instances where the distance d is below the threshold r.

Next, we define the logarithmic average over all the vectors of the $C_i^{d_E}(r)$ probability as

$$F^{d_E}(r) = \frac{1}{(N-d_E+1)} \sum_{i=1}^{N-d_E+1} \log\left(C_i^{d_E}(r)\right)$$

and ApEn of a corresponding time series is defined as

$$ApEn(d_E,\ r) = F^{d_E}(r) - F^{d_E+1}(r), \tag{4.4}$$

i.e., equation (4.4) measures the logarithmic likelihood that sequences of patterns that are close for d_E observations will remain close after further comparisons. Therefore, if the patterns in the sequence remain close to each other (high regularity), the ApEn becomes small, and hence, the time series data has a lower degree of randomness. High values of ApEn indicate randomness and unpredictability. But it should be considered that ApEn results are not always consistent, thus it depends on the value of r and the length of the data series. However, it remains insensitive to noise of magnitude if the values of r and d_E are sufficiently good, and it is robust to artifacts and outliers. Although ApEn remains usable without any models, it also fits naturally into a classical probability and statistics frameworks, and, generally, despite its shortcomings, it is still the applicable indicator of system stability, which significantly increased values may prognosticate the upcoming changes in the dynamics of the data.

4.2.3. Permutation entropy. According to this method, we need to examine "ordinal patterns" that consider the order among time series and relative amplitude of values instead of individual values. For evaluating PEn, at first, we need to consider a time series $\{x(i) \mid i = 1, \ldots, n\}$ which relevant details can be "revealed" in d_E-dimensional vector

$$\vec{X}(i) = [x(i), x(i+\tau), \ldots, x(i+(d_E-1)\tau)],$$

where $i = 1, 2, \ldots, N-(d_E-1)\tau$, and τ is an embedding delay of our time delayed vector. By the ordinal pattern, related to the time i, we consider the permutation $\pi_l(i) = (k_0, k_1, \ldots, k_{d_E-1})$ of $[0, 1, \ldots, d_E-1]$ where $1 \le l \le d_E!$ defined by

$$x(j+k_0\tau) \le x(j+k_1\tau) \le \ldots \le x(j+k_{d_E-1}\tau).$$

We will use ordinal pattern probability distribution as a basis for entropy estimation. Further, the relative frequencies of permutations in the time series are defined as

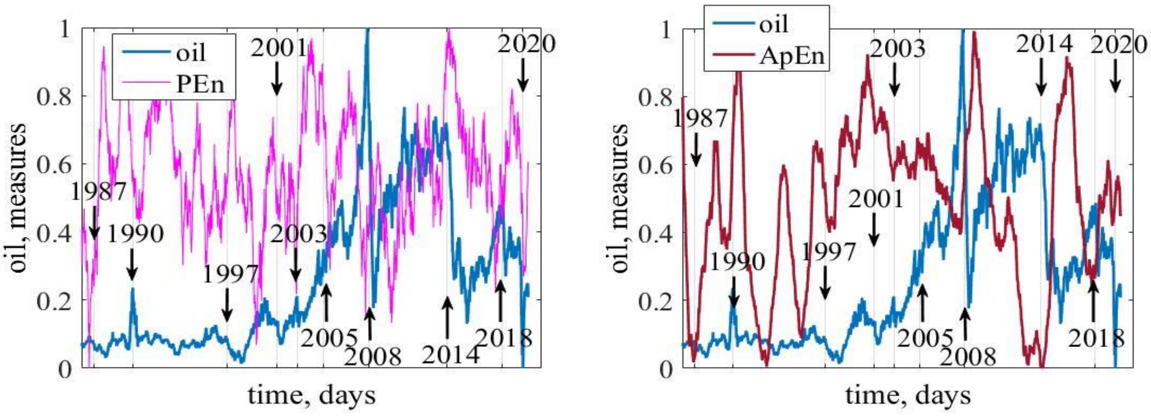
$$p(\pi_l) = \frac{\#\{s \mid s \le N-(d_E-1)\tau; (s) \text{ has type } \pi_l\}}{N-(d_E-1)\tau},$$

where the ordinal pattern probability distribution is given by $P = \{p_l(\pi_l) \mid l = 1, \ldots, d_E!\}$. The *Normalized Permutation entropy* (denoted by $E_s[P]$, where $0 \le E_s[P] \le 1$) of the corresponding time series presented as

$$E_s[P] = \frac{S[P]}{S_{\max}} = \frac{-\sum_{l=1}^{d_E!} p_l \log p_l}{S_{\max}},$$

whose $S_{\max} = \ln d_E!$ represents the maximum value of $E_s[P]$ (a normalization constant), and normalized entropy has a range $0 \le PEn \le 1$. Here, the maximal entropy possible value is realized when all $d_E!$ possible permutations have an equal probability of occurrence (more noise and random data). With the much lower entropy value, we get a more predictable and regular sequence of the data. Therefore, the PEn gives a measure of the departure of the time series from a complete noise and stochastic time series.

There must be predefined appropriate parameters on which PEn relying, namely, the embedding dimension d_E is paramount of importance because it determines $d_E!$ possible states for the appropriate probability distribution. With small values such as 1 or 2, parameter d_E will not work because there are only few distinct states. Furthermore, for obtaining reliable statistics and better detecting the dynamic structure of data, d_E should be relevant to the length of the time series or less [73]. For our experiments, $d_E \in \{3, 4\}$ and $\tau \in \{2, 3\}$ indicate the best results. Hence, in figure 4.4 we can observe the empirical results for permutation entropy, where it serves as an indicator-precursor of the possible shocks and crashes.



(a) (b)

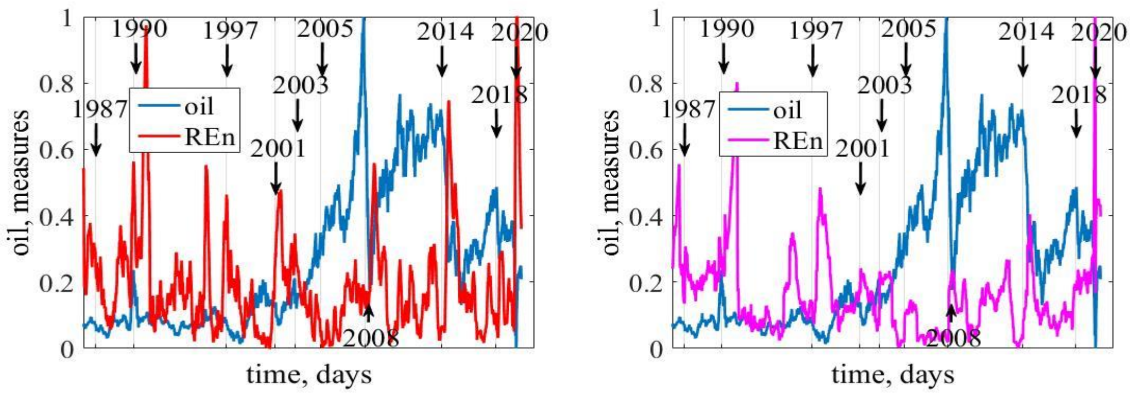
Figure 4.4. Pen (a) and ApEn (b) dynamics along with the oil price shocks and crashes.

Long before the shock (crash), the value of this types of entropy begins to decrease, the complexity of the system decreases. This measure, in our opinion, is one of the earliest precursors of the crisis.

4.2.4. Recurrence entropy. The corresponding measure of entropy is related to the recurrence properties that may be peculiar for the nonlinear complex system and important class of recurrence quantifiers are those that try to capture the level of complexity of a signal [37, 49, 192, 193]. In accordance with this study, the entropy diagonal line histogram (*REn*) is of the greatest interest which uses the Shannon entropy of the distribution of diagonal lines $P(l)$ to determine the complexity of the diagonal structures within the recurrence plot. One of the most know quantitative indicators of the recurrence analysis can be defined as

$$REn = -\sum_{l=l_{\min}}^{l=l_{\max}} p(l)\ln p(l) \quad \text{and} \quad p(l) = \frac{P(l)}{\sum_{l=l_{\min}}^{N} P(l)},$$

where $p(l)$ captures the probability of a diagonal line to have the exactly length l, and *REn* reflects the complexity of deterministic structure in the system. Further calculations were provided and presented in figure 4.5 for the initial oil time series and its normalized returns.



(a) (b)

Figure 4.5. REn dynamics along with initial time series (a) and normalized returns (b) of the entire time series of the oil price.

All of the informational measures of complexity take a prominent place among other measures of complexity. According to our empirical results, the complexity of the system changes in a specific way before corresponding shock or crash which unusual behavior is detected by LCZ and different versions of Shannon entropy. Their initial validity, transparency, easiness of usage could give us a new way of understanding complexity.

5. Fractals and multifractals

Mandelbrot was the first who highlighted the fractal structure of the market price fluctuations [130]. According to his pioneering work, they are poorly modeled and described by the so-called random walk hypothesis. Crude oil price systems are presented to be nonlinear complex systems that preserve chaotic and multiscaling features. Previously, in the field of hydrology, Hurst proposed the Rescaled Range (R/S) analysis that focuses on fractal structures [82, 83]. However, Lo [115] discovered that the R/S method is sensitive to short-term autocorrelation, which may lead to a bias error of nonstationary time series. The popular nowadays detrended fluctuation analysis method (DFA) was proposed by Peng et al. [155] to avoid significant long-range autocorrelation false detection [88]. Then, Kantelhardt et al. [90] extended DFA approach to its multifractal version (MF-DFA) that for a long time has been successfully applied for a variety of financial markets, such as stock [5, 77, 102, 104, 106, 109, 139, 199, 244], commodity [46. 74, 113, 126, 139], tanker [237], derivative [114], foreign exchange rates [30, 151, 153, 168, 213], and electricity markets [152]. An especially interesting application of multifractal analysis is measuring the degree of multifractality of time series, which can be related to the degree of efficiency of financial markets [143, 209, 216].

Podobnik and Stanley in order to study power-law cross-correlations between two non-stationary time series [159], extended standard version of DFA to the cross-correlation approach (DCCA). Guided by MF-DFA and DCCA approaches, Zhou [239] combined the multifractal and cross-correlation analyses into MF-DCCA [39]. Then a number of interesting methods have been proposed, such as the method of MF-PX-DFA and MF-PX-DMA [167], MF-X-DMA [87], MF-HXA [100], MF-X-PF [214], etc. Recently, the MF-DCCA method has become a widely applicable tool for the analysis of the multifractal characteristics of two cross-correlated nonstationary time series in the financial field such as the foreign exchange market [112, 226], the stock market [120, 215, 230], the crude oil market [120, 121, 217], carbon market [240, 241], and the commodity market [117]. The relationship between mass media and new media is examined by Zhang et al. with the MF-DCCA approach [236] and cross-correlation between investor sentiment proxies [235], i.e., fears [43] and Twitter happiness sentiment [234] is quantified.

Along with common multifractal methods, Sattarhoff and Gronwald [175] applied an intermittency coefficient for the evaluation of financial market efficiency. According to their study, the larger value of this measure, the more inefficient is a market.

Zhi-Qiang et al. [238] apply detrending moving average analysis and DFA of the WTI crude oil price (1983-2012) to investigate its efficiency. As statistical tests present, the market is inefficient if consider the whole period. When the time series is divided into three sub-series where such series are separated by some meaningful events as the Gulf War and the Iraq War. During the Gulf War, the efficiency of the crude oil market was reduced. Splitting it again, in the period of the North American Free Trade Agreement, the market seemed to be inefficient in the sub-periods of the Gulf War. With the sliding window approach, it can be observed that only when some turbulent event happens, such as the oil price crash in 1985, the market presents to be inefficient. The same results were obtained with the MF-DFA approach [172] where exactly the multifractal nature of WTI and Brent crude oil was studied. According to results, two markets become more and more efficient despite two Gulf Wars that, nevertheless, strongly affected the markets. Moreover, it is concluded that not only the broad fat-tail distribution and persistence are the reason for markets' multifractal structure, but also other factors.

This study covers three major international crude oil markets (WTI, Brent crude, and OPEC reference basket) from January 2, 2003, to January 2, 2014. The MF-DFA approach is applied to extract generalized Hurst exponent for each of the time series. This exponent is used to measure the multifractality degree which is used in turn to quantify the efficiency of three markets. The comparative results present that all of three markets present signs of multifractality. The most efficient is presented to be WTI while OPEC is the least. This implies a large monopoly power behind OPEC and reflects a high level of complexity of WTI and Brent which makes them very competitive.

Multifractal cross-correlation analysis (MFCCA) and detrended cross-correlation coefficient are applied to study statistical and multiscaling characteristics of WTI crude oil price in relation to the most traded currencies [218]. In most cases, the considered financial instruments are presented to follow the inverse cubic law. Crude oil, in the case of cross-correlation analysis, reveals multifractal organization, and, as it can be seen, the strongest ties to WTI express the oil extracting countries. The degree of these multifractal coupling varies over a studied period. Similar results are described for the pairs oil-gas, oil-CO_2, and gas-CO_2 [65]. Cross-correlations between these pairs obey a power-law and are weakly persistent. By employing the rolling window method, they analyze long-term and short-term market dynamics and obtain that exactly the global financial crisis has the most noticeable influence on the market dynamics. Moreover, the same type of procedure was applied for electricity and carbon markets [179] where multifractal characteristics are proved, and the reasons of multifractality are explained, and for analysis of the actual at this period coronavirus pandemic [86] where multifractal and cross-correlation properties between crude oil and selected agriculture future markets are studied.

