UDC 37.091.33:514:004.02

Ivan G. Lenchuk
PhD of Technical Sciences, Doctor of Pedagogical Sciences,
Professor, Professor at the Department of Algebra and Geometry
Zhytomyr Ivan Franko State University, Zhytomyr, Ukraine
ORCID ID 0000-0003-1923-9540
lench456@gmail.com

Oleksandr O. Mosiiuk
PhD in Pedagogical Sciences, Associate Professor,
Department of Computer Sciences and Information Technologies
Zhytomyr State Ivan Franko University, Zhytomyr, Ukraine
ORCID ID 0000-0003-3530-1359
mosxandrwork@gmail.com

CONSTRUCTIVE GEOMETRY IN IMPLEMENTATIONS OF MODERN 3D GRAPHICS

Abstract. 3D graphics are one of the crucial development trends of modern digital technologies. Engineering and manufacturing, architecture, design, cinematography, education, and the game industry are an incomplete list of industries where it is actively used. Specialists in 3D graphics are in high demand in the labor market. Their proper training presupposes high-quality knowledge of geometrical sciences, in particular – constructive geometry. Note that constructive geometry is an integral part of modern school mathematics education. That is why, even in the conditions of the school, the teacher should skillfully apply the demonstration capabilities of three-dimensional graphics. It will also encourage students of a comprehensive school to apply knowledge of constructive geometry in practice in the area of 3D modeling. This approach will make it possible to demonstrate the importance and interconnectedness of knowledge in geometry and computer science. Therefore, the article reveals the importance of interdisciplinary connections between the specified disciplines in the context of research, demonstration, and application aspects. In particular, the nuances of using the GeoGebra dynamic geometry complex for conducting computational experiments and creating spatial models based on tasks from a school spatial geometry course are described. After all, modern capabilities of software tools make it possible to demonstrate in real time all the transformations that took place during drawing modeling on the picture plane. The importance of the applied value of constructive geometry for 3D modeling reveals based on examples of solid and polygonal modeling of virtual spatial objects. In particular, the steps of creating a solid model of a pyramid, which is formed by cutting it off with a plane from a regular quadrangular pyramid, are illustrated by the basis of calculations and constructions, which are performed using techniques of constructive geometry. All stages are described and done using the TinkerCAD online modeling service tools. An example of using the Blender program for creating polygonal 3D models is also provided. In particular, the significant aspects of the part modeling process are presented in the example of a task from a drawing textbook. The importance of planimetric constructions in the process of performing high-precision polygonal modeling is also emphasized. The article contains many figures that illustrate the essential stages of modeling. The materials presented can be used to prepare lessons in either mathematics or computer science and can be used to conduct integrated classes that draw on both subjects. Possible prospects for further research on this topic are also presented.

Keywords: constructive geometry; computer graphics; 3D modeling; solid modeling; polygonal modeling; GeoGebra; TinkerCAD; Blender.

1. INTRODUCTION

Statement of the problem. The rapid development of digital technologies area provides unique opportunities for the involvement of modern tools of information presentation in the educational process for school mathematical disciplines. Thanks to three-dimensional graphics, a tool for visual demonstration with qualitative images of the most complex combinations of plane and space geometric figures has appeared. A prominent place in the creation of such educational materials is occupied by virtual and augmented reality technologies, which allow manipulating three-dimensional objects and demonstrating very important moments in educational practice.

However, no matter what progress modern graphic software has made, the skills and abilities associated with the graphic culture of displaying the necessary geometric shapes on a sheet of paper remain important. Therefore, students should understand the basic principles and approaches to the execution of both plane and space geometric images, dynamic planar additional drawings and transformations of objects. For support of this thesis, we present a number of convincing arguments.

Today, the vast majority of students are fascinated by realistic 3D models and characters in cinema, computer games and architectural visualizations. However, only a few people think about the fact that the first step in creating extraordinary compositions is a two-dimensional drawing on paper. Thus, according to the modern vision of the implementation of three-dimensional models, their development should include a number of important stages: concept-art, sculpting or modeling, retopology, UV-unwrapping, backing, texturing, rigging, animation [1]. And one of the most important of them is the concept-art itself, because at this stage a binary image of the future project is created in all possible details and standard projections.

An important part of the documentation necessary for the production of industrial products and even engineering STEM projects of students is a technical picture that is an axonometric image of an object, which is made by hand. All its elements are built on the basis of geometric rules [2, p. 127]. A technical picture allows us to visualize the shape of the designed thing or product.

At the same time, minimal attention is paid to the topic of constructions in geometry lessons in modern school. As a result, in the conditions of the global development of STEM education, graduates of schools and first-year students of universities of pedagogical profile unsystematically, uncertainly, and improperly perform drawings for theorems and problems not only in space, but also in plane geometry. In addition, students who are interested in engineering, 3D graphics or creating visual effects have difficulties presenting these ideas at the initial stage without the involvement of digital technologies.

The root cause of this state of affairs is the absence of graphic drawing operations in school educational programs. Traditionally, teachers do not teach students well-founded rules for performing qualitative images, constructivist methods, visual modeling of different geometric situations. Solving mainly the simplest computational problems (in one or two steps), they do not demonstrate any decisive addition drawings and transformations of figures on the way to the result.

In order to eliminate the specified problems in the educational sphere, we offer innovative, practically proven methods of teaching “the first of the sciences”, we substantiate with examples a constructive approach to its teaching, visualizing by qualitative drawing each geometric proposition, using ICT and applications [3, 4].

Analysis of recent researches and publications. Scientists in geometry are unanimous in the opinion that in the conditions of pedagogical process the discipline “Geometry should be geometric, and not only computational, algebraic or trigonometric. That is, the main object of research in geometry is a *figure*, and the main means of reasoning is a qualitative *drawing*.

The voiced thesis, its content component directly relates to the process of teaching students skillful, qualified solving of problems not below the average level of complexity. After reading and comprehending the condition of the task, the student to a certain extent subconsciously begins to perform the drawing. And only after that he (she) makes a plan, algorithmizes the process of solving the problem. The substantiation of a proposal in geometry is accompanied by logical considerations that directly follow from the visual picture. Yes, students should work, starting with a high-quality drawing or verbal visual description. Spatial representations of the situation, its strict understanding, appear with the drawing. The famous, truly great German mathematician (geometer) D. Hilbert back in the 19th century. wrote: “However, even now the visual understanding still has a primary role in geometry, and moreover, not only as something that has great evidential power, but also for understanding and evaluating the results of research.”

Modern empirical studies also confirm the importance of quality drawing in teaching geometry in general. Ya. Karpuz, E. Atasoy, having studied the perception of figures by students of the 9th grade using the Figure Apprehension Cognitive Processes Test (FACPT) method, came to the awareness that a significant number of respondents have difficulties in recognizing the various components of a given geometric figure, transforming verbal information into visual information and vice versa, formulating conclusions without the influence of the real look of the figure [5].

Thus, one of the ways to increase the effectiveness of the educational process in Euclidean geometry should be seen in the application of creative approaches to the organization of education and the balanced use of modern information technologies. That is why the research of many leading scientists and practitioners is devoted to the issues of creativity, the features of the application of visual literacy (in particular, with the help of digital information software), the use of spatial representations and imagination, unreal simulated images in education and in life, the culture of thinking and the expression of thoughts with high-quality images [6 - 9].

Nowadays, a very good assistant in teaching the modeling of geometric figures is a computer and proven, reliable applications developed by domestic and foreign scientists [10, 11].

The theory and practice of graphic culture of performing correct and visual images of bodies and their combinations, in conditions when students work with drawing tools or “by hand”, is sufficiently convincingly and fully covered in the author’s manual [12]. However, at the current level of implementation of ICT in the educational space of Ukraine, there is an urgent need for computerization of such developments.

Taking into account the above, we note that the use of leading digital graphics technologies allows us to reveal the full range of interdependencies in mathematics and demonstrate their applied value. In particular, virtual and augmented reality technologies [13, 14], which allow displaying and interacting with extremely complex spatial three-dimensional representations of graphic information, are extremely promising.

The aim of the research. In the context of the conducted analysis of research and publications, we consider it necessary to reveal on specific examples the importance of studying original constructive methods using 3D graphics programs in the process of practical visualization of stereometric constructions.

We have identified the following key tasks:

1) To emphasize the appropriate involvement of pedagogical software tools during the training of students in constructive space geometry.

2) To demonstrate the practical orientation of constructive geometric methods for the study of three-dimensional modeling and practical experiments with spatial figures.

2. RESULTS OF THE RESEARCH

2.1. Constructive binary modeling of spatial geometric shapes in GeoGebra

As already mentioned, any idea in the practical implementation of an engineering STEM project or the creation of a three-dimensional model for various spheres of human activity begins with a geometrically correctly constructed binary drawing. One of the most important ways that will allow students to develop the skills and abilities of operating with spatial objects is solving space geometric problems, where a qualitative drawing is considered the key to the result. Therefore, during teaching geometry the teacher is obliged to emphasize three requirements that should be followed by the students, namely: *correctness*, *clarity* and *simplicity* in the visual two-dimensional modeling of the condition of the formulated problem with a corresponding image on the picture plane, which is a blackboard, a notebook or a computer screen.

