

# Modelling of Trade Relations between EU Countries by the Method of Minimum Spanning Trees

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**Abstract**—The paper deals with the study of international flows of goods and services, modeling of the structure of trade relations between countries. The rapid development of international economic relations and the formation of various integration associations encourage the identification of structural links and their reformatting due to the influence of destabilizing factors. The study is based on one of the methods of cluster analysis, namely the method of construction of minimum spanning trees. The article analyzes the dynamics of European Union volumes for the period from 2000 to 2018. The influence of such factors as the integration of new member countries and the world financial crises on the structure of trade relations is substantiated. Changes in the structure of relations between the EU countries due to the influence of these factors are modeled. It is noted that there is a reformatting of links, especially in times of crisis, minimal spanning trees have more orderly structure. This can be used as an indicator of crisis phenomena. In addition, it has been proven that the accession of new member states to the EU leads to changes in the structure of trade relations that can be both positive and negative. The study has an applied nature and can be used in the future as a methodological basis for the development of effective mechanisms for reformatting trade relations between countries in the context of geo-economic transformations and global financial crises.

**Keywords**—cluster analysis, minimum spanning tree, cluster, European Union, international trade, import, export

## I. INTRODUCTION

International trade is one of the oldest and most important forms of implementation of the international division of labor and world economic relations. At the same time, it is the traditional and most developed form of international economic relations.

Over the past two centuries, the volume of international trade has increased markedly. Today, about a quarter of the overall world production is exported [1]. This has significantly affected the geo-economic structure of the world economy: new centers of economic power have emerged, regional integration associations and coalitions have been formed.

The integration of national economies into the world economic system has become one of the most important events of the last century. The development and deepening of the international trade relations became the root cause in the formation of integration associations. At the same time, the processes of integration in modern conditions contribute to a significant increase in exports and imports of goods and services between countries.

This is especially true of regional country groupings. Among these, the European Union has a special role to play. It is within the framework of this integration association that trade relations have reached the highest stage of their development. At the same time, modern threats and challenges significantly affect the stability of such ties. New members, not always with a high level of the economic development, enter this union from time to time. Or, like the UK, leave it [2]. Global financial crises and other external and internal factors also destabilize the situation.

In these circumstances, the studies aimed at identifying structural changes in the reformatting of trade relations between countries and identifying the causes of such transformations are of particular importance. One of the modern methods of economic and mathematical modeling, which works effectively on such objects of research, is cluster analysis and its specific tools: graph theory and methods of construction of minimum spanning trees. This

article is devoted to the modeling of trade ties between the EU countries by the methods of spanning trees.

## II. ANALYSIS OF PREVIOUS RESEARCHES

The analysis of scientific works shows that researchers have a lot of developed methods of modeling trade processes among countries. Their objects are individual countries, integration associations, certain groups of goods and services, etc.

In [3] regression models were used to link trade flows and gross products of all BRICs countries. The authors have developed a theoretical model that allows us to obtain endogenous macroeconomic models taking into account bilateral trade between all BRIC countries and foreign trade with the rest of the world. The Dixit-Stiglitz model was used to study the effects of international trade [4].

The gravitational model is quite common in the studies of the dynamics and forecasting of the processes of international trade. So in [5] testing and comparison of the classical and the gravity models of trade the example of the UK is carried out, and the authors recognize the applicability of the classical model in relation to the UK.

The study of the impact on the international trade of seafood as a type of goods, such factors as geographical distance among the countries, trade agreements, gross product and other macroeconomic variables using the gravitational model are given in [6]. The extended Frankel gravity model was used by the authors [7] in the study of India's bilateral external trade relations with ASEAN countries. The authors [8], based on the gravitational model, investigate the trade flows between China and its major world partners.

Factor model of the international trade in the Euro area based on the identification of the interaction between macroeconomic and trade variables was proposed in [9] and the effectiveness of its use for short-term forecasts was proved.

Recently publications have appeared with the use of network theory for modeling the processes of the international trade. Thus, the authors [10] consider international trade as a multilevel network.

The authors [11] analyze the stock markets of the USA and Sweden using special cluster structures in the networks. The obtained results allow us to conclude about the indicative role of the cluster structure in the identification of crisis phenomena.