The analysis of cross-correlations between between the major currency rates, Bitcoin, the DJIA, gold price, and the oil crude market was applied with MF-ADCCA method [63]. On its cross-correlation with the WTI, the Gold, and the DJIA, Bitcoin presented the greatest presence of multifractality. Bitcoin presented a different relationship between commodities and stock market indices, which had to be taken into consideration when investing. The reason is that over the years the currency was traded and over time, it has earned the trust of the community.

In similar way with our articles [20, 23, 64] where we applied the MF-DFA method to stock markets, cryptocurrency, and sustainability data, we use it here to explore the multifractal property of the oil price and construct reliable indicator for it.

5.1. Multifractal detrended fluctuation analysis (MF-DFA)

As an extension to the original DFA [156], the multifractal approach [90] estimates the Hurst exponent of a time series at different scales. Based on a given time series $\{x(i) \mid i = 1, \dots, N\}$, the MF-DFA is described as follows:

(i) The profile $Y(i)$ (accumulation) is defined as

$$Y(i) = \sum_{j=1}^{i} (g(j) - \langle g \rangle).$$

(ii) New time series $\{Y(i)\}$ is then divided into $N_s \equiv \text{int}(N/s)$ non-overlapping time segments of equal length s. Since the length of the time series is not always a multiple of s, for taking into account the remaining part and, therefore, to use all the elements of the sequence, the same procedure is repeated starting from the end of the profile $Y(i)$, and the total number of segments amount to $2N_s$. The data in each interval v are fitted with a polynomial Y_s^{fit} in order to eliminate the local trends, and the variance is computed as

$$F^2(v, s) = s^{-1} \sum_{i=1}^{s} \left[Y((v-1)s + i) - Y_v^{fit}(i) \right]^2, \quad \text{for} \quad s = 1, \dots, N_s$$

and

$$F^2(v, s) = s^{-1} \sum_{i=1}^{s} \left[Y(N - (v - N_s)s + i) - Y_s^{fit}(i) \right]^2, \quad \text{for} \quad s = N_s + 1, \dots, 2N_s.$$

(iii) Considering the variability of time series and the possible multiple scaling properties, we obtain the q^{th} order fluctuation function by averaging over all segments

$$F_q(s) = \left[\frac{1}{2N_s} \sum_{v=1}^{2N_s} \left[F^2(v,s) \right]^{q/2} \right]^{1/q} \quad (5.1)$$

At least, we determine the scaling behavior of the fluctuation function by analyzing $\log F_q(s)$ vs $\log s$ graphs for each value of q. Here, $F_q(s)$ is expected to reveal power-law scaling

$$F_q(s) \sim s^{h(q)}.$$

The scaling exponent $h(q)$ can be considered as generalized Hurst exponent, and with $q = 2$ we back to the standard DFA approach where $h(2) = H$ is the well-known Hurst exponent.

For $0.0 \le h(2) < 0.5$ we deal with anti-persistency where the process under study tends to decrease (increase) after a previous increase (decrease); $h(2) = 0.5$ corresponds to uncorrelated processes (random walk behavior) [77]; $0.5 < h(2) \le 1.0$ denotes persistency (process that tended to increase (decrease) for some period T, continue it for a similar period of time); $h(2) > 1.0$ responds for non-stationary processes (strong long-range correlations are presented).

For positive values of q, $h(q)$ describes the scaling behavior of time intervals with large fluctuations. Large fluctuations are usually characterized by smaller scaling coefficients of $h(q)$ for multifractal series. On the contrary, for negative values of q, time intervals with a small variance $F^2(v,s)$ will dominate. Thus, $h(q)$ will describe the scaling behavior of time intervals with small fluctuations.

(iv) Moreover, it is possible to characterize multifractality of a time series in terms of the multifractal scaling exponent $\tau(q)$ which is related to the generalized Hurst exponent $h(q)$ from the standard multifractal formalism and given by [146]:

$$\tau(q) = qh(q) - 1. \quad (5.2)$$

Here, equation (5.2) shows the scaling dependence of small fluctuations for negative $q's$ and large fluctuations for positives as the function of q moments. If equation (5.2) reflects linear dependence on q, the studied system is considered to be monofractal. Otherwise, if it has a non-linear dependence on q, then the system is multifractal.

(v) The different scalings are better described by the singularity spectrum $f(\alpha)$ which can be defined as

$$f(\alpha) = q[\alpha - h(q)] + 1 \quad \text{where} \quad \alpha = \frac{d\tau(q)}{dq} = h(q) + q\frac{dh(q)}{dq}$$

with α - the Hölder exponent or singularity strength and $f(\alpha)$ - fractal dimension with singularities α. Following the methods described above, we present results that reflect multifractal behavior of the oil price dynamics.

Figure 5.1(a) presents function $F_q(s)$ defined by equation (5.1). The slope changes dependently on q, which indicates the multifractal property of a time series. As it was pointed out, multifractality emerges not only because of temporal correlation, but also because crude oil price turns out to be fat-tailed [145], and this distribution could contribute to the multifractality of the time series. The same dependence can be observed in the remaining plots. The scaling exponent $\tau(q)$ remains non-linear, as well as generalized Hurst exponents that can serve as evidence that the oil market exhibits multifractal property.

The shape of $f(\alpha)$ resembles an inverted parabola (see figure 5.1(d)) which is an indicator of multifractal behavior; furthermore, the degree of complexity is straightforwardly quantified by the width of $f(\alpha)$, simply defined as $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$, where $\alpha_{\min}$ and $\alpha_{\max}$ correspond to the minimum and maximum values of α.

In the figure 5.2(a) we present the width of the spectrum of multifractality that changes over time accordingly to the sliding window approach. The whole figure consists of both a three-dimensional plot (singularity spectrum) and two-dimensional representation of its surface.

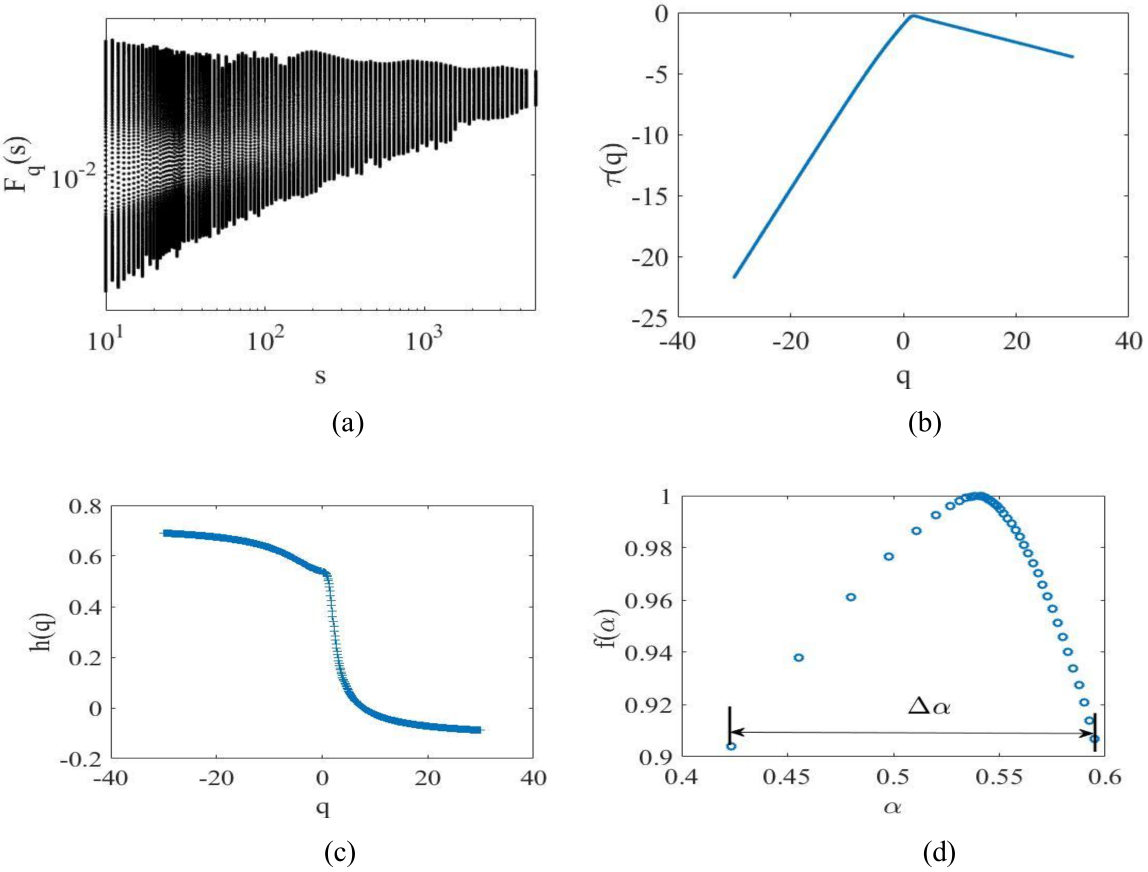
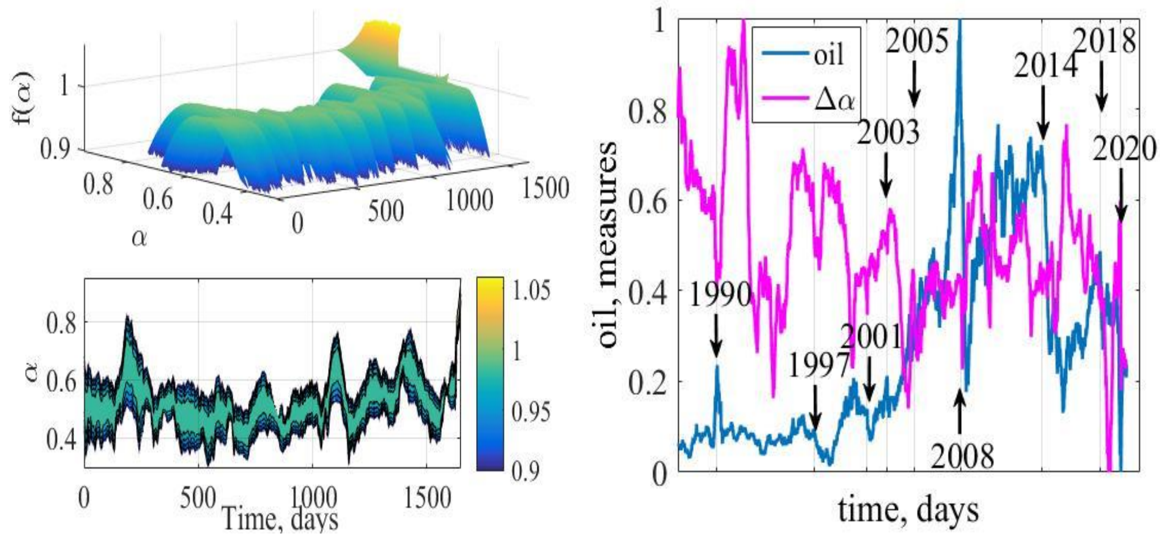


Figure 5.1. The fluctuation function $F_q(s)$ (a), multifractal scaling exponent $\tau(q)$ (b), $h(q)$ versus q of the oil price returns (c), and singularity spectrum $f(\alpha)$ (d) obtained from MF-DFA for the oil price time series.

In accordance with previous multiscaling indicators, $f(\alpha)$ remains constant if the system is monofractal. As can be observed, in figure 5.2(a) our series exhibits a simple multifractal scaling behavior, as $f(\alpha)$ changes dependently on α, i.e., it exhibits different scalings at different scales. Moreover, with the sliding window of 500 days and time step of 5 days, we understand that at different time periods oil price dynamics becomes more or less complex. The value of $\Delta\alpha$ gives a shred of additional evidence on it (see figure 5.2(b)). As we can see, the width of the singularity spectrum after the crisis starts to increase, which tells us that more violent price fluctuations are usually expected. With the decreasing width of the singularity spectrum, the series is expected to hold the trend. As the rule, it reaches its minimum before the collapse of the oil price.



(a) (b)

Figure 5.2. Changes in the spectrum of multifractality in time (a), and the comparison of the corresponding oil price time series and the width of the multifractality spectrum measure (b).

6. Lévy alpha-stable distribution

Modern financial markets are characterized by a rapid flow of information where a huge number of transactions between market participants in different time horizons happens. Such interconnectivity of different factors leads to uncertainty, abrupt and highly volatile changes, resulting in significant deviations of empirical data from normal distribution [13, 162].