Since the drawing for the task appears on the projection plane thanks to the operation of parallel projection of the imagined spatial object (figure), it is worth knowing the properties of the specified action and following them, because it will guarantee the *correctness* of the result. *Clarity* means that anyone (even an outsider) entering the audience and looking at the drawing should understand exactly which object is depicted on it. *Simplicity* in constructions means that a teacher or student, modeling space geometric situation in compliance with the first two requirements, works quickly, but necessarily with high quality.

It is very important to carefully conduct a preliminary analysis of the conditions of the task, thinking and planning how to perform a high-quality drawing. Work with the drawing, additional drawings and transformations that contribute to the algorithmization of the solution path largely depend on the thoroughness of the analysis and the chosen perspective, the angle of view of the imagined depicted object in the external parallel (orthogonal) projection, the rational, successful location of the body between the observer and the plane of projections.

Many years of work experience in the field of education proves that in the process of learning, when the teacher or student reproduces images of three-dimensional bodies or their combinations on the picture plane, it is very useful to follow the methodological recommendations sufficiently fully described in the manual [12, p. 76, 77].

In addition, it is very useful to understand the essence and acquire practical skills of performing high-quality space geometric drawings by the method of axonometric directions and conventional ratios. It is worth practicing drawing axes in axonometry: in rectangular isometry and dimetry, always have an ordinary circular compass (measuring compass) and use single segments ([12], chapters II and III]). This approach will allow you to independently create an algorithm for drawing a picture and effectively implement it with the help of graphic software of dynamic geometry to obtain qualitative space geometric images.

Projection drawings executed according to the given recommendations on the picture plane will be clear, correct and therefore, with the acquisition of practical skills, easy to construct. Their accuracy, aesthetic completeness, and overall quality will effectively contribute to finding a way to solve the problem. The rationalism and clarity of execution of the elements of geometric shapes will have a positive effect on the development of students' spatial ideas and imagination, the skills of visualizing thoughts and actions, interest in constructive operations, and therefore self-confidence.

Separately, we emphasize that in everyday practice it is not necessary to rely only on the method of axonometric directions and conventional ratios. Images made by this method are *guaranteed to be correct* and *the most visual*, however, at the beginning of learning, they are somewhat unusual for students and, to some extent, allegedly more difficult to construct. The teacher, who has many years of experience, has self-developed (borrowed) methods of high-quality imaging of space geometric bodies and their combinations.

Next, by means of visualization, we will solve several problems of a constructive and computational nature. In addition, we will demonstrate how dynamic geometry systems can be used for spatial visualization, and therefore, understanding of completed constructions.

Problem 1. *In the regular quadrangular pyramid SABCD, the height SO is equal to the side of the base AB, which is equal to a. Construct a section of the pyramid with a plane passing through the top of the base perpendicular to the opposite side edge. Find the volume of the pyramid, which is cut off from the given pyramid by the cutting plane.*

To get a high-quality drawing model of a regular quadrangular pyramid in the form of a drawing-picture[1], first we depict a square at its base. One of the properties of parallel projection indicates that a parallelogram of general arrangement and its varieties (rectangle, rhombus, square) are projected onto the picture plane by any parallelogram. We suggest that the square be represented by the parallelogram *ABCD* in four steps, which is not difficult to remember (Fig. 1, *a*): 1) at an angle of 10-15° to the horizon, draw a straight line and lay the *BC* side on it; 2) approximately at the angle 120° to *BC* from the point *B* draw the ray; 3) on the ray set the segment $BA \approx \frac{1}{2} BC$; 4) using well-known properties, we complete the triangle *ABC* to the parallelogram.

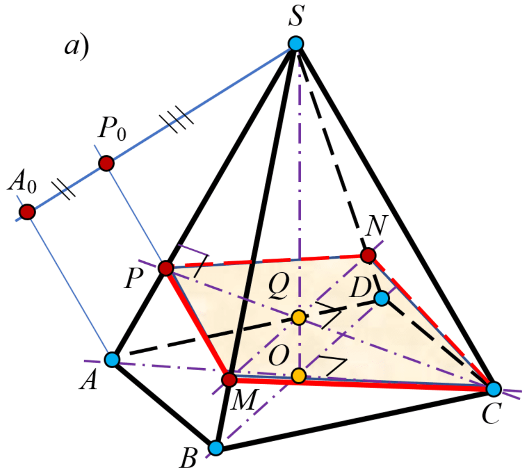

Fig. 1 (a). Drawing model of a regular quadrangular pyramid for problem 1

We fix the position of the top of the pyramid S on a vertical straight picture plane that is done on the figure.

We manually choose one of the vertices at the base of the pyramid, through which the cutting plane **Σ** will pass. Let it be point *C*. According to the condition of perpendicularity of a line and a plane, the section should be defined by two intersecting lines, each of which is perpendicular to the edge of *SA*. As one of these lines, we choose the perpendicular *CP* (*P* is its base), which still needs to be lowered from point *C* to the edge *SA*, the other line *MN* is convenient to draw in the plane of the axial section of the pyramid *SBD* parallel to *BD* through the point *Q*, where the segments *CP* and *SO* intersect, because *MN* ∥ *BD* by the second condition of parallelism of lines and *SA* ⊥ *BD* by the theorem about three perpendiculars.

Since the condition of the problem requires the construction of a section of the pyramid in the plane **Σ**, which is perpendicular to the edge *SA*, the following operations are performed in figures 1 and as in the model drawing[2].

In order to geometrically strictly construct the base *P* of the perpendicular *SR* to the edge *SA*, let's imagine that by moving in space we "lay" the pyramid diagonal *AC* on the picture plane. The side of the base of the pyramid is equal to *a*, therefore, the diagonal of the square is equal to $a\sqrt{2}$. The diagonal of the square at the base of the pyramid ($A'C' = AC = a\sqrt{2}$) we choose the segment in the image as the original length, which is a function of the parameter *a*. In the future, by rotating around the diagonal $A'C' = AC$, which is a zero-level line (belongs to the image plane), we perform two consecutive combinations with the picture plane: the square *ABCD* and the triangle *SAC*. In the first case, we constructively get the true side of the square in size *AB* = *SO* = *a* (Fig. 1, *b*); in the second, we will have an original triangle in shape and size *A'B'C'*, which makes it possible to put down the perpendicular *C'P'* from the point *C'* on the edge *S'A'* by the usual plane geometric technique (Fig. 1, *c*). All that remains it is the edge *SA* on (Fig. 1, *a*) divide by the point *P* in the ratio in which the point *P'* divides the side *S'A'* of the triangle *S'C'A'* and, finally, draw the needed perpendicular *CP*.

The image of the cross-section figure on (Fig. 1, *a*) is completed with the method already described above: we fix the point *Q* of the intersection of the perpendicular *SR* with the height of the pyramid *SO* and draw the straight line *MN* parallel to *BD*. Since the straight line *MN* belongs to the axial cross-section *SBD* of the pyramid, it cuts two more vertices of the cross-section on the edges *SV* and *SD* – points *M* and *N*, respectively. The quadrilateral *CMPN* is a

[1] Drawings-pictures are used to solve problems using the computational method.
[2] Model drawings are used to solve problems by constructive method.

deltoid, since the triangles *SPM* and *SPN* are equal (by two sides and the angle between them, which is obvious), and therefore, $PM = PN$ (similarly, we have that $CM = CN$); in addition, according to the theorem on the projection of a right angle, the angle between the diagonals of the section is equal to 90°.

Solving the problem computationally, it is enough to use a drawing-picture (Fig. 1, *a*). The point *P* on the edge *SA* can be chosen arbitrarily, remembering that the angle *SPC* is straight. We recommend conducting the algorithmization process on the way to finding a solution using an analytical method.

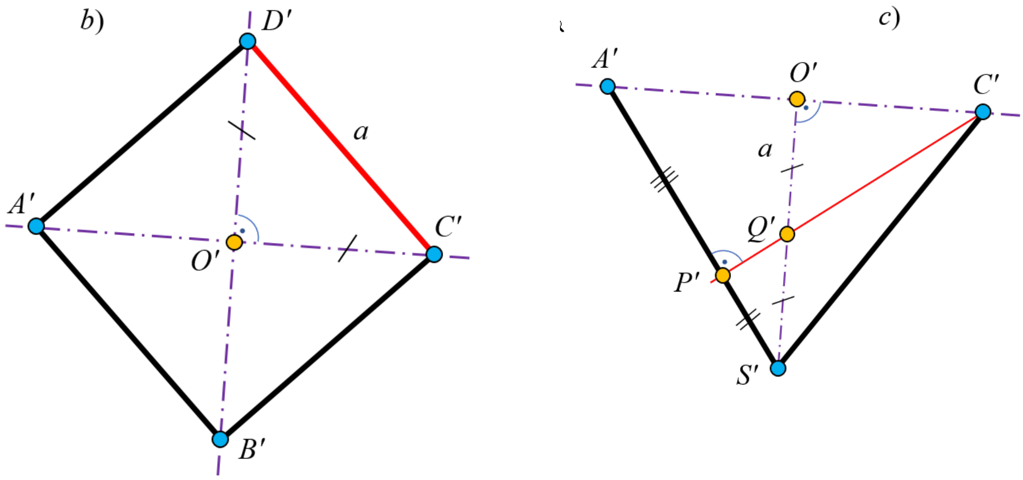

Fig. 1 (b, c). Sketches of the base ABCD of the pyramid and the triangle SAC

The volume of the *SCMPN* pyramid is calculated using the formula $V = \frac{1}{2} S_o \cdot h$, where $S_o = \frac{1}{2} CP \cdot MN$ and $h = SP$ is the height of the pyramid.

