Optimisation of Dynamic Loads of Rope Systems of Lifting Mechanisms of Bridge Cranes During Cargo Handling

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Abstract

Relevance. In this study, considerable attention is devoted to the analysis of dynamic loads that occur in the rope systems of lifting mechanisms of overhead cranes during start-up and braking, and the reduction of these loads. **Purpose.** To identify the magnitude and nature of changes in dynamic loads in the elements of lifting mechanisms of bridge cranes, a comprehensive dynamic analysis of the lifting mechanism of the bridge crane and its elastic elements (rope systems) was performed.

Methods. The dynamic analysis of the above mechanisms and systems was performed on well-grounded mathematical models of bridge-type cranes (single- and double-mass).

Results. The analysis of the obtained calculations of mathematical models of the mechanism of lifting the load of bridge cranes demonstrated that the dynamic loads applied to the structural elements and drive mechanisms are oscillatory and comparable to static loads. The analysis of the obtained calculations of mathematical models of the mechanism of lifting the load of bridge cranes demonstrated that the dynamic loads applied to the structural elements and drive mechanisms are oscillatory and comparable to static loads. To minimise the integral functionals, the methods of classical calculus of variations, mathematical physics and differential equations were used to model the dynamics of loading processes of rope systems and drives of bridge cranes, and the terminal (initial and final conditions of movement of such systems) were considered, which allowed solving the optimisation problem unambiguouslyThus, to reduce dynamic loads in structural elements (in particular, in ropes) during transients in such lifting mechanisms of bridge cranes, it is proposed to perform optimisation of the modes of movement of their drive mechanisms. An essential place in such optimisation is occupied by the choice of the optimisation criterion. Among such criteria, integral optimisation criteria were used. As such integral optimisation criteria, the RMS values of the current loads in the elastic elements (ropes) of overhead travelling cranes have been used.

Conclusions. Such integral criteria are integral functionalities that usually reflect undesirable properties of machines and their mechanisms, thus, they are subject to minimisation.

Keywords: dynamic optimisation, rope systems, lifting mechanisms, overhead cranes, cargo handling, transients

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Introduction

Lifting and transporting machines belong to machines with increased danger during lifting, transporting and installation works. The safe operation of these machines largely determines the magnitude and nature of the change in time of the acting loads on the structural elements and drive mechanisms. Particularly dangerous for the operation of lifting and transport machines are dynamic loads that change in time and can result in complex oscillatory processes in structural elements. Such fluctuations significantly affect the stability of the lifting machines and result in fatigue failure of their elements. All this affects the operational reliability of both lifting and transporting machines. The reliability of lifting and transport machines can be significantly improved by reducing dynamic loads on their structures and drive mechanisms. Particularly dangerous for lifting and transport machines are dynamic loads that occur during transient processes of movement, in particular, starting and deceleration of drive mechanisms. Thus, for example, a slight decrease in dynamic loads during start-up of up to 30% and deceleration of up to 40% of the mechanisms of portal cranes allowed increasing the overhaul cycle of these cranes from 2 to 5 times. Thus, in this study, considerable attention is devoted to the analysis of dynamic loads that occur in the rope systems of lifting mechanisms of overhead cranes during start-up and braking, and the reduction of these loads. To identify the magnitude and nature of changes in dynamic loads in the elements of lifting mechanisms of bridge cranes, a comprehensive dynamic analysis of the lifting mechanism of the bridge crane and its elastic elements (rope systems) was performed. The dynamic analysis of the above mechanisms and systems was performed on well-grounded mathematical models of bridge-type cranes (single- and double-mass).

The analysis of the obtained calculations of mathematical models of the mechanism of lifting the load of bridge cranes demonstrated that the dynamic loads applied to the structural elements and drive mechanisms are oscillatory and comparable to static loads. Thus, to reduce dynamic loads in structural elements (in particular, in ropes) during transients in such lifting mechanisms of bridge cranes, it is proposed to perform optimisation of the modes of movement of their drive mechanisms. An essential place in such optimisation is occupied by the choice of the optimisation criterion. Among such criteria, primarily, integral optimisation criteria were used. In the majority of cases, as such integral optimisation criteria, the RMS values of the current loads in the elastic elements (ropes) of overhead travelling cranes have been used. Such integral criteria are integral functionalities that usually reflect undesirable properties of machines and their mechanisms, thus, they are subject to minimisation. To minimise the integral functionals, the methods of classical calculus of variations, mathematical physics and differential equations were used to model the dynamics of loading processes of rope systems and drives of bridge cranes, and the terminal (initial and final conditions of movement of such systems) were considered, which allowed solving the optimisation problem unambiguously. As a result of the performed optimisation of the modes of movement of the drive mechanisms of lifting bridge cranes in the areas of transitional processes, the dynamic loads of the rope systems of bridge cranes are significantly reduced, the smoothness of the movement of the main elements of such mechanisms is achieved and oscillatory processes are practically eliminated.