At first, it was introduced by Bachelier that returns of stock prices are obeyed to a normal distribution and random walk hypothesis that considered the return over a time scale Δt to be the consequence of many independent critical-like phenomena [7]. Some stylized facts of daily returns [62, 94, 99] reveal that distributions are leptokurtic and, therefore, Gaussian distribution does not fit well to the data. Sornette and Lux [118] pronounce that the distribution of such data may be not only leptokurtic, but it can also be characterized by fat tails [19, 124, 125, 202]. Thus, it should belong to the class of fat-tailed distributions. For the majority of financial markets, it is currently accepted that the PDFs for fat tails of absolute normalized returns scale to a *power law*

$$f(x;\alpha,\beta,\gamma,\delta) \sim \begin{cases} c_\alpha \gamma^\alpha \alpha (1+\beta)|x|^{-(1+\alpha)} & \text{for } (x \to +\infty) \\ c_\alpha \gamma^\alpha \alpha (1-\beta)|x|^{-(1+\alpha)} & \text{for } (x \to -\infty) \end{cases} \tag{6.1}$$

and their CDF can be expressed as

$$\begin{cases} P(X > x) \sim c_\alpha \gamma^\alpha (1+\beta)|x|^{-\alpha} & \text{for } (x \to +\infty), \\ P(X < x) \sim c_\alpha \gamma^\alpha (1-\beta)|x|^{-\alpha} & \text{for } (x \to -\infty), \end{cases}$$

where c_α is a constant value $[\sin(\pi\alpha/2)\Gamma(\alpha)]/\pi$, and $\Gamma(\cdot)$ is the Gamma function.

The emergence of power-law behavior in price fluctuations is argued to be a consequence of underlying complex mechanisms, such as feedback effects and correlations in financial markets [71, 72, 160, 161]. Some theories associate this phenomenon with market impact and the distribution of large investors [61, 62], while other studies model the power-law behavior as a consequence of limited information and the true value of companies [96]. Such property is a symptom of self-organization and complexity which are prominent for economic systems. In Chakraborty et al. paper [31] it was

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established that currencies of several frontiers that are outside of *inverse cubic law* (with an exponent of $\alpha \approx 3$) belong to the Lévy-stable regime and are expected to be yet emerging and having sudden large changes such as crashes and critical events, while those of most developed exhibited *inverse cubic law*. Belov et al. [16], testing the adequacy of the family of stable distributions for financial modeling conclude that they are suitable but appropriate stability tests should be made before model application.

Tung-Li and Hai-Chin [211], fitting the daily returns into the Gaussian distribution and estimating its parameters, obtain that returns of oil market are slightly left-skewed, positive, and leptokurtic. After estimating the peak and the width of the volatility of the log-normal distribution, the crude oil market is presented to be volatile market.

Kateregga et al. [94] explores the theory behind the family of α- stable distributions, fitting them to financial asset log-returns data. Here, they present different techniques for estimating parameters of this type of distribution, and they argue that empirical characteristic function method performs better that maximum likelihood over a wide range of shape parameters, and it has better convergence. Besides, they apply the *t* location-scale distribution and compare it with the general stable distribution, showing that the former fails to capture skewness of the data.

Bautista and Mora [14] use value at risk (VaR) to quantify as best as possible the maximum price changes of three type of oil (Brent, WTI, and Mezcla Mexicana (MME)), considering GARCH models with three alternative distributions: stable, Student-t generalized, and normal in a period of high volatility. According to their results, the VaR-stable model is a more robust and accurate comparing to generalized asymmetric and normalized Student t-distributions.

Gunay and Khaki [75], relying on GARCH and APARCH models under different distributions, including alpha-stable, attempt to model volatility of futures for gas, brent oil, and heating oil. Their results confirm that the applicability of normal distribution to energy markets is narrow, and risk managers should consider alternative types for modelling fat tails in returns.

For some time, it has been considered that financial markets can be described in term of a normal distribution. Later, analysing cotton prices, Benoît Mandelbrot observed that in addition to being non-Gaussian, the form of a returns distribution remains stable for different time scales, and its tails are much heavier comparing to normal distribution [129]. Further empirical evidence allowed Mandelbrot and Fama to conclude that such empirical data are better fitted with the stable Lévy regime [111, 128, 129, 56]. The reasons to use so-called α-stable, stable Paretian or Lévy stable distributions are because the central limit theorem [70] points to the importance of using stable laws for properly normalized and centered sums of (IID) random variables. Also, that types of distribution are presented to be leptokurtic which is in consistency with the heavy tails and asymmetry of distribution.

6.1. Lévy's stable distribution properties

According to Nolan [150], the distribution can be described in terms of stability if its shape (up to scale and shift) retains under addition:

$$X_1 + X_2 + \cdots + X_n \overset{d}{=} c_n X + d_n \tag{6.2}$$

for some constants $c_n > 0$ and $d_n \in \mathbb{R}$, where $X_1, \ldots, X_n$ are independent, identical copies of X.

The class of all laws that satisfy condition (6.2) is presented by 4 parameters: $\alpha \in (0,2]$ is the *index of stability* or *characteristic exponent* where a smaller value of α corresponds to more severe tails of the distribution (much frequent and larger extreme events). The parameter $\beta \in [-1,1]$ is called the *skewness* parameter of the law. If $\beta = 0$, the distribution is symmetric. In the case when $\beta > 0$, it is skewed toward the right, otherwise to the left. The last two parameters stand for the scale $\gamma \in [0,\infty)$ and $\delta \in (-\infty,\infty)$ the location parameters of the distribution, which is also known as the mean or the measure of centrality. For the normal distribution γ is equal to the standard deviation. Since random variables X is characterized by four parameters, we will denote α-stable distribution by $S(\alpha,\beta,\gamma,\delta)$ and write

$$X \sim S(\alpha,\beta,\gamma,\delta). \tag{6.3}$$

As stable distributions do not have analytical expressions for either PDF (except for some cases: the Gaussian distribution where $\alpha = 2$; the Cauchy distribution where $\alpha = 1$ and $\beta = 0$; the Lévy distribution where $\alpha = 1/2$ and $\beta = 1$) or CDF, if a random variable follows (6.3), they can be described in terms of characteristic functions (CF) [97]:

$$\lambda(k) = \begin{cases} \exp\{i\delta k - \gamma^{\alpha}|k|^{\alpha}[1 - i\beta \operatorname{sgn}(k)\tan(\frac{\pi\alpha}{2})]\}, & (\alpha \neq 1), \\ \exp\{i\delta k - \gamma|k|[1 + i\beta \operatorname{sgn}(k)\frac{2}{\pi}\ln|k|]\}, & (\alpha = 1). \end{cases}$$

Thus, the inverse Fourier transform of the CF that can be expressed in the following form

$$f(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\exp(-ikx)\lambda(k)dk$$

allows us to reconstruct probability density function with a known characteristic function.

6.2. Methods for estimation of stable law parameters

For the last decades a variety of different approaches have been proposed to estimate parameters of stable law processes: the approximate maximum likelihood (ML) estimation [29, 149], quantiles method [57, 141], fractional lower order moment method [122, 177], method of log-cumulant [148], the logarithmic moment method [103] and more. In our study we rely on empirical characteristic function method as it is presented to be more accurate and computationally efficient comparing to other methods.

6.2.1. Empirical Characteristic Function Method. According to the method, our data $\{x_i \mid i = 1, 2, \ldots N\}$ are supposed to be ergodic [4]. Following this assumption, we obtain [212]:

$$\lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N}\exp(ikx_i) = \int_{-\infty}^{\infty}\exp(ikx)\rho(x)dx. \tag{6.4}$$

where $f(x)$ is some integrable function, and $\rho(x)dx$ is a measure in a space M .

Then, to consider characteristic functions, equation (6.4) comes out to be the following ergodic equality [212]:

$$\lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N}\exp(ikx_i) = \int_{-\infty}^{\infty}\exp(ikx)f(x)dx,$$

for which we have

$$\hat{\lambda}(k) = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\exp(ikx_i).$$

Consequently, for the empirical characteristic function $\lambda_N(k)$ following equation holds:

$$\hat{\lambda}_N(k) = \frac{1}{N}\sum_{i=1}^{N}\exp(ikx_i).$$

Then, according to Koutrouvelis'[97, 98] regression type from it can be derived that

$$\log(-\log(|\lambda(k)|^2)) = \log(2\gamma^{\alpha}) + \alpha\log(k). \tag{6.5}$$

The imaginary and real parts of $\lambda(k)$ are given by

$$\begin{cases} \lambda_{\mathrm{I}}(k) = \exp(-|\gamma k|^{\alpha}) \cdot i \sin\left[\delta k - |\gamma k|^{\alpha} \beta \operatorname{sgn}(k) \omega(k,\alpha)\right], \\ \lambda_{\mathrm{R}}(k) = \exp(-|\gamma k|^{\alpha}) \cdot \cos\left[\delta k - |\gamma k|^{\alpha} \beta \operatorname{sgn}(k) \omega(k,\alpha)\right], \end{cases}$$

where

$$\omega(k,\alpha) = \begin{cases} \tan\left(\dfrac{\pi\alpha}{2}\right), & \alpha \neq 1, \\ \dfrac{2}{\pi} \ln|k|, & \alpha = 1. \end{cases}$$

Suppose $\Upsilon(k) := \arctan(\lambda_{\mathrm{I}}(k) / \lambda_{\mathrm{R}}(k))$. Then, in the condition $\alpha \neq 1$, the ratio of two last equations, apart from considerations of principal values, leads to

$$\Upsilon(k) = \delta k - \beta \gamma^{\alpha} \tan\left(\frac{\pi\alpha}{2}\right) \operatorname{sgn}(k) |k|^{\alpha}.$$

Equation (6.5) depends only on α and γ, and it suggests that we estimate these parameters by regressing

$$y = \log\left(-\log|\lambda_N(k)|^2\right) = \log\left(2\gamma^{\alpha}\right) + \alpha \log(k)$$

on $\omega = \log(k)$ in the model

$$y_l = m + \alpha\theta_l + \varepsilon_l, \quad \text{for } l = 1, 2, \ldots, L,$$

where $y_l = \log(-\log(\hat{\lambda}(k_l))^2)$, $m = \log(2\gamma^{\alpha})$, $\theta_l = \log(k_l)$, and ε_l responds for an error term. The proposed real data set for L (see Koutrouvelis [97], Table I) is given by $k_l = \pi l / 25$ $(l = 1, \ldots, L)$.

With estimated and fixed parameters α and γ, the symmetric parameter β and location parameter δ can be obtained by linear regression estimation

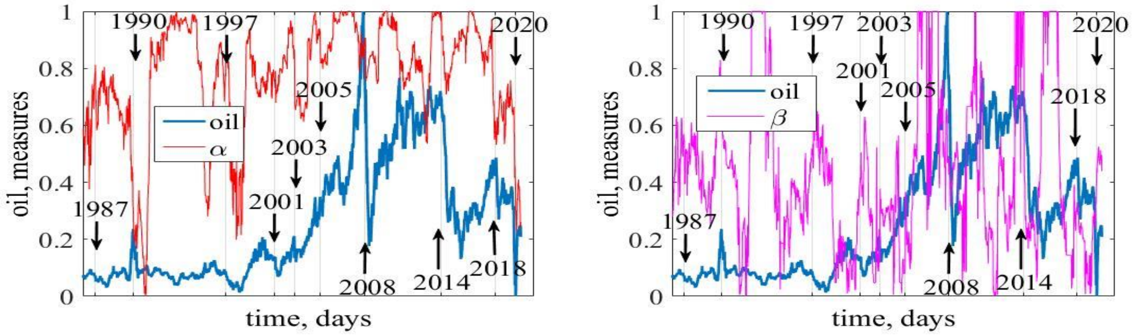
$$z_q = \delta k_q - \beta \gamma^{\alpha} \tan\left(\frac{\pi\alpha}{2}\right) \operatorname{sgn}(k_q) |k_q|^{\alpha} + \nu_q, \quad \text{for } q = 1, \ldots, Q,$$

where $z_q = \Upsilon_N(k_q) + \pi l_N(k_q)$, ν_q denotes an error term, and the proposed real data set for Q (see Koutrouvelis [97], Table II) is $k_q = \pi q / 50$ $(q = 1, 2, \ldots, Q)$.

6.3. Related studies and corresponding results

Recently, the use of dynamic indicators, precursors of crashes in stock markets using the parameters of a α-stable distribution was proposed by us in the papers [21, 22] and [23]. Moreover, analyzing only a crisis of 2008 using a limited set of stock indices [60], authors conclude β to be even more convincing indicator comparing to others. During our research, the opposite conclusion was made, i.e., both α and β are informative indicators that can be seen in figure 6.1.

Figures above align our confidence in the relation of shocks and crashes to the heavy tails of the distribution. We can see that our parameters start to decrease before corresponding crashes and shocks. Such characteristic behavior can serve as an indicator-precursor of such events that are related to fat-tailness in our data.



(a) (b)

Figure 6.1. Crude oil price time series and estimated for them parameters α (a) and β (b). Vertical arrows indicate crashes and shocks.

7. Network measures

Taking into account nonlinearity and nonstationarity of the studied systems, the complex network theory, which laid the foundation of a new network paradigm of synergetic [127], is seemed to be reasonable approach for studying multivariate properties of financial time series.