Therefore, the calculation process is reduced to the expression through the given parameter *a* of three segments $SP = h$, *PC* and *MN*. Also note that $PA = SA – h$.

Looking at (Fig. 1, *a*), we can see that the length of the segment *SP* is not difficult to find from the right triangle *SPC*, provided that the expressions for the two segments *PC* and *SC* are found. However, in the triangle *SOC* we will have $SC^2 = SO^2 + OC^2$, where $SO = a$ and $OC =$ $= a\sqrt{2}/2$, thus, $SC = SA = a\sqrt{\frac{3}{2}}$. In turn, the diagonal *PC* of the deltoid of the section *CMPN* divides the isosceles triangle *SAC* into two right triangles *PCS* and *PCA*, in which are unknown only the segments *PC* and $SP = h$. Using the Pythagorean theorem twice for the line segment *PC* in square (PC^2) and equating the right sides of the obtained equalities, we will obtain that $SP = h = a/\sqrt{6}$ and $PC = 2a/\sqrt{3}$.

It is obvious that it is expedient to express the segment *MN* in terms of parameter *a* from the similarity of the triangles *SMN* and *SBD*. Let’s write down the proportion $\frac{MN}{BD} : \frac{SQ}{SO}$, in which the segment *SQ* is unknown. The segment *SQ* is found from the similarity of two more right triangles *SRQ* and *SOA*: $\frac{SQ}{SP} : \frac{SA}{SO}$. Based on the available data and simple algebraic transformations of expressions, we consistently find that $SQ = a/2$, $MN = a\sqrt{2}/2$, $S_o = a^2/\sqrt{6}$ and $V = a^3/18$.

The problem has solved, and the given drawings clearly indicate the main stages of its solution. Now, with the help of the GeoGebra dynamic geometry system, we will supplement the 2D image with a spatial model.

Unlike graphical representation in programs, metric values are important. Let $a = 6$ units of length, then the calculated volume of the pyramid will be equal to 12 cubic units. Let’s check the calculations on the constructed model.

All constructions will be performed on a specialized three-dimensional “canvas” (in GeoGebra, this mode is designated as 3D Calculator).

At the initial stage, we set the *SABCD* pyramid according to the condition of the problem (Fig. 2, *a*). Next, we construct a cross-sectional plane that is perpendicular to the edge *SA*, and fix its points of intersection with the edges *SA*, *SB*, *SD* (Fig. 2, *b*). Visually, in real time, we observe the appearance of the *CMPN* section itself with the help of tools that provide the ability to rotate and move the figure in space. In the last step, in (Fig. 2, *c*), we single out the *SCMPN* pyramid.

In the GeoGebra program, thanks to the automation of a fairly large number of operations (building a pyramid by defined points; building a plane perpendicular to a given line; calculating distances, angles, areas and volumes of figures), it is possible for both the teacher and the student to check their conclusions, which were made while solving the problem. As can be seen from (Fig. 2, *c*) the volume of the *SCMPN* pyramid is equal to 12 cubic units, which, in turn, completely coincides with the analytical calculations. At the same time, the software independently analyzes the image and distinguishes the edges of the figure (depicted with a dashed line), which overlap with the faces and are invisible to the observer.

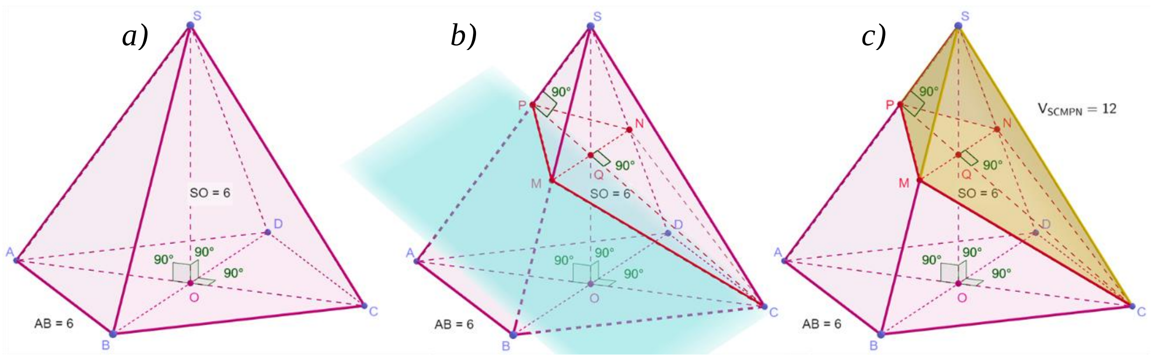

Fig. 2 (a, b, c). Stages of building a three-dimensional interactive pyramid model in the GeoGebra dynamic geometry system for problem 1

Similar empirical studies allow the demonstration of many geometric facts and mutual dependencies, which are quite difficult to imagine for the vast majority of students during the study of space geometry in senior school.

Now let's present on a spatial model all the stages of regular additions in a spatial figure, which the student usually has to perform in his imagination, which leads to the complication of understanding the transformations that take place on the picture plane. With that, let's consider the following problem.

Problem 2. *In the triangular pyramid SABD, the base is a right-angled triangle ABD with a right angle at the vertex A and legs AB = 3 and AD = 4. The height of the pyramid is projected to point A and is equal to the larger of the legs of the base. Drop a perpendicular from point A to the plane of face SBD and calculate its length.*

The drawing modeling of the formulated problem cannot be called special, since the right-angled triangle at the base of the pyramid is half of a rectangle (square), which is quite simply constructed (see problem 1). Using this example, it is quite convenient to demonstrate the constructions on the picture plane, which will be implemented as if they were performed in space (Fig. 3).

All constructions (Fig. 3, *a*, *c*, *e*, *g*, *i*), which are presented in the left column, reflect the stages of graphic operations on the picture plane. And the images (Fig. 3, *b*, *d*, *f*, *h*, *j*), presented on the right, simulate all the steps as if the observer sees both the pyramid itself and the picture plane on which the construction takes place. Let's reveal them in more detail.

As a first step, according to personal preference, we choose the upper-left half of *ABD* already depicted on the projection plane of the rectangle *ABCD*, and place the height *SA* of the pyramid vertically (Fig. 3, *a*). Figure 3, *b* shows how this stage is presented in GeoGebra.

First of all, you should put down the perpendicular from the vertex *A* to the face *ABD*, that is, find two straight lines in the plane of the specified face, to each of which the needed segment would be perpendicular[3]. One of these lines can be the hypotenuse *BD* at the base of the pyramid, another is the straight line *SP*, which belongs to the face *SBD* and is perpendicular to *AB*, which directly follows from the theorem about three perpendiculars. It is not difficult to construct a straight line *SP*. It is enough to draw the perpendicular *AP* from the vertex *A* of the right angle of the triangle *ABD* to its hypotenuse *BD*. If the problem is solved only computationally, point *P* on *BD* is chosen arbitrarily, remembering that it lies closer to the smaller of the legs.

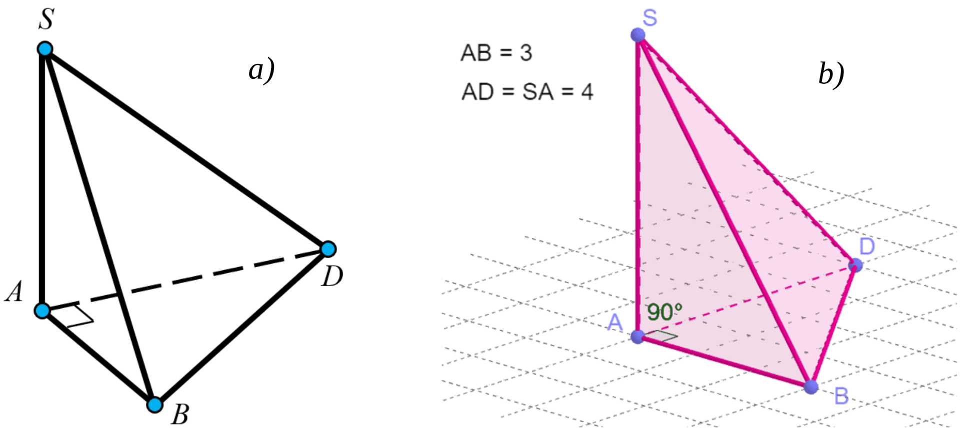

Fig. 3 (a, b). Stages of graphical construction of the perpendicular from point A to the plane of face SBD

By moving in space, we “lay” the *SABD* pyramid with the leg *AB* = *A'B'* = 3 units on the picture plane and, therefore, we qualify this leg as original. Next, by turning around the selected line of the zero level, we coincide the base *A'B'D'* of the pyramid with the plane of the copybook (board) and put down the perpendicular *A'P'* to the hypotenuse *B'D'* of the right triangle using the method of plane geometry (Fig. 3, *c*).