Hence, the above analysis of a sample of modern publications proves the relevance of the topic and the interest of scientists in the further development and application of new approaches to modeling trade relations between the countries within the framework of integration associations or in the world as a whole.

The aim of this work is to provide the results of modeling trade relations between the countries of the European Union by the method of minimum spanning trees and to analyze changes in the structure of these relations due to the influence of various socio-economic and political factors.

## III. METHOD AND MATERIALS

It is proposed to assess trade relations between countries and their change over time using the approach described in Rosario N. Mantegna [12]. This approach is based on the construction of a minimum spanning tree based on the graph obtained from the cross-correlation matrix and its analysis. It is essential in this method to study several time sequences simultaneously.

The pair correlation coefficient is used in work [12] as an indicator of quantitative assessment of the degree of similarity of the system elements.

$$c_{ij} = \frac{(y_i y_j) - (y_i)(y_j)}{\sqrt{((y_i^2) - (y_i)^2)((y_j^2) - (y_j)^2)}} \quad (1)$$

where  $i$  and  $j$  is the index of  $Y_i = \ln P_i(t) - \ln P_i(t-1)$  the  $P_i(t)$  variable, and is the value of the 1st variable at time  $t$ . The correlation coefficient  $c_{ij}$  is a matrix of size  $n \times n$ . It is known that the correlation coefficient can take values from -1 (completely uncorrelated pair) to 1 (fully correlated pair). The correlation coefficient matrix is a symmetric matrix with units on the main diagonal.

To understand and interpret the topological structure of the system under the study, we use a generalized metric defined by the formula

$$d(i, j) = \sqrt{2(1 - c_{ij})} \quad (2)$$

Under this definition  $d(i, j)$  numerically satisfies such axioms:

- $d(i, j) = 0$  if and only if  $i = j$ , that is, the axiom is satisfied under the condition of full correlation;
- $d(i, j) = d(j, i)$  - the second axiom is fulfilled because we have a matrix of cross-correlation coefficients and, respectively, a matrix of distances  $D$  is symmetric in values;
- $d(i, j) \leq d(i, k) + d(k, j)$ , which is numerically verified, the third axiom is satisfied.

The matrix  $D$  is used to determine the minimum spanning tree. To uniquely identify the taxonomy the subdominant ultrametric space will be considered that is the limiting topological space. In ultrametric space, the elements are placed in a hierarchical tree. The minimum spanning tree reflects the closest connections between the elements. The minimum spanning tree is represented graphically as a connected graph consisting of  $n$  vertices (nodes) and  $n - 1$  edges. The tree has the smallest length among all the trees, based on the sum of the distances between the two elements.

It is also shown in [12] that the minimum spanning tree reflects the hidden information in economic time series. The study used indicators of exports and imports of goods and services of the EU member States. The data cover the period from 2000 to 2018. In order to monitor the dynamics of structural changes in relations between the countries, this period is divided into the intervals. The first is 2000-2006 years, the EU included 25 countries. The second is 2007-

2012 years with the number of EU member States 27. The third is 2013-2018 with 28 countries as members of the Association. All data are obtained from the official statistics website [13].

IV. RESULT AND DISCUSSION

Within the framework of this approach, calculations were carried out in MatLab.

Fig. 1 calculation results for 25 EU member States are presented. We see the formation of one large and five small clusters. Poland is the top of a large cluster, which includes groups of countries with low (Luxembourg, Slovakia, Czech Republic) and medium (Italy) export volumes.

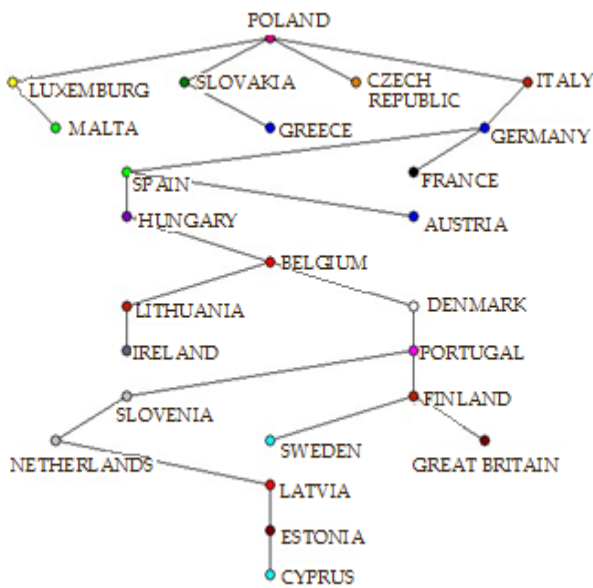


Fig. 1. Minimum spanning tree of dynamics of exports of goods of the EU countries for 2000-2006