Investigating such systems, we consider their topology, the distribution of nodes and edges, robustness, the effects of information dissemination, etc. [1, 12, 35, 147]. Moreover, such networks as electrical, transport, information, social, economic, biological, neural, etc. are also complex [19, 26, 146]. More frequently we can study networks that are interconnected with other networks and form hierarchy of different systems converted into network [25, 80].

Recently, the first papers using the spectral and topological characteristics of dynamic systems presented as networks have appeared. Thus, in [166], it has been investigated universal and non-universal allometric scaling behaviors in the visibility graphs of 30 world stock market indices. It has been established that the nature of such behavior is due to the returns distribution that is characterized by fat-tails, the nonlinear long-term correlation, and a coupling effect between the set of influential factors. Through some topological parameters (density, clustering, assortativity/disassortativity, centrality, and degree distribution) which are specific to complex networks are used to analyze the changing structure and evolution of global coal trade [198]. The coal trade network is presented to be complex (heterogeneous) in terms of connectedness in the network. There is an increasing trend overall, although connectivity has started declining since 2010. Centrality measure gives an overview of import/export shares, and, as it can be seen, Asian countries the biggest consumers of these energy resources according to network indicators.

Setting up the spot price fluctuation networks of heating oil spot and its futures price fluctuation network, the analysis of transformation characteristics between the modes was established [81]. Average path length, node strength, and strength distribution, betweenness, etc. were investigated, and the function of network similarity was established to analyze networks in more detail. The power law distributions of spot and futures price fluctuations present regularity and complexity in different periods. Moreover, these systems remain stable and correlated during usual trading days but become unstable in the phase of sharp fluctuation.

7.1. Methods of converting time series into graphs

For the past years, interesting algorithms have been developed for transformation the nonlinear time series into complex network in order to reveal its primal characteristics by using the topology of the network. Such alternative mathematical structure can be revealed with the usage of visibility graph (VG) [50, 105]. To reveal the probability of occurrence of similar states in network, techniques from RQA [50, 79, 133] can extracted, and then corresponding spectral and topological characteristics of such graph are calculated. Our previous studies consider the idea of such methods [80, 163, 201] for identification of critical and crisis phenomena in stock and crypto markets [194-196]. The recurrence diagram is easily transformed into an adjacency matrix, by which the spectral and topological characteristics of the graph are calculated [194]. Therefore, we will focus on algorithms of the VG [105] where each data point of a time series is considered to be a vertex, and an edge is putted between two vertexes if the following condition is satisfied

$$x(t_c) < x(t_a) + (x(t_b) - x(t_a))\frac{t_c - t_a}{t_b - t_a}. \quad (7.1)$$

The horizontal visibility graph is an alternative and much simpler algorithm (HVG) [50] in which a connection between two data points can be established if $x(t_a), x(t_b) > x(t_c)$ for all c such that $t_a < t_c < t_b$.

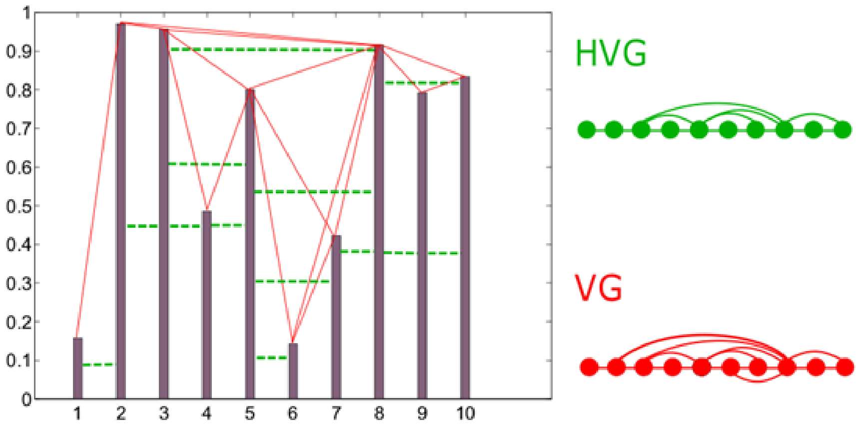


Figure 7.1. The idea of converting a time series into graph with visual graph and horizontal visual graph algorithms [84].

7.2. Indicators of Spectral and Topological graphs

Spectral theory of graphs is based on algebraic invariants of a graph - its spectra [207]. The spectrum of graph G is the set of eigenvalues $S_p(G)$ of a matrix corresponding to a given graph. For adjacency matrix A of a graph, there exists an characteristic polynomial $|\lambda I - A|$, which is called the characteristic polynomial of a graph $P_G(\lambda)$. The eigenvalues of the matrix A (the zeros of the polynomial $|\lambda I - A|$) and the spectrum of the matrix A (the set of eigenvalues) are called respectively their eigenvalues.

Important derivative characteristic of a spectral graph is the spectral radius - the largest absolute value (or complex modulus) of the graph eigenvalues (eigenvalues of the adjacency matrix) that is defined as

$$\rho(A) = \max_{1 \le i \le n} |\lambda_i|. \quad (7.2)$$

Among the topological measures, one of the most important is the node degree D - the number of edges attached to this node and its maximum degree D_m. For non-directed networks, the node's degree D_i is determined by the sum

$$D_i = \sum_j a_{ij}, \quad (7.3)$$

where the elements a_{ij} of the adjacency matrix.

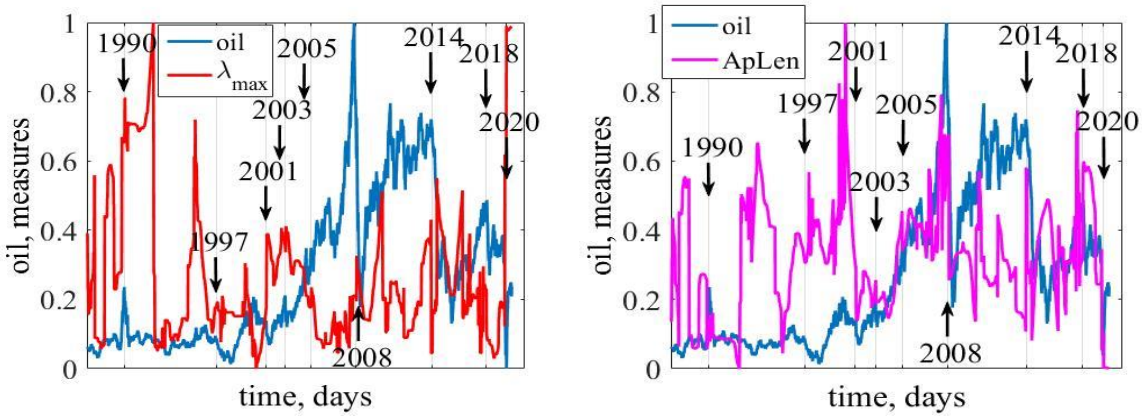
For characterizing the "linear size" of the network, there are useful concepts of mean $\langle l \rangle$ of the shortest paths. For a connected network of n nodes, the *average path length* (ApLen) is equal to

$$\langle l \rangle = \frac{2}{n(n-1)} \sum_{i>j} l_{ij}, \tag{7.4}$$

where l_{ij} - length of the shortest path between the nodes.

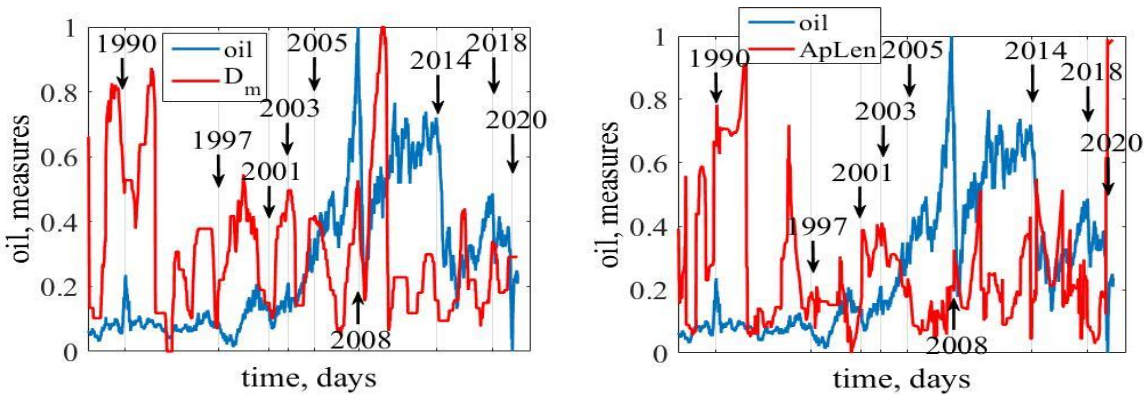
In our previous studies we have adapted, along with various quantitative measures of complexity [22, 23, 146, 183, 184, 188, 190, 191, 195], network complexity measures. In this work, from spectral measures, we consider important the largest eigenvalue of adjacency matrix ($\lambda_{\max}$ – figure 7.2(a)) and the average path length (ApLen - figure 7.2(b)) which are in accordance with equations (7.2) and (7.4). for cross-recurrent nodes. From the visibility graph, the maximum node degree (D_m - figure 7.3(a)) and the average path length (ApLen – figure 7.3(b)) is found.

Figures 7.2 and 7.3 demonstrates the asymmetric response of the spectral and topological measures of network complexity. For the complete series, the calculation parameters are as follows: window length of 500 days, step is of 5 days.



(a) (b)

Figure 7.2. Largest eigenvalue of adjacency matrix $\lambda_{\max}$ (a) and ApLen with recurrence diagrams for the crude oil price time series.



(a) (b)

Figure 7.3. Maximum node degree D_{m} (a) and ApLen (b) with VG for the crude oil price time series.

Corresponding spectral measures present that pre-crisis periods have the greatest complexity. During approaching shock or crash, the complexity of the system decreases, recovering after the end. ApLen shows an asymmetrical relationship. During crisis states, the length between nodes increases, indicating about approaching shock or crash.

8. Quantum precursors

In the first decade of the 21st century due to the lack of classical econophysics, the application of quantum algorithms in finance and economics was extended with the quantum concepts [78, 137, 174, 186].

It is pressing need for economists to discover completely extraordinary and complex models and analogies from interconnected disciplines that could deal with the emergence of uncertainty and randomness in financial field. While classical econophysics tries to reduce randomness by improving our conceptual tools, quantum econophysics integrates randomness in them. It has already been agreed that no-intuitive concepts such as uncertainty principle, system wave function, or superposition principle are appreciable for description of our world. Even classical econophysical notions of Hamiltonian or free energy could be applied to study the dynamics of financial markets. Therefore, reconsideration of uncertainty in macro and micro financial phenomena could improve our understanding the laws that govern financial systems, how to reproduce them, and how to resist the most undesirable ones.

Consequently, in this section, we would like to present the application of the Heisenberg uncertainty principle to the actual oil price dynamics [186, 189].

8.1. Economic analogue of the uncertainty principle

Modern theoretical physic, according to our study [174], provides ideas for new adequate and useful models in socio-economic phenomena and processes. Nowadays, such quantum-mechanical analogue as the uncertainty principle, can be applied for socio-economic processes.

Nowadays Heisenberg uncertainty principle is one of the cornerstones of quantum mechanics that deals rather with the uncertainty of quantum states [171, 173] than with the precision of measurements. Recent studies of uncertainty relations have been a topic of growing interest in such disciplines as quantum information and cryptography [17, 164].

The key concept of quantum physics (Heisenberg's uncertainty principle) can be described as the following [108]:

$$\Delta x \cdot \Delta \upsilon \geq \frac{\hbar}{2m_0}, \quad (8.1)$$

where Δx and $\Delta \upsilon$ standard deviations of position and momentum variables of a particle with mass m_0, and $\hbar = h / 2\pi$, h is Planck's constant. Considering values Δx and $\Delta \upsilon$ to be measurable when their product reaches their minimum, according to equation (8.1) we derive

$$m_0 = \frac{\hbar}{2 \cdot \Delta x \cdot \Delta \upsilon}, \quad (8.2)$$

i.e., the mass of the particle is taken via uncertainties of its position and momentum – time derivative of the same coordinate.

The main characteristic of physical laws and their constants is that they remain invariant at least $\sim 10^{11}$ years, while economic measurements are fundamentally relative, local in time, and the adequacy of the formalism used for their mathematical description should be revised each time as the states, tendencies, and perspectives of global, regional, and national economies change. Thus, continuous analysis and monitoring of stock indices, exchange rates, cryptocurrencies prices, spot prices, etc. become crucial.