This stage is extremely difficult for students to imagine, and therefore (Fig. 3, *d*) shows in detail how the leg *AB* of the base of the pyramid aligns with the picture plane, provided that the direction of the parallel projection is set by the “direction”.

Then, using the generalized theorem of Thales, we construct the point *P* on the side *BD* (Fig. 3, *e*, *f*) and connect the obtained point with the vertices of the pyramid *A* and *S*. The triangle *SAP* is right-angled, because the edge *SA* is perpendicular to the plane of the base of the pyramid.

To draw the perpendicular *AQ* from the vertex *A* to the hypotenuse *SP* of the triangle *SAP*, we perform similar operations, but now we take the leg *A'P'* as the axis of rotation (zero level), the length of which is now true according to the previous construction. Then, drawing the perpendicular *A'Q'* to *S'P'* (Fig. 3, *g*, *h*) and dividing the segment *SP* by the point *Q* in the ratio

[3] All references to the facts of elementary geometry are based on the textbooks of O. V. Pogorelov [15], [16].

in which the point Q' divides the segment $S'P'$, we obtain the needed image of the perpendicular dropped from point A on the plane of face SBD (Fig. 3, *i*, *j*).

Here it is worth noting that in (Fig. 3, *j*) first projected the segment SP onto the picture plane (the result of the segments $S''P''$) in the specified direction, divided it within the picture plane in the specified ratio and performed the inverse operation, which made it possible to find the necessary point Q on the apotheme SP.

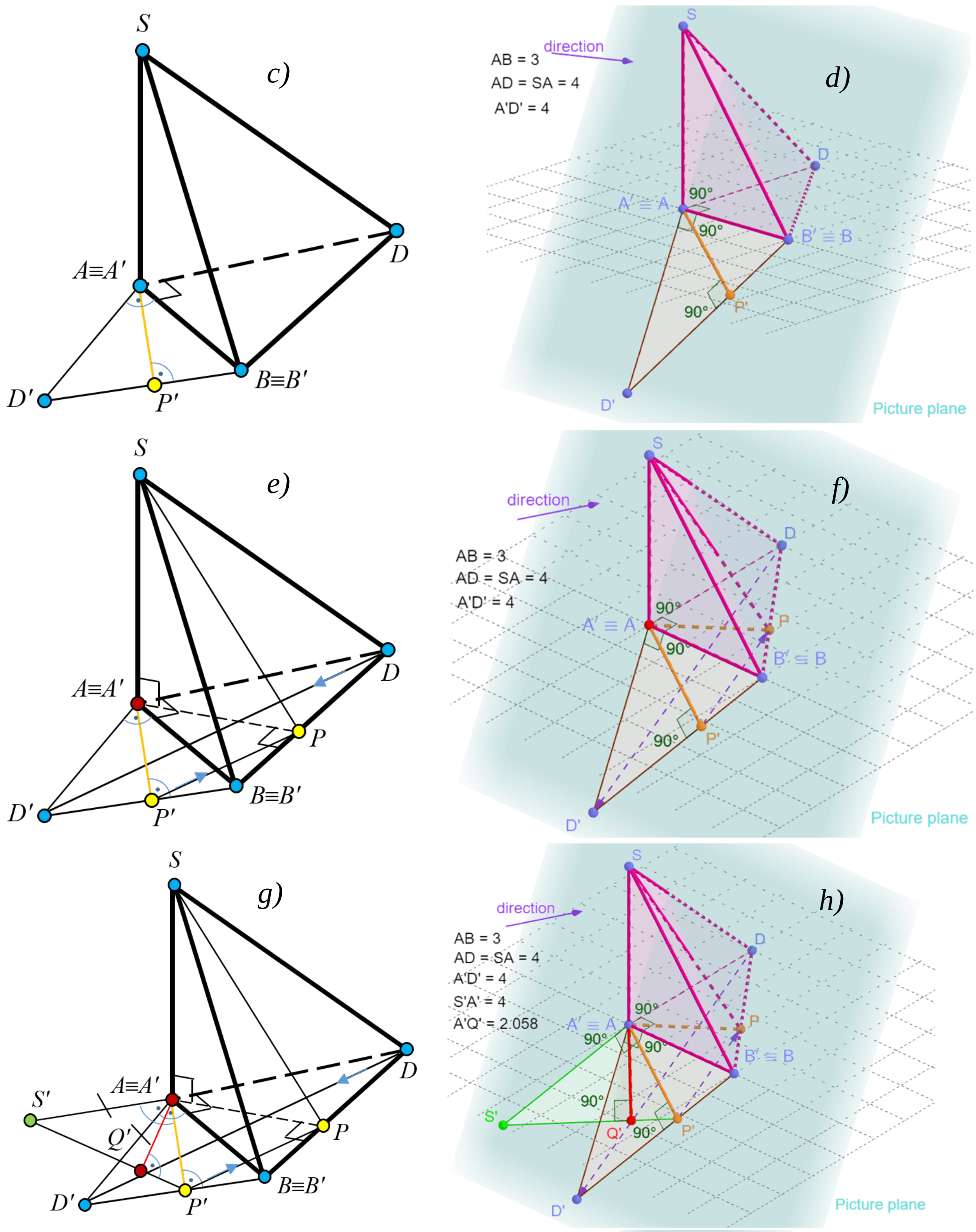

Fig. 3 (c, d, e, f, g, h). Stages of graphical construction of the perpendicular from point A to the plane of face SBD

We note that in order to obtain the original length (in 4 units) segment $A_0D_0 = A'D' = A'S'$, we used an external drawing (Fig. 4).

The computational stage of the problem is easy to carry out, since here we work only with metrically sized right-angled triangles. If we speak even more specifically, then exclusively with their average geometric values.

It is known that the leg of a right triangle is the mean proportional between its hypotenuse and its own projection on the hypotenuse. Thus, in a right triangle $D'A'B'$: $(A'B')^2 = D'B' \cdot P'B'$ $\Rightarrow P'B' = \frac{9}{5}$, $D'P' = \frac{16}{5}$. In addition, the height of a right triangle drawn from the vertex of a right angle is the average proportional between the projections of the legs on the hypotenuse: $(A'P')^2$ $= D'B' \cdot P'B' = \frac{12}{5}$.

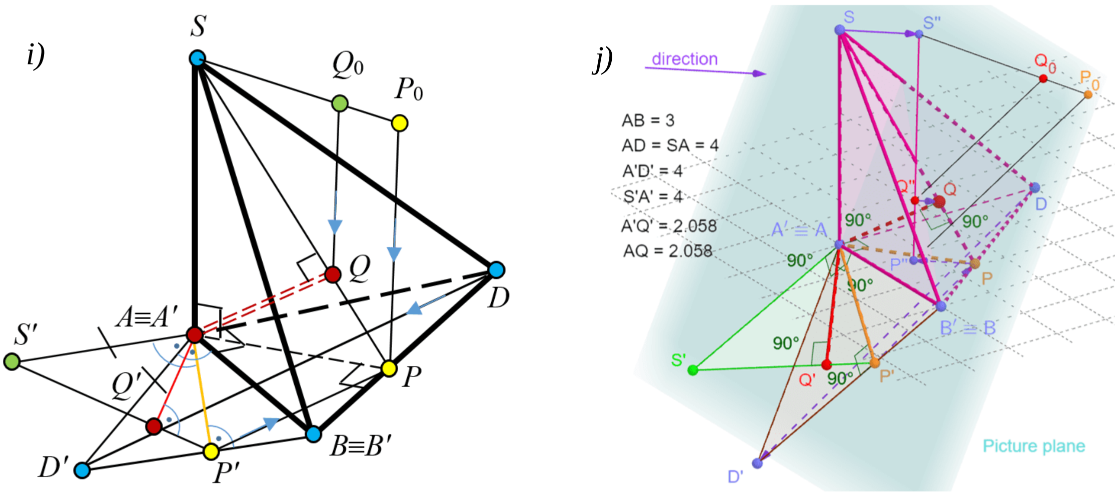

Fig. 3 (i, j). Stages of graphical construction of the perpendicular from point A to the plane of face SBD

Similar operations should be performed in a right-angled triangle $S'A'P'$, in which at the beginning, using the Pythagorean theorem, we find the length of its hypotenuse $S'P' = \sqrt{(S'A')^2 + (A'P')^2} = \frac{4\sqrt{34}}{5}$ and, finally, we obtain: $(A'P')^2 = S'P' \cdot Q'P'$, from that follows $Q'P'$ $= \frac{36}{5\sqrt{34}}$; $A'Q' = \frac{12}{\sqrt{34}} \approx 2{,}058$ units. If we look at (Fig. 3, *h*, *j*), then we can visually trace the process of finding the length of the needed segment, which is calculated thanks to the computational capabilities of the GeoGebra dynamic geometry system, and verify the correctness of the obtained result.

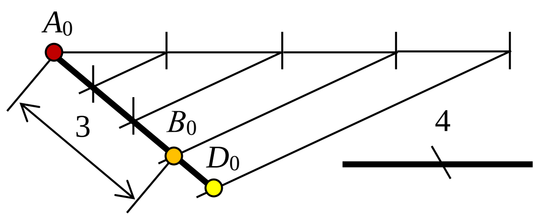

Fig. 4. Construction of the original length of segment AD on the external drawing

Thus, the given examples clearly demonstrate methodical possibilities in highlighting the relationships between figures in the school course of geometry in the space, which are extremely difficult for students to understand. In general, the tools of the mathematical graphic complex GeoGebra and the balanced actions of the teacher make it possible to significantly improve the understanding of the primary subject by schoolchildren, to interest them in the discipline and to develop the appropriate skills of mathematical (geometrical) visual thinking.