In 2007, this group included Bulgaria and Romania which are the countries with less developed economies in relation to other EU member States. At the same time, in 2007-2008 the global financial crisis took place, which affected all economic processes in the world. As a result, reformatting of the minimum spanning tree and the formation of new clusters occurred (Fig. 2).

First of all, we distinguish two large clusters, the tops of which are Italy and Germany. The cluster with the top Italy included Germany, France, Great Britain, and they are characterized by high volumes of exports, and Sweden, which has a low level of volumes. The cluster with the top of Germany includes the countries with medium (Spain) and low (Estonia, Greece) volumes of exports of goods. Bulgaria and Romania were in a cluster with a low level of exports of goods.

Croatia joined the EU in 2013. For the period 2013-2018 (Fig. 3) we have a different structure of relations. Seven clusters with peaks Greece, Slovakia, Austria, Slovenia, Romania, Lithuania, Germany were formed. The countries, included in the clusters with the peaks of Greece, Austria, Germany and Romania, are characterized by an increase in

the volume of exports of goods. A cluster with the top of Slovenia has uneven dynamics of changes in the values of this indicator. The UK is a part of this cluster, which may be a reflection of the process of its exit from the EU.

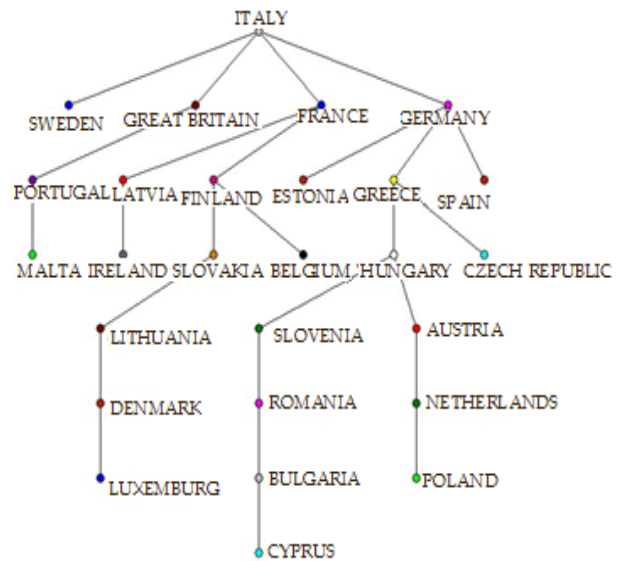


Fig. 2. Minimum spanning tree of dynamics of exports of goods of the EU countries for 2007-2012

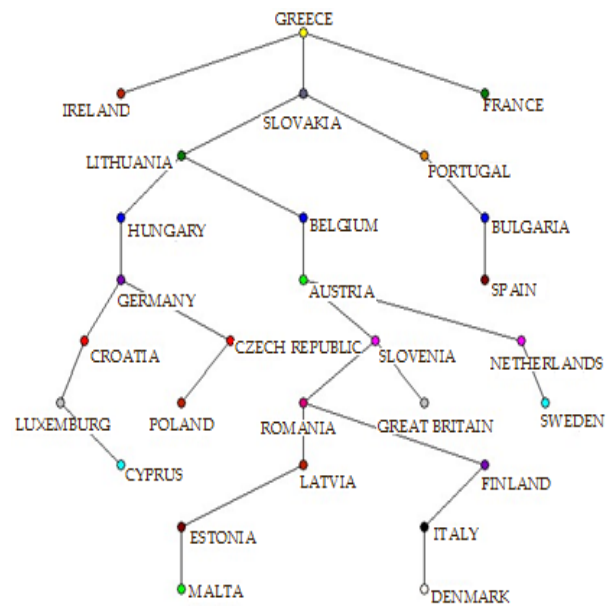


Fig. 3. Minimum spanning tree of dynamics of exports of goods of the EU countries for 2013-2018

Let us consider the results of calculations for the indicators of imports of goods. Also, the construction of minimum spanning trees occurred in similar periods. The period 2000-2006 is characterized by the formation of 6 clusters with peaks Latvia, Great Britain, Germany, Slovenia, Czech Republic, Hungary (Fig. 4).