Evaluating corresponding "economic mass", we suppose that

$$X_i(t_n),\ t_n = \Delta t_{\min} n,\ \text{for } n = 0,\ 1,\ 2, \ldots, N-1,\ \text{for } i = 1,\ 2, \ldots, K. \quad (8.3)$$

where K is a number of time series with N samples that lie on an equal minimal distance $\Delta t_{\min}$ one from each other. Then, we normalize our samples taking a natural logarithm of each one-dimensional trajectory of a certain "abstract particle" $x_i(t_n) = \ln X_i(t_n)$ to bring each series to the unified and non-dimensional representation. Then, registering its position after each time span $\Delta t_{\min}$, we estimate standard deviation of its position and momentum according to some time frame $\Delta T = \Delta N \cdot \Delta t_{\min} = \Delta N,\ \ 1 << \Delta N << N$. Accordingly, the "instantaneous" speed of i^{th} particle at t_n is defined as

$$\upsilon_i(t_n) = \frac{x_i(t_{n+1}) - x_i(t_n)}{\Delta t_{\min}} = \frac{\ln X_i(t_{n+1}) - \ln X_i(t_n)}{\Delta t_{\min}}, \quad (8.4)$$

with variance D_{v_i} and standard deviation $\Delta\upsilon_i = \sqrt{D_{\upsilon_i}}$.

Keeping an analogy with log-returns, after some transformations, we can write an uncertainty ratio for this trajectory [185]:

$$\frac{1}{\Delta t_{\min}}\left(< \ln^2 X_i(t_{n+1}) - \ln^2 X_i(t_n) >_{n,\Delta N} - \left(< \ln X_i(t_{n+1}) - \ln X_i(t_n) >_{n,\Delta N} \right)^2 \right) \square \frac{h}{m_i}, \quad (8.5)$$

where m_i - economic "mass" of an i^{th} series, and h is an economic Planck's constant.

Presented economic Planck's constant, unlike its physical analogue, may vary with different historical periods, averaging windows, and particular time series.

In recent research [20, 23, 183, 186, 187], the economic mass was tested as an indicator of crashes and critical events on stock, crypto and sustainability data. Here, we apply the model for the oil prices. During crashes and shocks economic mass noticeably decreases, indication about external changes in the market.

Obviously, m remains a good indicator-precursor even in this case. Value m is considerably reduced before a special market condition. Thus the market becomes more volatile and prone to changes.

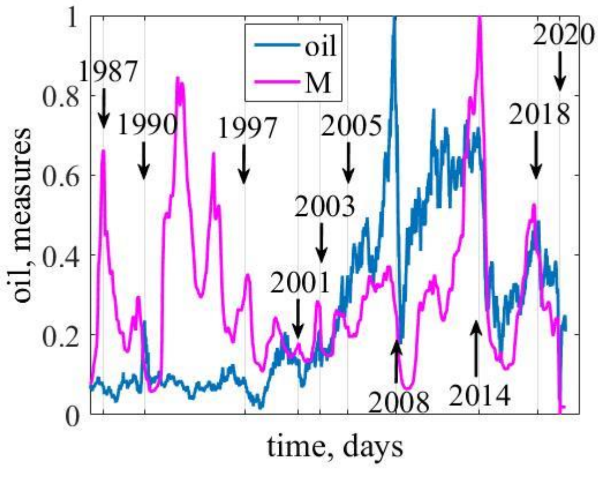


Figure 9.1. Dynamics of measure m for the oil price with the window size of 250 days and the step of 5 days.

9. Conclusions

Definitely, the factors from within and outside of the oil market universe are going to evolve all of them. The great influence will go from incumbents and policymakers, as well as from its competitors and ordinary users. Current mistrust on the part of the government may lead to the introduction of specific licensing requirements that may make these digital currencies less attractive. Similarly, the adaptation to them and acceptance of cryptocurrencies may lead to increasing demand for them. The current situation with coronavirus is of paramount importance and is of significant danger.

From the literature overview, we have understood that shocks and crashes do not disappear without a trace, but will also affect the fate of individuals. Thus, in order to give reliable, powerful, and simple indicators-precursors that are able to minimize further losses as a result of changes, we addressed the reach arsenal of the theory of complexity and the methods of nonlinear dynamics that can identify special trajectories in the complex dynamics and classify them. Following our research, we obtain informational, (multi-) fractal, recurrent, derived from alpha-stable distribution, based on complex networks, and quantum measures of complexity.

The obtained quantitative methods were applied to classified crashes of the oil market, where it was seen that these indicators can be used in order to protect yourself from the upcoming critical change. To draw some conclusions about its evolutions and factors that influence it, we pointed out the most influential critical changes in this market. The analysis of the oil market with the sliding (rolling or moving) window approach allowed us to draw some conclusions about its evolution and factors that influence it. Regarding empirical results, we have shown that some of the measures are very sensitive to the length of the sliding window and its time step. For example, if we consider two closest to each other events, a previous event that had much more volatility can have a great influence on the corresponding measure of complexity and spoil the identification of the next less influential, but important event. Thus, time localization is significant while calculating the measure of complexity. The less time localization and time step, the more corresponding changes are taken into account. For a much larger time window and its step, we can have less accurate estimations.

It turned out that most of the chosen measures of complexity respond in advance to the corresponding changes of complexity in the crude oil market and can be used in the diagnostic processes. Such measures can be presented as indicators or even indicators-precursors of the approaching shock or crash.

Relying on the information theory and its powerful toolkit, we emphasized four measures of complexity, such as the measure of Lempel-Ziv, classical Shannon entropy, and its three modifications (Approximate entropy, Permutation entropy and the entropy of diagonal line histogram, which methodology is based on recurrence quantification analysis). We referred to the complexity of the systems, how it was described in different studies, and what methods were applied to quantify its degree. Our results show that in the pre-shock or pre-crash period, the complexity of oil price starts to change, i.e., it starts to decrease, indicating that such events presented to be more predictable and corresponding patterns are more structured. Thus, the degree of predictability increases in times of such events.

Along with information theory, we referred to the multifractal properties of the crude oil market. As it was obtained with multifractal detrended fluctuation analysis, the scaling exponents remain non-linear, and the width of singularity spectrum changes in time that gives evidence that at different times (scales) oil time series exhibits more or less complex behavior, indicating that crude oil market exhibits multifractal properties. Applying the width of multifractality as an indicator of possible critical states we found that before the abnormal event, this measure starts to decrease that tell us that the series is expected to be more predictable and stable, while its dynamics after such events is increasing that present system to be more susceptible to fluctuations.

We considered that chaotic events can be related to the fat-tails and better described with non-Gaussian distributions, particularly, described by Lévy alpha-stable distribution and its four parameters. As it is still debatable whether the stable distribution is completely applicable or not, we addressed its group of stable parameters, and during tests, we emphasized that the characteristic parameters α and β are the best for serving as an indicator-precursor of possible shocks and crashes. Thus, is shown that such a complex system as the crude oil market, with growth and preferential attachments, is characterized by power-laws.

The analysis of the crude oil market with the measures from the recurrence quantification analysis revealed that its toolkit is suitable for distinguishing diverse market periods. Such measures as recurrence rate and determinism are presented to be great for detection of the periods of instability or relaxation.

Moreover, we have demonstrated the possibility of studying the complex energy market within the network paradigm. The time series can be presented as an economic network (visibility graph and recurrence diagram) with a set of both spectral and topological characteristics, which are sensitive to the critical changes in the oil market.

Addressing to quantum econophysics and its apparatus where appropriate measures of complexity were obtained. Such quantitative methods as the Heisenberg uncertainty approach have confirmed their effectiveness for studying the oil market. We found that economic “mass” is presented to be effective due to its robustness, computational efficiency, and simplicity.

Apparently, the impact of the different shocks and crashes was reflected in the crude oil market, as well as the coronavirus pandemic and therefore, the dynamics of past events, as well as of the subsequent could be identified in advance using the appropriate indicators of the theory of complexity. In our further studies, we are going to aim our view on exploring and analyzing other methods from the econophysics as the random matrix theory or the theory of chaos. Moreover, the research in the field of artificial intelligence, machine, and deep learning does not remain without attention.

References

[1] Albert R and Barabasi A-L 2002 Statistical Mechanics of Complex Networks *Reviews of Modern Physics* **74** 47–97

[2] Aloui C, Hamdi M, Mensi W and Nguyen D 2012 Further evidence on the time-varying efficiency of crude oil markets *Energy Stud. Rev.* **19** 39-51

[3] Amigo J M 2010 Permutation Complexity in Dynamical Systems. Ordinal Patterns, Permutation Entropy and All That (Berlin: Springer-Verlag) p 280

[4] Arnold V I and Avez A 1968 *Ergodic problems of classical mechanics* (N.-Y./Amsterdam: The Mathematical Physics Monograph Series)

[5] Aslam F; Mohti W and Ferreira P 2020 Evidence of Intraday Multifractality in European Stock Markets during the Recent Coronavirus (COVID-19) Outbreak. *Int. J. Financial Stud.* **8** 31

[6] Ausloos M, Grech D, Matteo T Di, Kutner R, Schinckus C and Stanley H E 2020 *Econophysics and sociophysics in turbulent world* (*Phys.* A vol 531) ed M Ausloos et al (Amsterdam: Elsevier)

[7] Bachelier L 1900 Théorie de la Spéculation *Ann. Sci. Ecole Norm* **3** 21-86

[8] Bandt C 2020 Order patterns, their variation and change points in financial time series and Brownian motion *Stat Papers* **61** 1565-1588

[9] Bandt C and Pompe B 2002 Permutation Entropy: A natural complexity measure for time series *Phys. Rev. Lett.* **88** 174102

[10] Bariviera A F, Zunino L and Rosso O 2016 Crude oil market and geopolitical events: an analysis based on information-theory-based quantifiers *Fuzzy Economic Rev.* **21** 41-51

[11] Barkoulas J T, Chakraborty A and Ouandlous A 2012 A metric and topological analysis of determinism in the crude oil spot market *Energy Economics* **34** 584–91

[12] Barrat A, Barthelemy M and Vespignani A 2008 *Dynamical processes on complex networks* (UK: Cambridge University Press)

[13] Barunik J, Vacha L and Vosvrda M 2010 Tail Behavior of the Central European Stock Markets during the Financial Crisis *Czech Economic Rev.* **4** 281-294

[14] Bautista R Sand Mora J A 2020 Value at risk in the oil sector: an analysis of the efficiency in the measurement of the risk of the α-stable distribution versus the generalized asymmetric Student-t and normal distributions *Contaduría y Administración* **65** 19

[15] Bell S and Morse S 2018 Sustainability Indicators Past and Present:What Next? *Sustainability* **10**

[16] Belov I, Kabašinskas A and Sakalauskas L 2006 A Study of Stable Models of Stock Markets *Information Technology and Control* **35** ISSN 1392 – 124X

[17] Berta M, Christandl M, Colbeck R, Renes J and Renner R 2010 The uncertainty principle in the presence of quantum memory *Nature Phys.* **6** 659-662
[18] Bhaduri S N 2014 Applying approximate entropy (ApEn) to speculative bubble in the stock market *J. Emerging Market Finance* **13** 43–68
[19] Bianconi G 2015 Interdisciplinary and physics challenges in network theory *Europhysics Let.* **111** 56001
[20] Bielinskyi A, Semerikov S, Serdiuk O, Solovieva V, Soloviev V and Pichl L 2020 Econophysics of sustainability indices *Machine Learning for Prediction of Emergent Economy Dynamics, Proceedings of the Selected Papers of the Special Edition of International Conference on Monitoring, Modeling & Management of Emergent Economy (M3E2-MLPEED 2020) (Ukraine: Odessa)* vol 2713 (Germany: CEUR Workshop Proceedings) In press
[21] Bielinskyi A, Semerikov S, Solovieva V and Soloviev V Levy's stable distribution for stock crash detecting *The 8th Int. Conf. on Monitoring, Modeling & Management of Emergent Economy (M3E2 2019) (SHS Web of Conf.* vol 65*)* ed A Bielinskyi (EDP Sciences, 2019)
[22] Bielinskyi A, Soloviev V, Semerikov S and Viktoria Solovieva 2019 Detecting Stock Crashes Using Levy Distribution *Experimental Economics and Machine Learning for Prediction of Emergent Economy Dynamics: Proc. of the Selected Papers of the 8th Int. Conf. on Monitoring, Modeling & Management of Emergent Economy (M3E2 2019) (Ukraine: Odessa)* vol 2422 (Germany: CEUR Workshop Proceedings) pp 420-433
[23] Bielinskyi A, Semerikov S, Serdyuk O and Soloviev V 2020 The Econophysics of Cryptocurrency Crashes: An Overview *The Int. Conf. on History, Theory and Methodology of Learning (ICHTML 2020) (Urkaine: Kryvyi Rih)* (SHS Web of Conferences) In press
[24] Blanc J L, Pezard L and Lesne A 2011 Delay independence of mutual-information rate of two symbolic sequences *Phys. Rev.* E **84** 036214
[25] Boccaletti S et al. 2014 The structure and dynamics of multilayer networks *Phys. Rep.* **544** 122
[26] Boccaletti S, Latora V, Moreno Y, Chavez M and Hwang D-U 2006 Complex networks: Structure and dynamics *Phys. Rep.* **424** 175–308
[27] Boltzmann L 1872 Weitere studien uber das wärmegleichgewicht unter gasmolekülen *Sitzungsberichte Akademie der Wissenschaften* **66** 275-370
[28] Bonchev D 2009 Information Theoretic Complexity Measures (*Encyclopedia of Complexity and Systems Science* vol 1) ed R A Meyers (New York: Springer) chapter 6 pp 4820-4838
[29] Brorsen B W and Yang S R 1990 Maximum likelihood estimates of symmetric stable distribution parameters *Communications in Statistics-Simulation and Computation* **19** 1459-1464
[30] Caraiani P and Haven E 2015 Evidence of multifractality from CEE exchange rates against Euro*Phys.* A **419** 395–407
[31] Chakraborty A, Easwaran S and Sinha S 2016 An "inverse square law" for the currency market: Uncovering hidden universality in heterogeneous complex systems. Available at arXiv:1606.06111[q-fin.ST]
[32] ChenS, Carlini M, Krasnogorskaya N andScalia M 2013 Mathematical Problems in Sustainable Energy and Environment *Mathematical Problems in Engineering* **2013**
[33] Cirstea S D, Moldovan-Teselios C, Cirstea A, Turcu A C and Darab C P 2018 Evaluating Renewable Energy Sustainability by Composite Index *Sustainability* **10**
[34] Clausius R and Hirst T 1867 *The Mechanical Theory of Heat: With Its Applications to the Steam-engine and to the Physical Properties of Bodies* (London: John van Voorst) p 376
[35] Cohen R and Havlin S. 2010 *Complex networks. Structure, robustness and function* (New York, NY: Cambridge University Press)
[36] Colangelo G, Clurana F M, Blanchet L C, Sewell R J and Mitchell M W 2017 Simultaneous tracking of spin angle and amplitude beyond classical limits *Nature* **543** 525-528
[37] Corso G, Prado T, Lima G and Lopes S 2017 A novel entropy recurrence quantification analysis. Available at arXiv:1707.00944v1 [stat.OT]