2.2. The use of solid and polygonal modeling programs as a visual demonstration of the practical orientation of constructive geometry

The GeoGebra dynamic geometry system is a high-quality software tool that allows us to constructively demonstrate the geometric facts that are studied in the school course of geometry. It is important to note that in this case, schoolchildren operate with imaginary models that are depicted on a plane or generated and stored in the memory of a computer (tablet) and have no direct practical application. At the same time, studying should contain a significant applied focus along with rigorous inferences and skills to operate models in imagination [17, 18]. That is why the use of three-dimensional computer modeling programs in teaching mathematics is one of the important aspects of forming a comprehensively developed student's personality. In addition, it allows you to successfully implement numerous interdisciplinary connections between mathematics, computer science, drawing, etc.

A number of important facts are given to confirm these theses. In particular, the key element of thorough knowledge of space geometry is the course of plane geometry, which students study in grades 7-9. In turn, in the 9th grade, according to the 2017 program, students are introduced to the topic of "3D graphics" [19] on computer science lessons. They learn to create virtual 3-dimensional objects using solid and polygonal modeling approaches, imagine a spatial figure in different projections on the picture plane, operate with such geometric transformations in space as parallel translation, rotation around a defined axis, similarity transformation (object scaling); get acquainted with spatial Cartesian coordinates, etc.

Separately, we note that for specialized schools with in-depth study of disciplines of the technical (engineering) cycle in grades 7-8 (10-11), the study of drawing is provided, which is also important in the formation of spatial visual thinking, imagination and ideas, which helps to navigate in three-dimensional space during mastering programs for 3D modeling [20]. Also, the optional module "3-dimensional modeling" [21] is offered for senior school students in the informatics course. Thus, school education programs establish the basis not only for senior school students to acquire a complex of knowledge, abilities and skills of constructive geometric modeling, but also for their practical application, and this contributes to the formation of practically significant competencies for every comprehensively developed personality: the ability to design and implement algorithmic and heuristic activities on mathematical material; classify and construct geometric figures on a plane and in space; to set their properties according to ready-made drawings; depict spatial figures and their elements; perform additions on images; measure the metric parameters of objects (angles, distances), find their quantitative values (areas and volumes); find out positional characteristics in the placement of individual elements of geometric figures, etc. [22].

Now let's consider examples of other programs, already purely applied and, to some extent, professional three-dimensional modeling tools that can be studied in computer science classes. Immediately, it is worth noting at an important point that each software of this class is oriented towards the formation of a certain resulting object: a 3D model intended for printing on a 3D printer; processing using a machine with numerical software control; used in architectural rendering, cinematography, or the computer game industry, and therefore not well suited for demonstrating geometric interrelationships between shapes. If we need to show students classical constructive approaches in building figures, we should turn to dynamic geometry systems, in particular the GeoGebra complex.

At the beginning, we will demonstrate the use of one of the simplest three-dimensional modeling tools – the TinkerCAD online service. Here, the creation of a three-dimensional object is based on the usual operations on sets – union and difference. Thanks to them, as well as tools that implement parallel transfer, rotation around a specified axis (for all three-dimensional modeling programs, the function of rotating the object around each of the coordinate axes tied to the local Cartesian coordinate system is implemented), and scaling, it is possible to create a digital model for the vast majority of spatial figures and their combinations studied at school. In fact, a solid-state approach to the formation of a digital model of an object is implemented in a regular Internet browser.

For example, consider the construction of the pyramid *SCMPN*, the volume of which was calculated in problem 1. So, the length of the side of the base of the pyramid *SABCD* will be equal to 6. At the initial stage, we place the pyramid itself and set its dimensions (Fig. 5, *a*). Next, for ease of positioning, rotate the figure around the vertical axis (that is, the height of the pyramid) by 45° (Fig. 5, *b*). Then we cut off the lower part of the pyramid so that its component with the top *C* remains. Since the three-dimensional editor does not have a tool that allows us to perform cross-sections of geometric bodies with a plane, we will use the following sequence of actions.

1. We add a parallelepiped whose dimensions exceed the base of the pyramid and make its reflection translucent. This means that when grouped with an object that will be painted in a solid color (like a pyramid), the common part of the two shapes will be cut off from the latter.

2. Place the parallelepiped so that one of its faces coincides with the cross-sectional plane. To do this, we need to calculate the angle of rotation.

Let's turn to the additional drawing (Fig. 1, *c*). $\Delta A'C'P' \sim \Delta A'S'O'$ at two angles, thus, $\angle A'C'P' = \angle A'S'O'$. So, to find the value of the angle of rotation, it is enough to calculate the tangent of the angle $A'S'O'$ in the right triangle $A'S'O'$. The calculations will be as follows: $\angle A'C'P' = \angle A'S'O' = arctg\left(\frac{A'O'}{O'S'}\right) = arctg\left(\frac{3\sqrt{2}}{6}\right) \approx 35{,}26°$. It remains only to turn the parallelepiped to the calculated angle (Fig. 5, *c*).

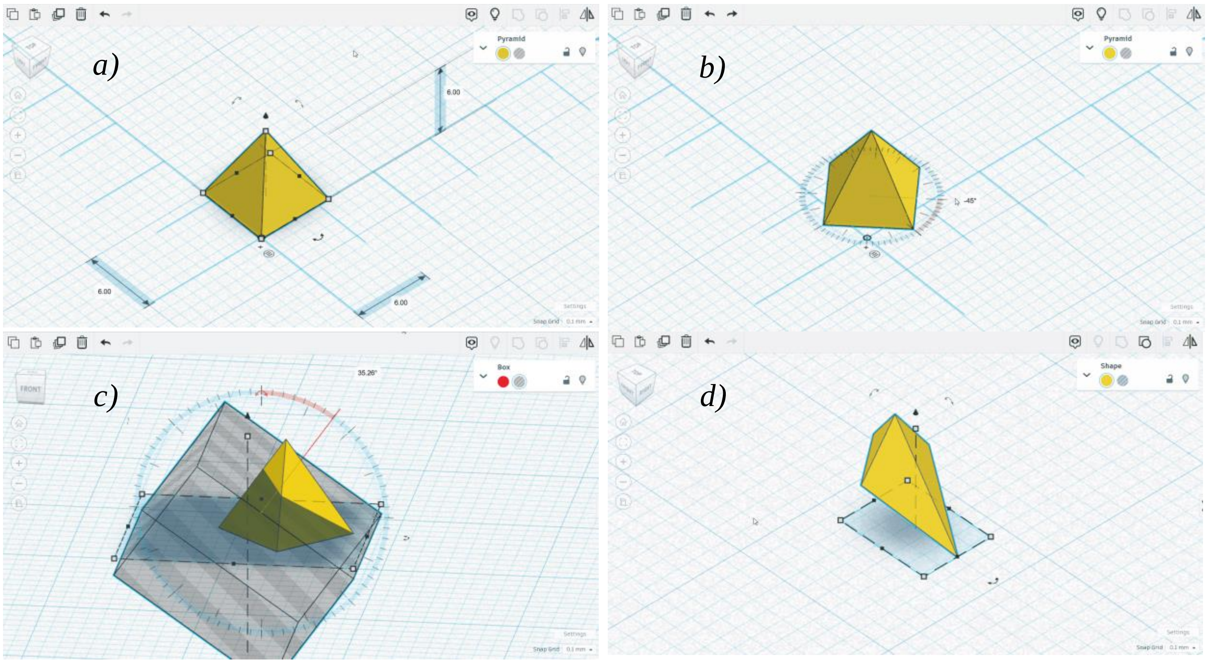

Fig. 5 (a, b, c, d). The main stages of building the model of the SCMPN pyramid from problem 1 in the TinkerCAD online solid modeling service

The last step of obtaining the shape needed is to group two objects: a pyramid and a translucent parallelepiped (Fig. 5, *d*).

Note that in the TinkerCAD program, which has a well-developed and adapted functionality for solid modeling, it was necessary to use the additional drawing (Fig. 1, *c*) to determine the angle of rotation for the parallelepiped. This fact, by the way, symbolically demonstrates the importance of the constructive component of geometry for the creation of three-dimensional models.

Further operations with the constructed model may be different. Thanks to the export of the created simple project in such a format as *stl*, it is possible to print this object on a 3D printer and empirically verify the calculations made, having previously taken the corresponding dimensions. In addition, such models can be visualized with the help of other programs and used in the preparation of scientific work.

If we consider the given examples, including the last construction in the TinkerCAD environment, then an important part of solving the problem is the analysis of the shape of the represented figure. Thus, in problem 1, a critically important condition for both the execution of the geometric construction and the analytical finding of the volume was the identification of a number of facts: the base of a regular pyramid is a square, and the section passes through the top of the base and is perpendicular to the edge. For problem 2, the defining conditions, which set both the shape of the figure and indicate the path to its solution, were the following statements: the base is a right-angled triangle and the top of the pyramid is projected orthogonally to point A – the top of the right angle. It is these facts that allow you to fully imagine the shape of the figure, identify the main steps for its display on a sheet of paper, and solve the problem.