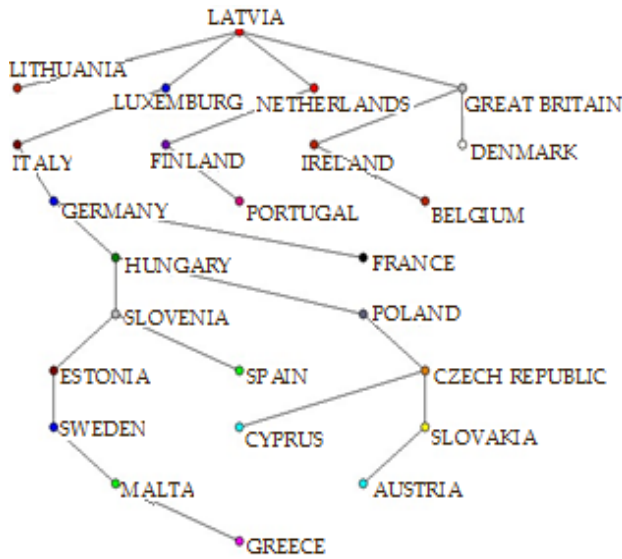


Fig. 4. Minimum spanning tree of dynamics of imports of goods of the EU countries for 2000-2006

In 2007-2012, the number of clusters remained, but the vertices changed (Fig. 5). In 2009 there was a sharp decrease in the volume of imports of goods in all countries, which may be a consequence of the crisis of 2007-2008. This is reflected in the structure of the minimum spanning tree. The clusters are small and the tree graph is quite simple.

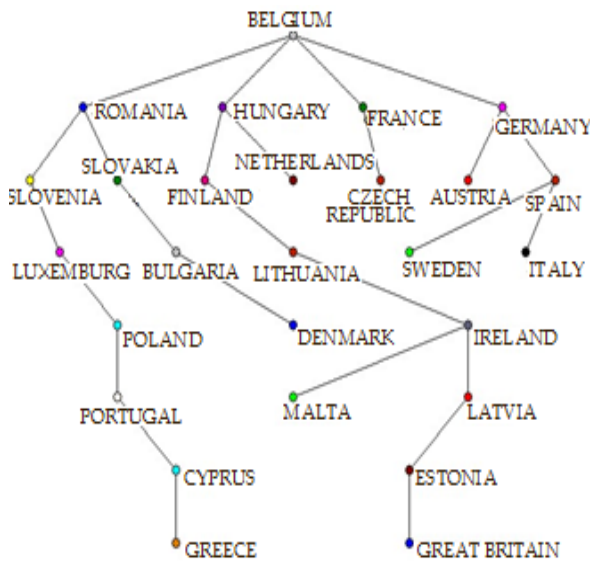


Fig. 5. Minimum spanning tree of dynamics of imports of goods of the EU countries for 2007-2012

The period 2013-2018 is characterized by the modern composition of the EU member States. At this stage (Fig. 6) we see the formation of two large clusters with peaks Slovenia and Ireland. The countries belonging to the cluster with the top of Ireland have an average level of imports of goods with the same dynamics of its change during the study period. The cluster with the top of Slovenia includes countries with low (Slovenia, Sweden), medium (Austria, Netherlands) and high (France) levels of imports of goods with a tendency to increase.

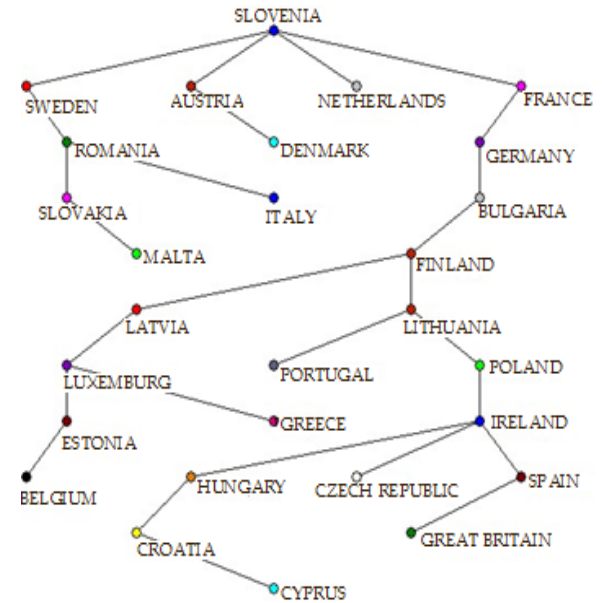


Fig. 6. Minimum spanning tree of dynamics of imports of goods of the EU countries for 2013-2018