[38] Costa M, Chung-Kang Peng C-K and Goldberger A 2008 Multiscale Analysis of Heart Rate Dynamics: Entropy and Time Irreversibility *Measures Cardiovascular Engineering* **8** 88-93
[39] Costa N, Silva C and Ferreira P 2019 Long-Range Behaviour and Correlation in DFA and DCCA Analysis of Cryptocurrencies. *Int. J. Financ. Stud*. **7** 51
[40] Da Silva S 2015 Financial market efficiency should be gauged in relative rather than absolute terms *J. Stock Forex Trad.* **4** 140
[41] Da Silva S, Matsushita R and Gigio R 2008 The relative efficiency of stockmarkets *Economics Bulletin* **7** 12
[42] Da Silva S, Taufemback C and Giglio R 2011 Algorithmic complexity theory detects decreases in the relative efficiency of stock markets in the aftermath of the 2008 financial crisis *Economics Bulletin* **31** 1631–47
[43] Da Z, Engelberg J and Gao P 2014 The sum of all FEARS investor sentiment and asset prices *Rev. Financ. Stud.* **28**
[44] Danilchuk H and Soloviev V 2015 Dynamics of graph spectral entropy in financial crisis Socio-economic aspects of economics and management **2** 227–234
[45] Database of Spot Prices (Crude Oil in Dollars per Barrel, Products in Dollars per Gallon). Available at https://www.eia.gov/dnav/pet/pet_pri_spt_s1_d.htm
[46] Delbianco F., Tohmé F, Stosic T and Stosic B 2016 Multifractal behavior of commodity markets: Fuel versus non-fuel products *Phys.* A **457** 573– 580.
[47] Delgado-Bonal A 2019 Quantifying the randomness of the stock markets *Sci Rep* **9** 12761
[48] Delgado-Bonal A and Marshak A 2019 Approximate Entropy and Sample Entropy: A Comprehensive Tutorial *Entropy* **21** 541
[49] Derbentsev V, Semerikov S, Serdyuk O, Solovieva V and Soloviev V 2020 Recurrence based entropies for sustainability indices *The Int. Conf. on Sustainable Futures: Environmental, Technological, Social and Economic Matters (ICSF 2020)* (*E3S Web of Conferences* vol 166) (EDP Sciences, 2020) p 7
[50] Donner R V, Small M, Donges J. F, Marwan N, Zou Y, Xiang R and Kurths J 2010 Recurrence-based time series analysis by means of complex network methods *Int. J. Bifurcation and Chaos* **21** 1019–46
[51] Duan W-Q and Stanley H 2010 Volatility, irregularity, and predictable degree of accumulative return series *Phys. Rev.* E **81** 066116
[52] Eckmann J P, Kamphorst S O and Ruelle D 1987 Recurrence plots of dynamical systems *Europhys. Lett.* **5** 973–977
[53] Eckmann J-P and Ruelle D 1985 Ergodic theory of chaos and strange attractors *Rev. Mod. Phys.* **57** 617-656
[54] Eom C, Oh G and Jung W S 2008 Relationship between efficiency and predictability in stock price change *Phys.* A **387** 5511–17
[55] Estevez-Rams E, Lora Serrano R, Aragon Fernandez B and Brito Reyes I 2013 On the non-randomness of maximum lempel ziv complexity sequences of finite size *Chaos* **23** 023118
[56] Fama E F 1965 The behavior of stock market prices *J. Business* **38** 34-105
[57] Fama E F and Roll R 1971 Parameter estimates for symmetric stable distributions *J. Am. Stat. Assoc*. **66** 331-38
[58] Faure P and Korn H 1998 A new method to estimate the Kolmogorov entropy from recurrence plots: its application to neuronal signals *Phys.* D **122** 265–279
[59] Ferreira P and L C Loures 2020 An Econophysics Study of the S&P Global Clean Energy Index *Sustainability* **12** 9
[60] Fukunaga T and Umeno K 2017 Universal Lévy's stable law of stock market and its characterization. Available at: arXiv:1709.06279 [q-fin.ST]
[61] Gabaix X 2009 Power laws in economics and finance *Annual Rev. of Economics* **l** 255–294
[62] Gabaix X, Gopikrishnan P, Plerou V and Stanley H E 2003 A theory of power law distributions in financial market fluctuations *Nature* **423** 267-270

[63] Gajardo G, Kristjanpoller W D and Minutolo M 2018 Does bitcoin exhibit the same asymmetric multifractal cross-correlations with crude oil, gold and djia as the euro, great british pound and yen? *Chaos, Solitons and Fractals* **109** 195–205
[64] Ganchuk A, Derbentsev V and Soloviev V N 2006 Multifractal Properties of the Ukraine Stock Market. Available at arXiv:physics/0608009 [physics.data-an]
[65] Gang-Jin W, Chi X, Shou C and Feng H 2014 Cross-Correlations between Energy and Emissions Markets: New Evidence from Fractal and Multifractal Analysis *Mathematical Problems in Engineering* **2014** 13
[66] Gao J and Cai H 2000 On the structures and quantification of recurrence plots *Phys. Lett.* A **270** 75-87
[67] Gibbs J 1914 *Elementary principles in statistical mechanics: developed with especial reference to the rational foundation of thermodynamics* (Yale University Press)
[68] Giglio G and Da Silva S 2009 Ranking the stocks listed on Bovespa according to their relative efficiency *University Library of Munich, Germany, MPRA Paper* **3** 2133–42
[69] Giglio R, Matsushita R, Figueiredo A, Gleria I and Da Silva S 2008 Algorithmic complexity theory and the relative efficiency of financial markets *Europhys. Lett.* **84** 48005
[70] Gnedenko B V and Kolmogorov A N 1954 *Limit Distributions for Sums of Independent Random Variables* (Addison-Wesley)
[71] Gopikrishnan P, Meyer M, Amaral L A N and Stanley H E 1998 Inverse cubic law for the probability distribution of stock price variations *The Eur. Phys. J.* B **3** 139–40
[72] Gopikrishnan P, Plerou V, Amaral L A N, Meyer M and Stanley H E 1999 Scaling of the distribution of fluctuations of financial market indices *Phys. Rev.* E **60** 5305–5316
[73] Gu R 2017 Multiscale Shannon Entropy and its application in the stock market *Phys.* A **484** 215-24
[74] Gu R, Chen H and Wang Y 2010 Multifractal analysis on international crude oil markets based on the multifractal detrended fluctuation analysis *Phys.* A **389** 2805–15
[75] Gunay S and Khaki A R 2018 Best Fitting Fat Tail Distribution for the Volatilities of Energy Futures: Gev, Gat and Stable Distributions in GARCH and APARCH Models *J. Risk Financial Manag.* **11** 30
[76] Halvin S, and Cohen R 2010 *Complex networks. Structure, robustness and function* (New York: Cambridge University Press)
[77] Hasan R and Mohammad S M 2015 Multifractal analysis of asian markets during 2007-2008 financial crisis *Phys.* A **419** 746-761
[78] Hidalgo E G Quantum Econophysics. arXiv: physics/0609245v1 [physics.soc-ph]
[79] Hongli N, Jun W and Cheng L 2018 Analysis of crude oil markets with improved multiscale weighted permutation entropy *Phys.* A **494** 389–402
[80] Hua X, Minggang W and Weiguo Y 2020 Information Linkage between Carbon and Energy Markets: Multiplex Recurrence Network Approach *Complexity* **2020** 12
[81] Huan C, Lixin T, Minggang W and Zaili Z 2017 Analysis of the Dynamic Evolutionary Behavior of American Heating Oil Spot and Futures Price Fluctuation Networks *Sustainability* **9** 574
[82] Hurst H E 1951 Long term storage capacity of reservoirs *ASCE Transactions* **116** 770-808
[83] Hurst H E 1957 A suggested statistical model of some time series which occur in Nature *Nature* **180** 494
[84] Iacovacci J and Lacasa L 2016 Sequential motif profile of natural visibility graphs *Phys. Rev.* E **94** 052309
[85] International Atomic Energy Agency 2005 *Energy Indicators for Sustainable Development: Guidelines and Methodologies* (Vienna: IAEA) p 171
[86] Jian W, Wei S and Junseok K 2020 Analysis of the impact of COVID-19 on the correlations between crude oil and agricultural futures *Chaos Solutions Fractals* **136**
[87] Jiang Z-Q and Zhou W-X 2011 Multifractal detrending moving-average cross-correlation

analysis *Phys. Rev.* E **84** 016106
[88] Jiang Z-Q, Xie W-J and Zhou W-X 2014 Testing the weak-form efficiency of the WTI crude oil futures market *Phys.* A **405** 235-244
[89] Joshua S, Richman J and Moorman R 2000 Physiological time-series analysis using approximate entropy and sample entropy *Am. J Physiol. Heart Circ. Physiol* **278** 2039-2049
[90] Kantelhardt J W, Zschiegner S A, Koscielny-Bunde E, Havlin S, Bunde A and Stanley H E 2002 Multifractal detrended fluctuation analysis of nonstationary time series *Phys.* A **316** 87-114
[91] Kantz H and Schreiber T 2003 *Nonlinear Time Series Analysis (2nd ed.)*. (UK: Cambridge University Press)
[92] Kanwal M S, Grochow J A and Ay N 2017 Comparing information-theoretic measures of complexity in Boltzmann machines *Entropy* **19** 310
[93] Kapica J 2012 Entropy analysis of energy price movement *Teka. Commission of motorization and energetics in agriculture* **12** 101–104
[94] Kateregga M, Mataramvura Sand Taylor D | Zhang X (Reviewing Editor) 2017 Parameter estimation for stable distributions with application to commodity futures log-returns *Cogent Economics & Finance* **5** 13118813
[95] Kolmogorov A N 1965 Three approaches to the quantitative definition of information *Int. J. Computer Mathematics* **2** 157-168
[96] Kostanjcar Z and Jeren B 2013 Emergence of Power-Law and Two-Phase Behavior in Financial Market Fluctuations *Advances in Complex Systems* **16** 1350008.
[97] Koutrouvelis I A 1980 Regression-type estimation of the parameters of stable laws *J. Amer. Statist. Assoc.* **75** 918-28
[98] Koutrouvelis I A 1981 An iterative procedure for the estimation of the parameters of stable laws *Commun. Statist.- Simula.* **10** 17-28
[99] Krężołek D 2015 The application of alpha-stable distributions in portfolio selection problem - the case of metal market *Studia Ekonomiczne* **247** 56-68
[100] Kristoufek L 2011 Multifractal height cross-correlation analysis: A new method for analyzing long-range cross-correlations *Europhys. Lett.* **95** 68001
[101] Kristoufek L and Vosvrda M 2013 Commodity futures and market efficiency. Available at arXiv:1309.1492v1 [q-fin.ST]
[102] Kumar S and Deo N 2009 Multifractal properties of the Indian financial market *Phys.* A **388** 1593-1602
[103] Kuruoğlu E E 2001 Density parameter estimation of skewed α-stable distributions *IEEE T.Signal Proces*. **49** 2192
[104] Kwapień J et al. 2005 Components of multifractality in high-frequency stock returns *Phys.* A **350** 466-474
[105] Lacasa L, Luque B, Ballesteros F, Luque J and Nuño J C 2008 From time series to complex networks: the visibility graph *PNAS* **105** 4972–4975
[106] Lahmiri S 2017 Multifractal analysis of moroccan family business stock returns *Phys.* A **486** 183-191
[107] Lahmiri S and Bekiros S 2020 The impact of covid-19 pandemic upon stability and sequential irregularity of equity and cryptocurrency markets *Chaos, Solitons & Fractals* **138** 109936
[108] Landau L D and Lifshitis E M 1975 *The classical theory of fields. Course of theoretical physics* (England: Butterworth-Heinemann/Oxford)
[109] Lee J W, Lee K E and Rikvold P A 2006 Multifractal behavior of the Korean stock-market index KOSPI *Phys.* A **364** 355-361
[110] Lempel A and Ziv J 1976 On the complexity of finite sequences *IEEE Transactions on Information Theory* **22** 75–81
[111] Levi P 1924 Théorie des erreurs. La loi de Gauss et les lois exceptionnelles *Bulletin de la Société Mathématique de France* **52** 49-85