In this context, it is worth noting that the ability of student to analyze a geometric shape is not only of theoretical importance (especially during solving theoretical problems), but is also a necessary condition for high-quality technical design. This is due to the fact that before creating a sketch of a part or a concept of an object for modeling, a specialist mentally imagines its shape, divides it into already familiar components (polyhedra, solids of rotation, etc.) and, based on convincing statements, forms a sequence of steps for reflection of the idea on the picture plane.

Thus, constructive geometry, which is an important part of mathematics education, establish the foundation for the development of geometric design skills, which are extremely important in work on STEM projects in engineering or in 3D modeling (studied in the school computer science course).

Confirmation of these statements will be an example that will demonstrate the importance of analyzing the geometric shape when creating a three-dimensional model. Let's consider the process of modeling in the freely distributed three-dimensional graphics complex Blender. To demonstrate the modeling process, we will choose a task that is presented in the textbook "Drawing" [23, p. 78]. Let's rethink it in the context of building a polygonal model. The task itself will look like this.

Based on the image presented in the figure, create a model of the object. The dimensions of the object have to be fully maintained (Fig. 6).

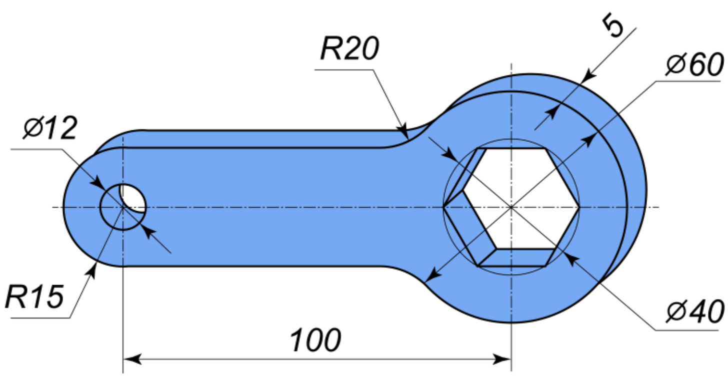

Fig. 6. Drawing for the task in the "Drawing" textbook

Let's analyze the front view of the presented image (Fig. 7). The outline of the object is a contour in which the left semicircle touches two horizontal segments at diametrically opposite points. These parallel segments are externally conjugated by small arcs of circles with a right circle. There are also two holes in the object. The center of the round hole, 12 mm in diameter, coincides with the center of the smaller semicircle, and the center of the hole in the shape of a regular hexagon merges with the center of the larger circle. In addition, the centers of both holes belong to the axis of symmetry of the part. Thus, to create the necessary model, the first step is to make a flat image of the part (main view), and then, with the help of a classic extrusion operation or a special modifier, convert the 2D image into a spatial object in the Blender program.

So, to begin with, let's create a two-dimensional outline of the projection. There are two ways to do it. The first involves the constructive construction of a projection in a vector editor, followed by saving it in SVG format and importing it into the three-dimensional graphics editor Blender. The second predicts the use of a mathematical apparatus to construct a flat outline on one of the coordinate planes. We will consider the second approach in more detail (Fig. 7).

Of all the elements in Blender, it is the most difficult to accurately represent the conjugation of straight lines and a great circle. In order to reproduce it, it is necessary to know the location of the points of conjugation, the coordinates of the centers of the circles of conjugation and the value of the arc *KL* in radians or degrees. Geometrically, the center of the arc of the conjugation circle is located at the intersection of the line *l*, which is parallel to the horizontal segment and is located at a distance of the conjugation radius $R_S = 20$ from it and the circle *w* with the center at the point O_1 and the radius $R_1 + R_S$. $R_1 = 30$.

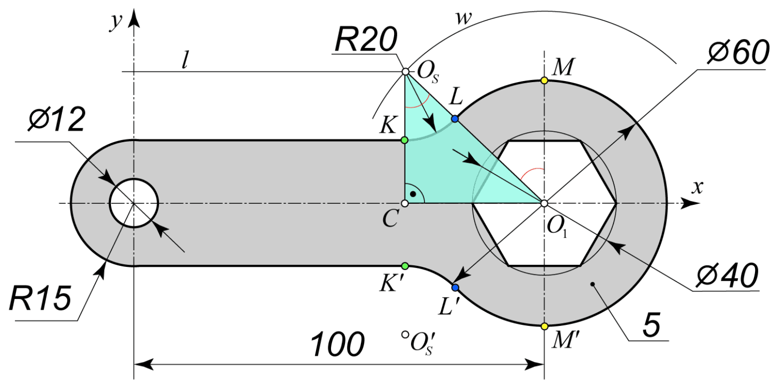

Fig. 7. Geometric additional drawings on the drawing of the model being created

Considering the triangle O_SO_1C, we notice that it is right-angled with the hypotenuse $R_1 + R_S$ and the leg $b + R_S$, $b = 15$. To accurately position the center of the conjugation circle and the points of contact in the triangle O_SO_1C, it is necessary to calculate the length of the leg O_1C and, in fact, the angle O_1O_SC. We can find the corresponding values in two ways: purely geometrically, by constructing a triangle, and analytically. Let's consider them both.

In the first case, to implement the necessary geometric constructions, it is worth performing an analysis of the construction problem. From the considerations described above, it is known that the triangle O_SO_1C is right-angled, in which the hypotenuse $O_1O_S = R_1 + R_S$ and the leg $O_SC = b + R_S$ are given. So, to carry out the construction, it is enough to use two sets of points: the set of the points from which the hypotenuse is visible at an angle of 90°, and a circle with the center at one of the vertices and a radius equal to the known leg. As usual, all constructions for the accuracy of execution should be carried out in the GeoGebra dynamic geometry system (Fig. 8), as this will allow geometric constructions to achieve accuracy even to the fifth decimal place, which will significantly increase the quality of the created three-dimensional model.

The second way involves calculating the angle based on trigonometric functions and dependencies in a right triangle. In the triangle O_SO_1C ($\angle C = 90°$) $\cos\angle O_1O_SC = \frac{O_SC}{O_SO_1}$, thus, $\angle O_1O_SC = arccos\,\frac{O_SC}{O_SO_1}$.

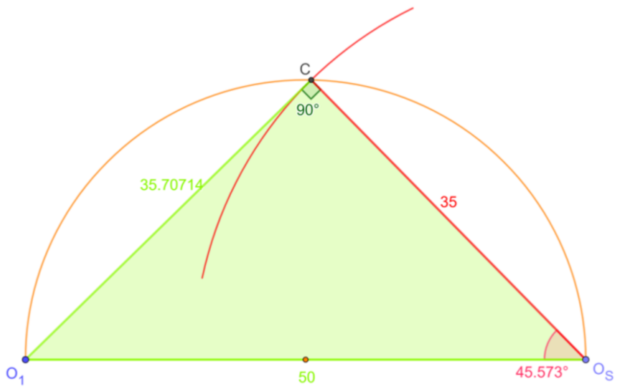

Fig. 8. Construction of ΔOSO1C in the GeoGebra dynamic geometry system

To find the leg O_1C, we use the Pythagorean theorem $O_1C = \sqrt{O_1O_S{}^2 - O_SC^2}$. By substituting the values, we get the following results: $O_1C \approx$ 35,707142143, $\angle O_1O_SC \approx$ 45,572995999°. It testifies to the effectiveness of the constructive solution of geometric problems in the Blender program.

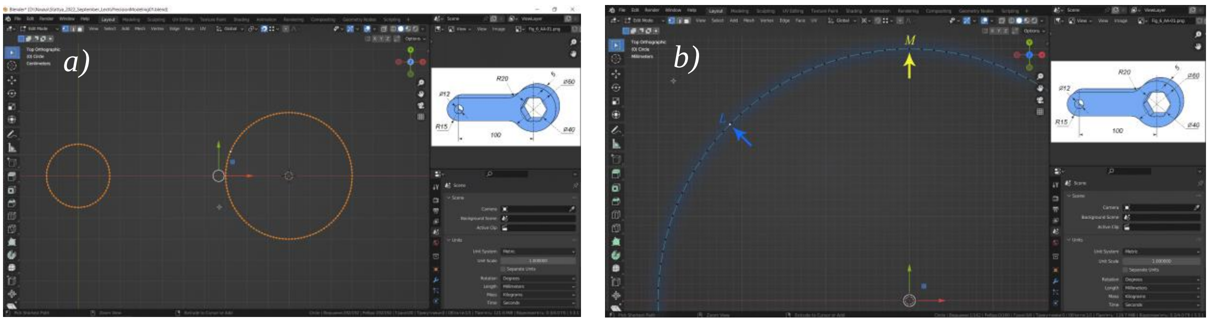

Fig. 9 (a, b). The main stages of building a digital three-dimensional model of the object in Blender (the polygonal border of the object is highlighted in blue)

Since the spatial Cartesian coordinate system is the basis for positioning in virtual space, it is important to specify coordinates for key points. To do this, align the origin of the coordinates with the center of the smaller hole, place the abscissa axis along the axis of symmetry of the part, and the ordinate place perpendicular to it. In this case, the coordinates of the circle with the center at the point O_1(100; 0) and the centers for the conjugation circles O_S (64.292857857; 35) and O'_S (64.292857857; –35) are determined quite easily. O'_S is the center of the conjugation circle, symmetrical about the axis of symmetry of the object.