To study the clustering of EU member States in terms of volumes of import-export of services, the period 2010-2017 was chosen.

Minimum spanning tree of service exports (Fig. 7) has two powerful clusters with peaks Germany and the Netherlands. These clusters include countries with low (Estonia, Malta, Romania, Slovakia, Lithuania), medium (Finland, Ireland, Austria, Spain) and high (France, Netherlands, Germany) levels of services exports. The cluster with the top of the Netherlands includes countries with low (Slovenia, Poland) and medium (Sweden, Italy) levels. The countries of these two clusters are characterized by an increase in the volume of services exports.

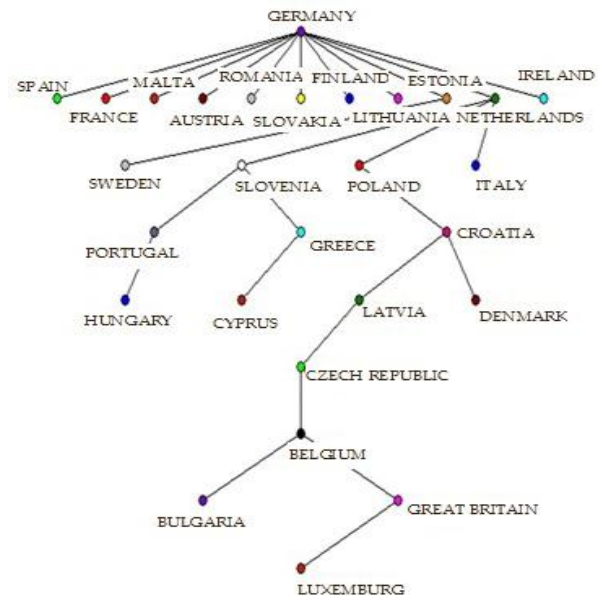


Fig. 7. Minimum spanning tree of dynamics of exports of services of the EU countries for 2010-2017



Germany is also the top of the largest cluster in the minimum spanning tree (Fig. 8). This cluster includes the countries with different levels of imports of services. The second largest is a cluster with the top of Poland. This cluster is formed by countries with low levels of imports of services. However, a common characteristic of the cluster countries is the gradual increase in imports of services.

Thus, the application of the method of minimum spanning trees at successive intervals of historical time series with indicators of exports and imports of goods and services of the EU allows us to track the dynamics of changes in the structure of relations between countries that occurred during this period. At the same time, graphical visualization helps to make a quick analysis of the formed clusters, to identify the countries belonging to a particular cluster, that is, to determine their homogeneity.

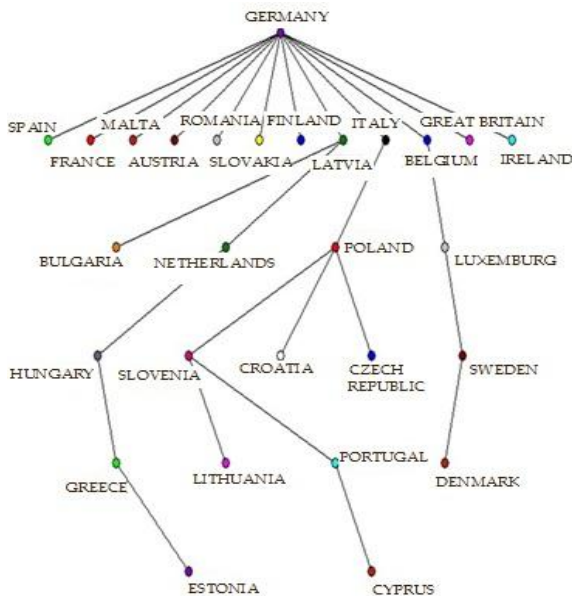


Fig. 8. Minimum spanning tree of dynamics of imports of services of the EU countries for 2010-2017