[112] Li J, Lu X and Zhou Y 2016 Cross-correlations between crude oil and exchange markets for selected oil rich economies *Phys.* A **453** 131-143
[113] Li Z and Lu X 2011 Multifractal analysis of china's agricultural commodity futures markets *Energy Procedia* **5** 1920–1926
[114] Lim G, Kim S, Lee H, Kim K and Lee D-I 2007 Multifractal detrended fluctuation analysis of derivative and spot markets *Phys.* A **386** 259–266
[115] Lo A W 1989 Long-term memory in stock market prices*Econometrica* **59** 1279-1313
[116] Lovász L. 1996 *Information and Complexity (How To Measure Them?) (The Emergence of Complexity in Mathematics, Physics, Chemistry and Biology)* ed B Pullman (Pontifical Academy of Science/Vatican City/Princeton University Press) pp 65-80
[117] Lu X, Li J, Zhou Y and Qian Y 2017 Cross-correlations between RMB exchange rate and international commodity markets *Phys.* A **486** 168-182
[118] Lux T and Sornette D 2002 On rational bubbles and fat tails *J. Money Credit Bank* **34** 589–610
[119] Ma F, Wei Y and Huang D 2013 Multifractal detrended cross-correlation analysis between the chinese stock market and surrounding stock markets *Phys.* A **392** 1659-1670
[120] Ma F, Wei Y, Huang D and Zhao L 2013 Cross-correlations between west texas intermediate crude oil and the stock markets of the bric *Phys.* A **392** 5356-5368
[121] Ma F, Zhang Q, Peng C and Wei Y 2014 Multifractal detrended cross-correlation analysis of the oil-dependent economies: evidence from the west texas intermediate crude oil and the gcc stock markets *Phys.* A **410** 154-166
[122] Ma X Y and Nikias C L 1995 Parameter estimation and blind channel identification in impulsive signal environments *IEEE T. Signal Proces* **43** 2884-97
[123] Mahmoud I, Naoui K and Jemmali H 2013 Study of speculative bubbles: The contribution of approximate entropy.*Int. J. of Econ. and Financial* **3** 683–693
[124] Malevergne Y, Pisarenko V and Sornette D 2005 Empirical distributions of stock returns: between the stretched exponential and the power law? *Quant. Financ.* **5** 379–401
[125] Malevergne Y, Pisarenko V and Sornette D 2011 Testing the Pareto against the lognormal distributions with the uniformly most powerful unbiased test applied to the distribution of cities *Phys. Rev.* E **83** 036111
[126] Mali P and Mukhopadhyay A 2014 Multifractal characterization of gold market: a multifractal detrended fluctuation analysis *Phys.* A **413** 361–372
[127] Malinetskiy G G 2013 Teoriya samoorganizatsii. na poroge iv paradigmy *Computer research and modeling* **5** 315–366 [in Russian]
[128] Mandelbrot B B 1960 The pareto-lévy law and the distribution of income *Int. Econ. Rev.* **1** 79–106
[129] Mandelbrot B B 1963 The Variation of certain speculative prices *The Journal of Business* **36** 394-419
[130] Mandelbrot B B 1983 *The fractal geometry of nature* (New York: Freeman)
[131] Mantegna R N and Stanley H E 2000 *An Introduction to Econophysics: Correlations and Complexity in Finance* (Cambridge: Cambridge University Press)
[132] Marchuk G I, Aloyan A E and Kondratyev K Ya 2015 Mathematical Models and Simulation in Environment *Mathematical models of life support systems* **1** 10
[133] Marsh P 2012 The New Industrial Revolution: Consumers, Globalization and the End of Mass Production (UK: London/Yale University Press)
[134] Martina E, Rodriguez E, Escarela-Perez R and Alvarez-Ramirez J 2011 Multiscale entropy analysis of crude oil price dynamics*Energy Economics* **33** 936-947
[135] Marwan N, Romano M C, Theil M and Kurths J 2007 Recurrence plot for the analysis of complex systems *Phys. Rep.* **438** 237–329
[136] Marwan N, Wessel N, Meyerfeldt U, Schirdewan A and Kurths J 2002 Recurrence plot based measures of complexity and its application to heart rate variability data. *Phys. Rev.* E **66** 026702

[137] Maslov V P 2002 Econophysics and quantum statistics *Mathematical Notes* **72** 811-818
[138] Mastroeni L, Vellucci P and Naldi M 2019 A reappraisal of the chaotic paradigm for energy commodity prices *Energy Economics* **82** 167-178
[139] Matia K, Ashkenazy Y and Stanley H E 2003 Multifractal properties of price fluctuations of stocks and commodities *Europhys. Lett.* **61** 422
[140] Matilla-García M 2007 Nonlinear dynamics in energy futures *The Energy J.* **28** 7–29
[141] McCulloch J H 1986 Simple consistent estimators of stable distribution parameters *Communication in Statistics - Computation and Simulation* **15** 1109-36
[142] McNally S, Roche J and Caton S 2018 Predicting the price of bitcoin using machine learning *Proc. of the 2018 26th Euromicro International Conference on Parallel, Distributed and Network-based Processing* (PDP) pp 339–343
[143] Mensi W, Tiwari A K and Yoon S-M 2017 Global financial crisis and weak-form efficiency of islamic sectoral stock markets: an mf-dfa analysis *Phys.* A **471** 135–146
[144] Mensi W, Aloui C, Hamdi M and Nguyen D 2012 Crude oil market efficiency: An empirical investigation via the Shannon entropy *Économie internationale* **129** 119-137
[145] Moshiri S and Foroutan F 2006 Forecasting nonlinear crude oil futures prices *The Energy J.* **27** 81–96
[146] Newman M 2003 The structure and function of complex networks *SIAM Rev.* **45** 167–256
[147] Newman M, Watts D and Barabási A-L 2006 *The Structure and Dynamics of Networks* (Princeton and Oxford: Princeton University Press)
[148] Nicolas J M 2002 Introduction aux statistique de deuxi'emeesp'ece: application des logs-moments et des logs-cumulants' al'analyse des lois d'images radar *Trait du Signal* **19** 139-167
[149] Nolan J P 2001 Maximum likelihood estimation of stable parameters (*Lévy Processes: Theory and applications),* ed O E Barndorff-Nielsen, S I Resnick and T Mikosch (Birkhäuser, Boston, MA) pp 379-400
[150] Nolan J P 2003 Modeling Financial Data with Stable Distributions *Handbooks in Finance* (*Handbook of Heavy Tailed Distributions in Finance* vol 1) ed T Svetlozar (North Holland: Elsevier) chapter 3 pp 105-130
[151] Norouzzadeh P and Rahmani B 2006 A multifractal detrended fluctuation description of Iranian rial–US dollar exchange rate *Phys.* A **367** 328–336
[152] Norouzzadeh P, Dullaert W and Rahmani B 2007 Anti-correlation and multifractal features of spain electricity spot market *Phys.* A **380** 333–342
[153] Oh G, Eom C, Havlin S, Jung W-S, Wang F, Stanley H and Kim S 2012 A multifractal analysis of asian foreign exchange markets *The European Phys. J.* B **85** 214
[154] Ott E, Sauer T and Yorke J A 1994 *Coping with chaos* (New York: Wiley Interscience)
[155] Peng C-K, Buldyrev S V, Havlin S, Simons M, Stanley H E and Goldberger A L 1994 Mosaic organization of dna nucleotides *Phys. Rev.* E **49** 1685-89
[156] Pincus S and Kalman R E 2004 Irregularity, volatility, risk, and financial market time series. *Proc. of the National Academy of Sciences* **101** 13709-714
[157] Pincus S M 1991 Approximate entropy as a measure of system complexity *Proc. Natl. Acad. Sci. USA* **88** 2297-2301
[158] Pincus S M and Goldberger A L 1994 Physiological time-series analysis: what does regularity quantify? *American Journal of Physiology-Heart and Circulatory Physiology* **266** H1643–H1656
[159] Podobnik B and Stanley H E 2008 Detrended cross-correlation analysis: a new method for analyzing two nonstationary time series *Phys. Rev. Let.* **100** 084102
[160] Podobnik B, Horvatic D, Petersen A M and Stanley H E 2009 Cross-correlations between volume change and price change *Proc. of the National Academy of Sciences of the United States of America* **106** 22079–084
[161] Podobnik B, Matia K, Chessa A, Ivanov P C, Lee Y and Stanley H E 2001 Time evolution of

stochastic processes with correlations in the variance: stability in power-law tails of distributions *Phys.* A **300** 300-9
[162] Podobnik B, Valentincic A, Horvatic D and Stanley H E 2011 Asymmetric Lévy flight in financial ratios *PNAS* **108** 17883-88
[163] Poincaré H 1890 Sur le problème des trois corps et les équations de la dynamique. *Acta mathematica* **13** 270
[164] Prevedel R, Hamel D R, Colbeck R, Fisher K and Resch K J 2011 Experimental investigation of the uncertainty principle in the presence of quantum memory *Nature Phys.***7** 757-761
[165] Pueyo S 2014 Ecological Econophysics for Degrowth *Sustainability* **6** 3431-83
[166] Qian M C, Jiang Z Q and Zhou W X 2010 Universal and nonuniversal allometric scaling behaviors in the visibility graphs of world stock market indices *Phys.* A **43** 335002
[167] Qian X-Y, Liu Y-M, Jiang Z-Q, Podobnik B, Zhou W-X and Stanley H E 2015 Detrended partial cross-correlation analysis of two nonstationary time series influenced by common external forces *Phys. Rev.* E **91** 062816
[168] Qin J, Lu X, Zhou Y and Qu L 2015 The effectiveness of china's rmb exchange rate reforms: an insight from multifractal detrended fluctuation analysis *Phys.* A **421** 443–454
[169] Razmjoo A A and Sumper A 2019 Investigating energy sustainability indicators for developing countries *Int. J. Sustainable Energy Planning and Management* **21** 59-76
[170] Razmjoo A A, Sumper A and Davarpanah A 2019 Development of sustainable energy indexes by the utilization of newindicators: A comparative study *Energy Rep*. **5** 375-383
[171] Rodriguez E B and Aguilar L M A 2018 Disturbance-disturbance uncertainty relation: the statistical distinguishability of quantum states determines disturbance *Scientific Reports* **8** 4010
[172] Rongbao G, Hongtao C and Yudong W 2010 Multifractal analysis on international crude oil markets based on the multifractal detrended fluctuation analysis *Phys.* A **389** 2805-15
[173] Rozema L A, Darabi A, Mahler D H, Hayat A, Soudagar Y and Steinberg A M 2012 Violation of heisenberg's measurement-disturbance relationship by weak measurements *Phys. Rev. Lett.* **109** 100404
[174] Saptsin V and Soloviev V 2009 Relativistic quantum econophysics - new paradigms in complex systems modelling. Available at: arXiv:0907.1142v1 [physics.soc-ph]
[175] Sattarhoff C and Gronwald M 2018 How to measure financial market efficiency? a multifractality-based quantitative approach with an application to the european carbon market *Discussion Paper in Economics* **18** ISSN0143-4543
[176] Shannon C E 1948 A mathematical theory of communication *The Bell System Technical J.* **27** 379-423
[177] Shao M and Nikias C L P 1993 Signal processing with fractional lower order moments: stable processes and their applications *Proc. of the IEEE* **81** 986-1010
[178] Shaobo H, Chunbiao L, Kehui S and Sajad J 2018 Multivariate Multiscale Complexity Analysis of Self-Reproducing Chaotic Systems *Entropy* **20** 556
[179] Shaohui Z and Tian Z 2019 Multifractal Detrended Cross-Correlation Analysis of Electricity and Carbon Markets in China *Mathematical Problems in Engineering* **2019** 13
[180] Shuen-De W, Chiu-Wen W, Shiou-Gwo L, Kung-Yen L and Chung-Kang Peng 2014 Analysis of complex time series using refined composite multiscale entropy*Phys. Lett.* A **378** 1369-74
[181] Siegenfild A F and Bar-Yam Y 2020 An Introduction to Complex Systems Science and Its Applications *Complexity* **2020** 16
[182] Sole R V and Valverde S 2004 Information theory of complex networks: on Evolution and Architectural constraints *Lecture Notes in Phys.* **650** 189-207
[183] Soloviev V and Belinskij A 2018 Complex systems theory and crashes of cryptocurrency market. *Information and Communication Technologies in Education, Research, and Industrial (Communications in Computer and Information Science 2019* vol 1007*)* ed V Ermolayev et al (Cham: Springer) pp 276-297