Now let's build a three-dimensional model step by step. Activate the isometric display in the 3D View window and switch to the top view and construct the flat outline of the part. With this, first we construct the small and large circles (Fig. 9, *a*), then we build the conjugation arcs: the points marked in Fig. 8 with the letters *M* and *M'* are turned by the calculated angle (Fig. 9, *b*) counterclockwise (point *M*) and clockwise (point *M'*). In fact, in this way, we build the points of contact of the conjugation circles and the large circle of the part.

Next, we rotate the obtained points (Fig. 9, *c*) by the same angle around the centers of the conjugation circles (clockwise only for the first point, and counterclockwise for the second). In this case, the points of contact of the conjugation circles with segments are constructed. If we use calculations with an accuracy of 5 decimal places, then the obtained points will be at the same level as the diametrically opposite points of a small semicircle, and therefore it is enough to connect them to get the external contour of the projection of the part on the *xOy* plane (Fig. 9, *d*).

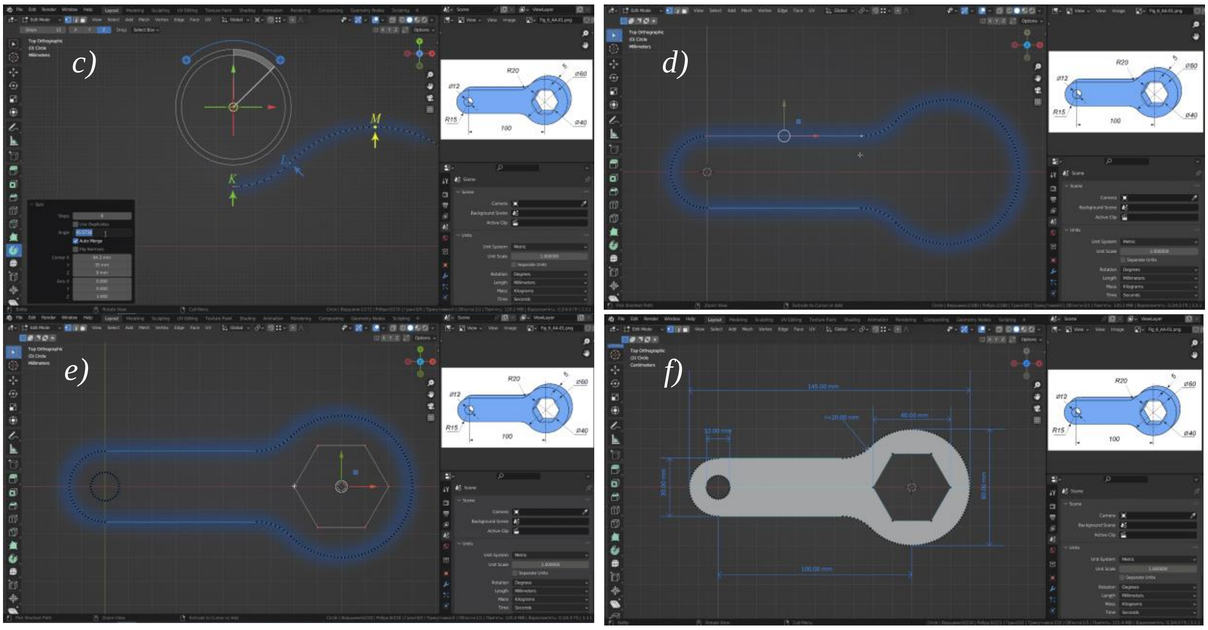

Fig. 9 (c, d, e, f). The main stages of building a digital three-dimensional model of the object in Blender (the polygonal border of the object is highlighted in blue)

We determine the places for the small and hexagonal holes (Fig. 9, *e*) and fill the space in such a way as to form a flat projection of the object (Fig. 9, *f*). The last operation is to apply the Solidfy modifier to quickly add volume to the figure (Fig. 10). The same result can be achieved using the extrusion operation.

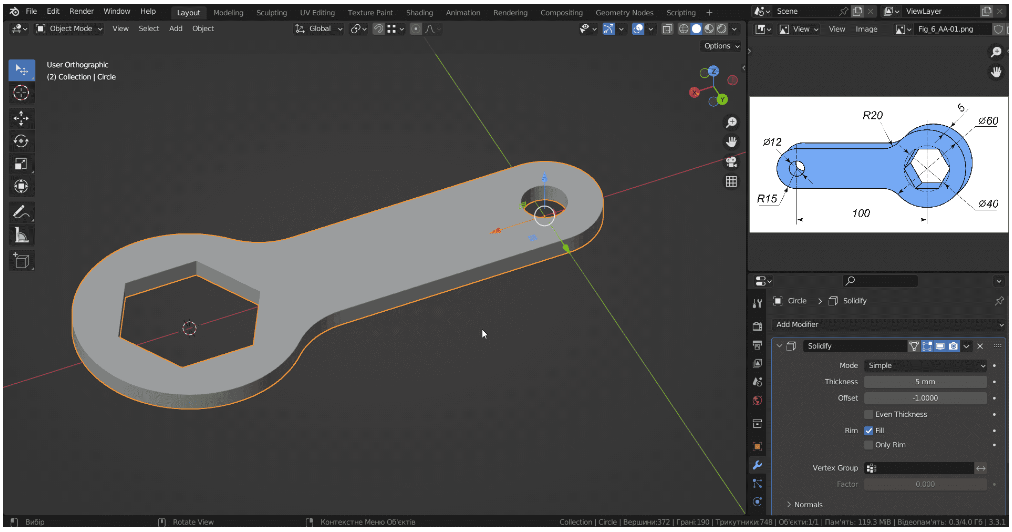

Fig. 10. Digital three-dimensional model of the object in Blender

As a result of all operations, we will get a three-dimensional model, which was presented in the figure. Moreover, it is made on a 1:1 scale and with high accuracy. Now you can perform a number of actions with it: print it on a 3D printer or use it to create a more complex virtual assembly of a student's STEM project.

3. CONCLUSIONS AND PROSPECTS FOR FURTHER RESEARCH

In the condition of a geometric problem, the figure used to solve it is imaginary, it practically does not exist. Therefore, we use a correct and clear drawing that provides a path to the result. It is not so easy to make a high-quality image from ideas, it is an extremely difficult task for a student without experience since the drawing is a graphic model of geometric structure, which is a meaningful component of the task. Similar drawing operations should be studied for quite a long time, which is lacking in school.

We are sure that learning geometry problems in space are possible only under conditions of professional mastery of algorithms for performing operations borrowed from drawing. After all, standardized axonometric projections guarantee the most necessary correctness and clarity of the images. There is another (proven above) method, easier to use, that is at the heart of the work of many generations of teachers and practicing scientists. Many years of work with students convinces us that both of the specified ways of construction are easy to practice to automatism. However, the latter (artificial) method is not suitable for algorithmization and computer visualization. Therefore, in this situation, it is better to use reliable, proven pedagogical software tools.

However, for a professional teacher who teaches geometry, drawing, informatics or any other discipline of the natural cycle, it is very important to develop students' spatial ideas and logic of reasoning, and, when appropriate, also to give examples of the application of undeniable regularities in the applied field of human activity. Without acquiring the listed abilities, students will not be able to correctly perceive the environment, orient in it, to think qualitatively and truly in geometric images. The teacher's need for a visual, pedagogically balanced demonstration of some useful nuances of practical modeling of the objects of the surrounding world, his own achievements and developments, or questions of theory (practice) arises quite often. In such a situation, it is rational to use a computer, a learning aid and available multimedia devices.

The progress of students in geometric and computer science education, the acquired knowledge, skills, and abilities, as well as the developed ability to analyze the form, positional and metric characteristics of imagined objects, is an important basis for the high-quality implementation of engineering STEM projects. In particular, they can use this knowledge for the creation of binary images three-dimensional shapes, and their various combinations employing solid-state or polygonal modeling.

To demonstrate the important role of shape analysis in the creation of three-dimensional models, we intend in the future to work out in detail the process of modeling geometric objects in the three-dimensional graphics complex Blender, with a rethinking of the construction of their polygonal models. We also plan to generalize the experience gained and methodical approaches to teaching students both constructive geometry and three-dimensional modeling.