The predecessor scientists conducted a comprehensive analysis of dynamic loads in the structural elements of overhead cranes during their start-up and deceleration [1-3]. However, such an analysis has significant disadvantages, since the driving forces that result in the necessary movement of the lifting mechanisms of bridge cranes were not established, and the problem was reduced to the analysis and synthesis of the modes of movement of mechanisms in the presence of specific kinematic terminal conditions [4]. This, in turn, has resulted in inaccuracies and frankly vast formulas that describe the optimal modes of movement (of ropes with loads) in such mechanisms, which has no practical use, since it does not evaluate the usual engineering indicators of movement in such systems (for example, dynamism coefficients). To eliminate such disadvantages, this study was performed, which allowed determining the coefficients of dynamism in the transient modes of functioning of lifting mechanisms and rope systems of overhead travelling cranes for typical methods of lifting loads: "from weight" and "with pickup"/"from the base" [5].

The purpose of the research is to substantiate the modes of starting the lifting mechanisms of overhead travelling cranes, which minimise dynamic loads in rope systems in different ways of lifting the load: "from the weight", "with pickup"/"from the base".

Materials and Methods

In the work, the dynamic load on the load-gripping devices was evaluated in two variants – lifting the load: "from weight" and "with pickup"/"from the base". In the first variant, it is assumed that the load is at a specific (small) height above the base, and the static load acting on the load gripping devices is equal to the weight of the load Q_l. Dynamic load P_{dyn} occurs at the initial moment of deceleration of the descending load when the brakes are applied [4-6].

In the second variant of loading, it is assumed that the load is on any base, the ropes are sagging, and, accordingly, at this moment, the load on the load-catching devices is zero.

In determining the dynamic load, the masses, m_r and m_l are attributed to the periphery of the drum,

and the mass m_l is determined proportionally to the square of the ratio of the number of branches of the cargo chain hoist, which are wound (or winding) on the drum, to the total number of branches on which the load hangs [1].

When determining the starting modes of the lifting mechanisms of overhead cranes, the results of the following indicators were considered: excessive driving force, rope speed and rigidity of the supporting structure, mass of the engine rotor and load,

kinetic and potential excessive force, system stiffness and time, load dynamics, oscillation frequency, conditions and mode of movement, etc. When assessing the dynamic load on load-gripping devices, the following should be considered: under conditions of proper operation, mainly only the vertical dynamic load during the operation of the load-lifting mechanism is significant, since, during the operation of the crane movement mechanisms and the rotation of its slewing part, it does not exceed 5-6% of the static load (Fig. 1) [1; 2].
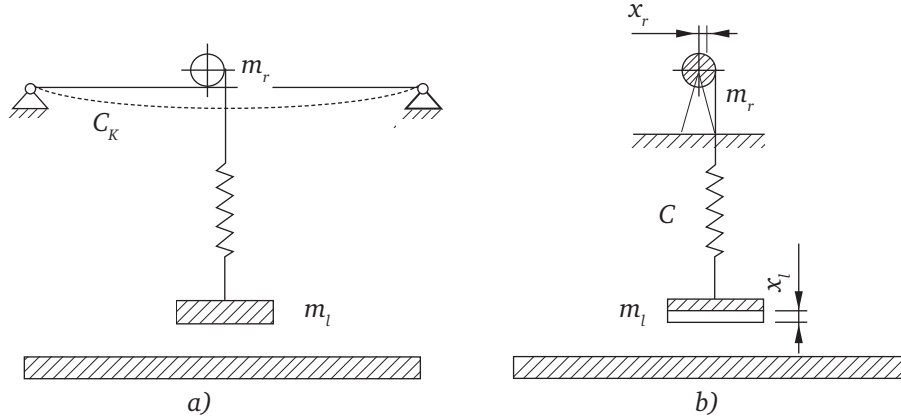

Figure 1. Scheme of dynamic loading of a loading device when lifting a load "from weight"
Note: a) on an overhead crane; b) design scheme
Source: [1; 2]

There are two options for lifting the load: "from weight" and "with pickup"/"from the base" [4-6]. In both cases, the dynamic coefficient (K_d) is determined by the dependence:

$$K_d = 1 + P_{dyn}/Q_l , \qquad (1)$$

where: P_{dyn} – in the first case is a function of the excess driving force and the stiffness of the supporting structure and in the second case is a function of the rope speed and the stiffness of the supporting structure.

Results

When designing in the case of lifting a load: "from weight" the crane is modelled by a two-mass oscillating system [7-9]. In this system, the stiffness of the ropes and the crane C_K structure are replaced by the reduced stiffness C_p, and the system itself consists of two masses – m_r (mass of the rotor of the engine and the reduced masses of the elements of the lifting mechanism) and m_l (mass of the load), connected by stiffness C:

$$C_p = \frac{C_K}{(C+C_K)} . \qquad (2)$$

When moving x_r mass m_r and x_l mass m_l, the kinetic and potential energy are respectively:

$$\begin{cases} K = \frac{m_r}{2} \cdot (\dot{x}_r)^2 + \frac{m_l}{2} \cdot (\dot{x}_l)^2; \\ P = C \cdot \{(x_r - x_l)^2/2\} \end{cases} \qquad (3)$$

For the mass, m_r the driving force is the weight of the cargo Q_l ($Q_l = m_l \cdot g, g$ – acceleration of free fall, $g=9.81$ m/s²), and the excessive force of the engine T_{exc}; for the mass m_l – weight of the cargo Q_l, acting in the same way as the inertial force of the cargo during lifting.