### V. CONCLUSIONS

Consequently, the construction of minimum spanning trees allowed us to study the structural and dynamic changes in trade relations of the EU member States. It is proved that the entry into the EU of new countries and the world financial crises make certain transformations in the tree graphs. As a result, there is a reformatting and formation of new relations between the countries. It is obvious that as a result of the entry of the new country into the integration association, the volume of trade operations will change. Joining the Union of a country with a developed economy will create a powerful cluster of countries with similar

characteristics which will lead to a change in the structure of relations in general in the Union and changes in the volume of trade operations. Due to the accession to the integration association of the country with a low level of economic development, the minimum tree will also change its configuration, but such a country will enter the cluster of countries with similar indicators. The formed cluster can become less powerful, it is weaker, there will be a decrease and redistribution of exports and imports of goods and services.

### REFERENCES

- [1] E. Ortiz-Ospina, D. Beltekian, and M. Roser, "Trade and Globalization," 2019. [Online]. Available: <https://ourworldindata.org/trade-and-globalization>. Accessed on: Jun 25, 2019.
- [2] L. O. Kybalyk and H. B. Danylchuk, "Analysis of the consequences of the UK exit from the EU through network-entropy methods," *Int. Relat., Ser. Econ. Sci.*, vol. 11, 2017. [Online]. Available: [http://journals.iir.kiev.ua/index.php/ec\\_n/article/view/3196](http://journals.iir.kiev.ua/index.php/ec_n/article/view/3196). Accessed on: June 25, 2019.
- [3] S. A. Makhov, "BRICS trade dynamic model," KIAM Preprint no. 128, Moscow, 2016. doi: <https://doi.org/10.20948/prepr-2016-128>.
- [4] N. A. Tabachkova, and V. V. Krachkovskiy, "Design of processes of international agglomeration," *Sci. Bul. Uzhhorod Univ., Ser. Int. Econ. Relat. World Econ.*, no. 6, part 3, 2016, pp. 74-77.
- [5] P. Minford, and Y. Xu, "Classical or Gravity? Which Trade Model Best Matches the UK Facts?," *Open Econ. Rev.*, 2018, 29: 579. doi: <https://doi.org/10.1007/s11079-017-9470-z>.
- [6] F. Natale, A. Borrello, and A. Motova, "Analysis of the determinants of international seafood trade using a gravity model," *Mar. Policy*, vol. 60, pp. 98-106, Oct. 2015. doi: <https://doi.org/10.1016/j.marpol.2015.05.016>.
- [7] S. L. Chakravarty, and R. Chakrabarty, "A Gravity Model Approach to Indo-ASEAN Trade-fluctuations and Swings," *Procedia-Social Behav. Sci.*, vol. 133, pp. 383-391, 15 May 2014. doi: <https://doi.org/10.1016/j.sbspro.2014.04.205>.
- [8] G. M. Caporalea, A. Sova, and R. Sova, "Trade flows and trade specialisation: The case of China," *China Econ. Rev.*, vol. 34, pp. 261-273, Jul. 2015. doi: <https://doi.org/10.1016/j.chieco.2015.03.010>.
- [9] M. Modugno, D. Antonello, and O. Chiara, "A Global Trade Model for the Euro Area," *Finance Econ. Discussion Ser.* 2015-013. Washington: Board of Governors of the Federal Reserve System, 2015. doi: <http://dx.doi.org/10.17016/FEDS.2015.013>.
- [10] B. Shen, J. Zhang, and Q. Zheng, "Exploring multi-layer flow network of international trade based on flow distances," arXiv:1504.02361v1 [physics.soc-ph] 9Apr 2015.
- [11] A. A. Kocheturov, M. V. Batsyn, and P. M. Pardalos, "Dynamics of Cluster Structures in Stock Market Networks," *J. NEA*, no. 4(28), 2015, pp. 12-30.
- [12] R. N. Mantegna, "Hierarchical Structure in Financial Markets," arXiv: cond-mat / 9802256v1 [cond-mat.stat-mech] 24Feb 1998.
- [13] European Statistical Office. [Online]. Available: <https://ec.europa.eu/eurostat/>. Accessed on: May 28, 2019