REFERENCES (TRANSLATED AND TRANSLITERATED)

[1] A. Tereshchuk, "3D Character Development Pipeline", *n-ix.com*, Oct. 2022. [Online]. Available: https://www.n-ix.com/game-vr-studio/3d-character-development-pipeline/#title9. (in English)

[2] V. E. Mykhaylenko, V. V. Vanin, and S. M. Kovalev, *Engineering and computer graphics*: *manual*, Kyiv, Karavela, 2010, 360 p. (in Ukrainian)

[3] V. H. Kremen Ed., *National report on the state and prospects of education development in Ukraine: monograph*, Kyiv, Ukraine, 2021. Oct. 2022. [Online]. Available: https://lib.iitta.gov.ua/726223/ . (in Ukrainian)

[4] V. Yu. Bykov, O. Yu. Burov, and A. M. Gurzhii, *Theoretical and methodological principles of informatization of education and practical implementation of information and communication technologies in the educational sphere of Ukraine: monograph,* V. Yu. Bykov, S. G. Litvynova, V. I. Lugovii, Eds., Kyiv, 2019, 213 p. (in Ukrainian)

[5] Ya. Karpuz, and E. Atasoy, "Investigation of 9th grade students' geometrical figure apprehension", *European Journal of Educational Research,* vol. 8, no. 1, pp. 285-300, 2019, doi: https://doi.org/10.12973/eu-jer.8.1.285. (in Ukrainian)

[6] V. H. Kremen, and V. V. Ilyin, "Presentation of visual literacy in the educational process and its explication in the culture of thinking", *Information Technologies and Learning Tools*, vol. 75, no. 1, 2020, pp. 1–12, doi: https://doi.org/10.33407/itlt.v75i1.3660. (in Ukrainian)

[7] L. I. Bilousova, and N. V. Zhytyenyova, "Functional approach to the use of technology of visualization for intensification of learning process", *Information Technologies and Learning Tools*, vol. 57, no. 1, pp. 38–49, 2017, doi: https://doi.org/10.33407/itlt.v57i1.1525. (in Ukrainian)

[8] T. I. Koval, and O. P. Besklinskaya, "Use of visualization facilities to create electronic educational resources in the process of teaching mathematical disciplines in universities", *Information Technologies and Learning Tools*, vol. 77, no. 3, 2020, pp.145–161, doi: https://doi.org/10.33407/itlt.v77i3.3411. (in Ukrainian)

[9] R. S. Palais, "The Visualization of Mathematics: Towards a Mathematical Exploratorium»", *Notices of the American Mathematical Society*, vol. 46, no. 6, pp. 647–658, 1999. (in English)

[10] M. I. Zhaldak, Yu. V. Goroshko, and Ye. F. Vinnichenko, *Mathematics with a computer: manual,* Kyiv, Ukraine**,** 2015. (in Ukrainian)

[11] F. Botana, M. Abánades, and J. Escribano. "Using a Free Open Source Software to Teach Mathematics", *Computer Applications in Engineering Education*, vol. 22, no. 4, pp. 728 – 735, 2014, Oct. 2022. [Online]. Available: https://core.ac.uk/download/pdf/33103181.pdf. (in English)

[12] I. G. Lenchuk, *Constructive stereometry in tasks: a manual for students of mathematics majors in higher pedagogical educational institutions*, Zhytomyr, Ukraine, 2010. (in Ukrainian)

[13] M. Uygur , T. Y. Yelken, and C. Akay, "Analyzing the Views of Pre-Service Teachers on the Use of Augmented Reality Applications in Education", *European Journal of Educational Research*, vol. 7, no. 4. October 2018, pp 849-860, doi: https://doi.org/10.12973/eu-jer.7.4.849. (in English)

[14] F. Bakri, H. Permana, S. Wulandari, and D. Muliyati, "Student worksheet with AR videos: Physics learning media in laboratory for senior high school students", *Journal of Technology and Science Education,* vol. 10, no. 2, 2020, pp. 231-240, doi: http://dx.doi.org/10.3926/jotse.891. (in English)

[15] O. V. Pohorielov, *Geometry. Planimetry: manual for the 7-9 years of secondary school*, Kyiv, Ukraine, 1998. (in Ukrainian)

[16] O. V. Pohorielov, *Geometry: Stereometry: manual for the 10 – 11 years of secondary school*, Kyiv, Ukraine, 1998. (in Russian)

[17] D. H. Tong, T.-T. Nguyen, B. Ph. Uyen, L. K. Ngan, L. T. Khanh, and P. T. Tinh, "Realistic mathematics education's effect on students' performance and attitudes: a case of ellipse topics learning", *European Journal of Educational Research*, vol. 11, no. 1, January 2022, pp 403-421, doi: https://doi.org/10.12973/eu-jer.11.1.403. (in English)

[18] F. Bakri, H. Permana, S. Wulandari, and D. Muliyati, "Student worksheet with AR videos: Physics learning media in laboratory for senior high school students", *Journal of Technology and Science Education,* vol. 10, no. 2, 2020, pp. 231-240, doi: http://dx.doi.org/10.3926/jotse.891. (in English)

[19] Informatics. 5-9 grades. The curriculum for secondary schools, *mon.gov.ua*, Oct. 2022. [Online]. Available: https://mon.gov.ua/storage/app/media/zagalna%20serednya/programy-5-9-klas/onovlennya-12-2017/8-informatika.docx. (in Ukrainian)

[20] Curriculum. Drawing. 5-9 grades. For specialized schools with in-depth study of technical (engineering) cycle subjects, *mon.gov.ua*, Oct. 2022. [Online]. Available: https://mon.gov.ua/storage/app/ media/navchalni-programi-kursiv-za-viborom-fakultativiv/11092019/ kreslennya-7-8-kl%203.doc. (in Ukrainian)

[21] Informatics. Curriculum of an elective and mandatory subject for students of 10-11 grades of secondary schools. Standard level, *mon.gov.ua*, Oct. 2022. [Online]. Available: https://mon.gov.ua/ storage/app/media/zagalna%20serednya/programy-10-11-klas/2018-2019/informatika-standart-10-11.docx. (in Ukrainian)

[22] Mathematics curriculum (Algebra and beginnings of analysis, and Geometry) for students of 10-11 grades of general educational institutions. Standard level, *mon.gov.ua*, Oct. 2022. [Online]. Available: https://mon.gov.ua/storage/app/media/zagalna%20serednya/programy-10-11-klas/2018-2019/matematika.-riven-standartu.docx. (in Ukrainian)

[23] V. K. Sydorenko, *Drawing: manual for the 11 years of secondary school*, Kyiv, Ukraine, 2011. (in Ukrainian)

The material has been sent to the editorial office 10.01.2023

КОНСТРУКТИВНА ГЕОМЕТРІЯ В РЕАЛІЗАЦІЇ СУЧАСНОЇ 3D ГРАФІКИ

Ленчук Іван Григорович
кандидат технічних наук, доктор педагогічних наук,
професор, професор кафедри алгебри та геометрії
Житомирський державний університет імені Івана Франка, м. Житомир, Україна
ORCID ID 0000-0003-1923-9540
lench456@gmail.com

Мосіюк Олександр Олександрович
кандидат педагогічних наук, доцент кафедри комп'ютерних наук та інформаційних технологій
Житомирський державний університет імені Івана Франка, м. Житомир, Україна
ORCID ID 0000-0003-3530-1359
mosxandrwork@gmail.com

Анотація. 3D графіка є одним з найважливіших трендів розвитку сучасних цифрових технологій. Інженерія та виробництво, архітектура, дизайн, кінематограф, освіта та ігрова індустрія – це неповний перелік галузей, у яких вона активно використовується. Фахівці ж з 3D-графіки надто затребувані на ринку праці. Їх належна підготовка передбачає якісне знання геометричних освітніх компонент, зокрема – з конструктивної геометрії. Зауважимо, що конструктивна геометрія є невід'ємною частиною сучасної шкільної математичної освіти. Саме тому ще в школі потрібно вміло застосовувати демонстраційні можливості тривимірної графіки та заохочувати учнів до практичного застосування знань з конструктивної геометрії у сфері 3D моделювання. Такий підхід дасть змогу наочно продемонструвати важливість і взаємопов'язаність знань з геометрії та інформатики. Тому у статті розкривається важливість міждисциплінарних зв'язків між зазначеними дисциплінами в контексті дослідницького, демонстраційного та прикладного аспектів. Зокрема на основі задач зі шкільного курсу стереометрії розкриваються нюанси застосування комплексу динамічної геометрії GeoGebra для проведення обчислювальних експериментів і створення просторових моделей. Адже сучасні можливості програмних засобів дають змогу в режимі реального часу продемонструвати всі перетворення, які відбувались під час рисункового моделювання на картинній площині. Важливість прикладного значення конструктивної геометрії для 3D моделювання розкривається на прикладах твердотільного та полігонального моделювання віртуальних просторових об'єктів. Зокрема на основі розрахунків і побудов, які виконуються з використанням прийомів конструктивної геометрії, ілюструються етапи створення твердотільної моделі піраміди, що утворена шляхом її відсікання площиною від правильної чотирикутної піраміди. Усі етапи описані й виконані за допомогою інструментів онлайн-сервісу моделювання TinkerCAD. Наведено приклад використання програми Blender для створення полігональних 3D моделей. Зокрема суттєві аспекти процесу моделювання деталі представлено на прикладі завдання з підручника креслення. Також підкреслено важливість планіметричних побудов під час виконання високоточного полігонального моделювання. Стаття супроводжується значною кількістю рисунків, які ілюструють важливі етапи моделювання. Представлені матеріали можуть бути використані для підготовки окремих уроків з математики та інформатики і проведення інтегрованих уроків на основі цих предметів. Також подаються можливі перспективи подальших досліджень з даної тематики.

Ключові слова: конструктивна геометрія; комп'ютерна графіка; тривимірне моделювання; твердотільне моделювання; полігональне моделювання; GeoGebra; TinkerCAD; Blender.