[184] Soloviev V and Belinskij A 2018 Methods of nonlinear dynamics and the construction of cryptocurrency crisis phenomena precursors *Proc. of the 14th Int. Conf. on ICT in Education, Research and Industrial Applications. Integration, Harmonization and Knowledge Transfer. Volume II: Workshops* (*Ukraine: Kyiv*) vol 2104 (Germany: CEUR Workshop Proceedings) pp 116–127
[185] Soloviev V and Saptsin V 2011 Heisenberg uncertainty principle and economic analogues of basic physical quantities. Available at arXiv:1111.5289v1 [physics.gen-ph]
[186] Soloviev V and Serdiuk O 2020 Quantum econophysical precursors of cryptocurrency crashes *Cherkasy University Bulletin: Applied Mathematics. Informatics* **1** 3-16
[187] Soloviev V N and Romanenko Y V 2017 Economic analog of heisenberg uncertainty principle and financial crisis. *Proc. in 20th Int. Conf. SAIT 2017* (*Ukraine: Igor Sikorsky Kyiv Polytechnic Institute*) (ESC “IASA” NTUU) pp 32-33
[188] Soloviev V N, Bielinskyi A, Serdyuk O, Solovieva V and Semerikov S 2020 Lyapunov Exponents as Indicators of the Stock Market Crashes *Proc. of 16th Int. Conf. on ICT in Research, Education and Industrial Applications* (*ICTERI 2020*) (*Workshops 2020* vol 2) ed Zholtkevych G, Sokolov O et al. (Germany: CEUR Workshop Proceedings) p 16 In press
[189] Soloviev V N, Yevtushenko S P and Batareyev V V 2019 Comparative analysis of the cryptocurrency and the stock markets using the Random Matrix Theory *Computer Science & Software Engineering: Proc. of the 2nd Student Workshop(Ukraine: Krivyi Rih)* vol 2546 (Germany: CEUR Workshop Proceedings) pp 87-100
[190] Soloviev V, Belinskij A and Solovieva V 2019 Entropy analysis of crisis phenomena for DJIA index. *Proc. of the 15th International Conference on ICT in Education, Research and Industrial Applications. Integration, Harmonization and Knowledge Transfer. Vol. II: Workshops (Ukraine: Kherson)* vol 2393 (Germany: CEUR Workshop Proceedings) pp 434– 449
[191] Soloviev V, Semerikov S and Solovieva V 2020 Lempel-ziv complexity and crises of cryptocurrency market *III International Scientific Congress Society of Ambient Intelligence 2020 (Advances in Economics, Business and Management Research* vol 129) (Atlantis Press) pp 299-306
[192] Soloviev V, Serdiuk O, Semerikov S and Kiv A 2020 Recurrence plot-based analysis of financial-economic crashes *Machine Learning for Prediction of Emergent Economy Dynamics, Proceedings of the Selected Papers of the Special Edition of International Conference on Monitoring, Modeling & Management of Emergent Economy (M3E2-MLPEED 2020)* (Ukraine: Odessa) vol 2713 (Germany: CEUR Workshop Proceedings) In press
[193] Soloviev V, Serdiuk O, Semerikov S and Kohut-Ferens O 2019 Recurrence entropy and financial crashes *Proc. of the 2019 7th Int. Conf. on Modeling, Development and Strategic Management of Economic System (MDSMES 2019) (Advances in Economics, Business and Management Research* vol 99*)* (Atlantis Press) pp 385-88
[194] Soloviev V, Solovieva V and Tuliakova A 2019 Visibility graphs and precursors of stock crashes *Neuro-Fuzzy Technologies of Modeling in Economy* **8** 3–29
[195] Soloviev V, Solovieva V, Tuliakova A, Hostryk A and Pichl L 2020 Complex networks theory and precursors of financial crashes *Machine Learning for Prediction of Emergent Economy Dynamics, Proceedings of the Selected Papers of the Special Edition of International Conference on Monitoring, Modeling & Management of Emergent Economy (M3E2-MLPEED 2020) (Ukraine: Odessa)* vol 2713 (Germany: CEUR Workshop Proceedings) In press
[196] Soloviev V and Tuliakova A 2016 Graphodinamical Research Methods for Complexity of Modern Stock Markets *Neuro-Fuzzy Technologies of Modeling in Economy* **5** 152–179
[197] Sornette D 2003 *Why Stock Markets Crash* (USA: Princeton/Princeton University Press)
[198] Soyyiğit S, Topuz H and Halil Ö 2020 An Alternative View to the Global Coal Trade: Complex Network *Approach Studies in Business and Economics* **15** 270-288
[199] Suárez-García P and Gómez-Ullate D 2014 Multifractality and long memory of a financial index *Phys.* A **394** 226-34

[200] Suganthi L 2020 Sustainability indices for energy utilizationusing a multi-criteria decision model *Sustainability and Society* **10**
[201] Takens F 1981 Detecting strange attractors in turbulence *Dynamical Systems and Turbulence, Lecture Notes in Mathematics* **898** 366–81
[202] Taleb N N 2020 On the statistical differences between binary forecasts and real world payoffs. Available at arXiv:1907.11162 [q-fin.GN]
[203] TalebN N, Bar-Yam Y and Cirillo P 2020 On single point forecasts for fat-tailed variables. Available at arXiv:2007.16096 [physics.soc-ph]
[204] Tang Y, Kurths J, Lin W, Ott E and Kocarev L 2020 Introduction to Focus Issue: When machine learning meets complex systems: Networks, chaos, and nonlinear dynamics *Chaos* **30** 063151
[205] Thiel M, Romano M C and Kurths J 2003 Analytical description of recurrence plots of white noise and chaotic processes *Appl. Nonlinear Dyn.* **11** 20–30
[206] Thiel M, Romano M C, Kurths J, Meucci R, Allaria E and Arecchi F T 2002 Influence of observational noise on the recurrence quantification analysis *Phys.* D **171** 138–152
[207] Thurner S, Hanel R and Klimek P 2018 Introduction to the Theory of Complex Systems (UK: Oxford/Oxford Univ.press) p 448
[208] Ting L L et al. 2015 Complexity Measurement Based on Information Theory and Kolmogorov Complexity *Artificial Life* **21** 205-24
[209] Tiwari A, Albulescu C and Yoon S 2017 A multifractal detrended fluctuation analysis of financial market efficiency: Comparison using Dow Jones sector ETF indices *Phys.* A **483** 182–92
[210] Tsallis C 2009 *Introduction to Nonextensive Statistical Mechanics, Approaching a Complex World* (New-York: Springer)
[211] Tung-Li Shih and Hai-Chin Yu Probability Distribution of Return and Volatility in Crude Oil Market Available at http://www.jgbm.org/page/21%20Tung-Li%20Shih%20.pdf
[212] Umeno K 2016 Ergodic transformations on r preserving cauchy laws *Nonlinear Theory and Its Applications* **7** 14-20
[213] Wang D-H, Yu X-W and Suo Y-Y 2012 Statistical properties of the yuan exchange rate index *Phys.* A **391** 3503-12
[214] Wang J, Shang P and Ge W 2011 Multifractal cross-correlation analysis based on statistical moments *Fractals* **20** 271-9
[215] Wang Y, Wei Y and Wu C 2010 Cross-correlations between chinese a-share and b-share markets *Phys.* A **389** 5468-78
[216] Wang Y, Wei Y and Wu C 2011 Analysis of the efficiency and multifractality of gold markets based on multifractal detrended fluctuation analysis *Phys.* A **390** 817-27
[217] Wang Y, Wei Y and Wu C 2011 Detrended fluctuation analysis on spot and futures markets of west texas intermediate crude oil *Phys.* A **390** 864-75
[218] Wątorek M, Drożdż S, Oświęcimka P and Stanuszek M 2019 Multifractal cross-correlations between the world oil and other financial markets in 2012–2017 *Energy Economics* **81** 874-885
[219] Webber Jr C L and Zbilut J P 1994 Dynamical assessment of physiological systems and states using recurrence plot strategies *J. Appl. Physiol.* **76** 965-73
[220] Webber Jr C L and Zbilut J P 2005 Recurrence quantification analysis of nonlinear dynamical systems *Tutorials in Contemporary Nonlinear Methods for the Behavioral Sciences* 26-94
[221] Webber Jr C L, Ioana C and Marwan N 2016 Recurrence plots and their quantifications: expanding horizons 2016 *Proc. of the 6th Int. Symposium on Recurrence Plots (France: Grenoble)* vol 180 (Heidelberg: Springer) p 387
[222] Webber Jr C L, Ioana C and Marwan N 2016 Recurrence plots and their quantifications: expanding horizons 2016 *Proc. of the 6th Int. Symposium on Recurrence Plots (France: Grenoble)* vol. 180 (Heidelberg: Springer) p 387
[223] Wei-Shing C and Sheng-Yu C 2012 Application of Permutation Entropy and Statistical Complexity Measure in Crude Oil Price Time Series. Available at

http://cyc2012.dyu.edu.tw/pdf/D-5-利用排列熵與統計複雜量測於原油價格時間序列之研究(陳偉星).pdf
[224] World Energy Council 2020 World Energy Trilemma Index Available at https://www.worldenergy.org/assets/downloads/World_Energy_Trilemma_Index_2020_-_REPORT.pdf
[225] Wu J and T Wu 2012 *Sustainability indicators and indices: an Overview* (*Handbook of Sustainable Management*) ed C N Madu and C Kuei (London: Imperial College Press) chapter 4 pp 65-86
[226] Xie C, Zhou Y, Wang G and Yan X 2017 Analyzing the cross-correlation between onshore and offshore rmb exchange rates based on multifractal detrended cross-correlation analysis (MF-DCCA) *Fluctuation and Noise Letters* **16** 1750004
[227] Yali Z and Jun W 2017 Nonlinear complexity of random visibility graph and Lempel-Ziv on multitype range-intensity interacting financial dynamics *Phys.* A **482** 741-756
[228] Yi Y and Pengjian S 2014 Weighted multiscale permutation entropy of financial time series *Nonlinear Dynamics* **78** 2921-2939
[229] Yingchao Z, Lean Y and Kaijian H 2015 Wavelet Entropy Based Analysis and Forecasting of Crude Oil Price Dynamics *Entropy* **17** 7167-7184
[230] Yue P, Xu H-C, Chen W, Xiong X and Zhou W-X 2017 Linear and nonlinear correlations in the order aggressiveness of chinese stocks *Fractals* **25** 1750041
[231] Yun K et al. 2012 Decreased cortical complexity in methamphetamine abusers *Psychiatry Research: Neuroimaging* **201** 226–232
[232] Zanin M, Zunino L, Rosso O and Papo D 2012 Permutation entropy and its main biomedcal and econophysics applications: a review *Entropy* **14** 1553
[233] Zbilut J P and Webber C L Jr 1992 Embeddings and delays as derived from quantification of recurrence plots *Phys. Lett.* A **171** 199–203
[234] Zhang W, Wang P, Li X and Shen D 2018 Twitter's daily happiness sentiment and international stock returns: evidence from linear and nonlinear causality tests *J. Behavioral and Experimental Finance* **18** 50-3
[235] Zhang Z, Zhang Y, Shen D and Zhang W 2018 The cross-correlations between onlinesentiment proxies: evidence from google trends and twitter *Phys.* A **508** 67-75
[236] Zhang Z, Zhang Y, Shen D and Zhang W 2018 The dynamic cross-correlations between mass media news, new media news, and stock returns *Complexity* **2018**
[237] Zheng S and Lan X 2016 Multifractal analysis of spot rates in tanker markets and their comparisons with crude oil markets *Phys.* A **444** 547–59
[238] Zhi-Qiang J, Wen-Jie X and Wei-Xing Z 2012 Testing the weak-form efficiency of the WTI crude oil futures market. Available at arXiv:1211.4686v1 [q-fin.ST] 20 Nov 2012
[239] Zhou W-X 2008 Multifractal detrended cross-correlation analysis for two nonstationary signals *Phys. Rev.* E **77** 066211
[240] Zhuang X, Wei Y and Ma F 2015 Multifractality, efficiency analysis of chinese stock market and its cross-correlation with wti crude oil price *Phys.* A**430**(C) 101-13
[241] Zhuang X, Wei Y and Zhang B 2014 Multifractal detrended cross-correlation analysis of carbon and crude oil markets *Phys.* A **399** 113-125
[242] Zou Y, Donner R V, Marwan N, Donges J F and Kurths J 2019 Complex network approaches to nonlinear time series analysis *Phys. Rep.* **787** 97
[243] Zozor S, Ravier P and Buttelli O 2005 On lempel-ziv complexity for multidimensional data analysis *Phys.* A **345** 285–302
[244] Zunino L, Figliola A, Tabak B M, Pérez D G, Garavaglia M and Rosso O A 2009 Multifractal structure in latin-american market indices *Chaos, Solitons & Fractals* **41** 2331-2340