In cranes with the drive of the load-lifting mechanism from a three-phase current motor, the excess force T_{exc} can be considered constant (with other laws of change of the driving force, and, accordingly, the excess force, the dynamism of the lifting process will be less) [1]. The equations of motion have the type:

$$\begin{cases} m_r \cdot \ddot{x}_r + C \cdot (x_r - x_l) = Q_l + T_{exc}; \\ m_l \cdot \ddot{x}_l - C \cdot (x_r - x_l) = -Q_l, \end{cases} \qquad (4)$$

where: $(x_r - x_l)$ – can be denoted by ξ. Then:

$$\dot{\xi} = (\dot{x}_r - \dot{x}_l); \ \ddot{\xi} = (\ddot{x}_r - \ddot{x}_l). \qquad (5)$$

In the new notation (ξ) and (5) instead of (4) have:

$$\begin{cases} m_r \cdot \ddot{x}_r + C \cdot \xi = Q_l + T_{exc}; \\ m_l \cdot \ddot{x}_l - C \cdot \xi = -Q_l. \end{cases} \Leftrightarrow \begin{cases} \ddot{x}_r + \frac{C}{m_r} \cdot \xi = \frac{Q_l + T_{exc}}{m_r}; \\ \ddot{x}_l - \frac{C}{m_l} \cdot \xi = -Q_l/m_l. \end{cases} \qquad (6)$$

After subtracting from the first equation of system (6) the second equation of the same system obtained an inhomogeneous differential equation for ξ:

$$\ddot{\xi} + \Omega^2 \cdot \xi = \frac{Q_l + T_{exc}}{m_r} + \frac{Q_l}{m_l}, \Omega^2 = C \cdot \left(\frac{1}{m_r} + \frac{1}{m_l}\right). \quad (7)$$

For the case of lifting the load "from weight" use the following initial conditions to solve (7):

$$\xi|_{t=0} = \frac{Q_l}{C}; \ \dot{\xi}\big|_{t=0} = 0. \quad (8)$$

The general solution of (7) is found in the following type:

$$\xi(t) = A_1 \cdot \cos \Omega t + A_2 \cdot \sin \Omega t + \frac{1}{\Omega^2} \cdot \left\{\frac{Q_l + T_{exc}}{m_r} + \frac{Q_l}{m_l}\right\}. \quad (9)$$

Considering the conditions (8), from (9) have:

$$A_1 = \frac{Q_l}{C} - \frac{1}{\Omega^2} \cdot \left\{\frac{Q_l + T_{exc}}{m_r} + \frac{Q_l}{m_l}\right\}; \ A_2 = 0. \quad (10)$$

Then the solution of $\xi(t)$ (9) is given as follows:

$$\xi(t) = \left(\frac{Q_l}{C}\right) \cdot \cos \Omega t + \frac{1}{\Omega^2} \cdot \left(\frac{Q_l + T_{exc}}{m_r} + \frac{Q_l}{m_l}\right) \cdot (1 - \cos \Omega t). \quad (11)$$

Present (11) in a more convenient form for analysis:

$$\xi(t) = \left(\frac{Q_l}{C}\right) \cdot \cos \Omega t + \frac{2}{\Omega^2} \cdot \left\{\frac{Q_l + T_{exc}}{m_r} + \frac{Q_l}{m_l}\right\} \cdot \sin^2\left(\frac{\Omega t}{2}\right). \quad (12)$$

The force in the elastic element (in the elastic link), which can be considered as the impact of the load on the load-carrying devices:

$$P_{LC} = C\xi = Q_l \cdot \cos \Omega t + \frac{2C}{\Omega^2} \cdot \left\{\frac{Q_l + T_{exc}}{m_r} + \frac{Q_l}{m_l}\right\} \cdot \sin^2\left(\frac{\Omega t}{2}\right). \quad (13)$$

Its value is variable and is a function of system stiffness, C and time t. The maximum value of force in an elastic link P_{LCmax} occurs at:

$$\cos \Omega t = -1 \Rightarrow \Omega t = (2n - 1) \cdot \pi, n \in N, \quad (14)$$

that is, in moments of time:

$$t* = \frac{(2n-1) \cdot \pi}{\Omega}, n \in N, \quad (15)$$

$$P_l \frac{2 \cdot m_l}{(m_r + m_l)}_{exc\,LCmax}. \quad (16)$$

Since the excess force $T_{exc} = \phi \cdot Q_l$ (where ϕ – is the proportionality factor [1]), then:

$$P_l \left\{1 + 2\phi \cdot \frac{m_l}{m_l + m_r}\right\}_{LCmax}, \quad (17)$$

and the coefficient of dynamism has the form:

$$K_d|_{t=t*} = \frac{P_{LCmax}}{Q_l\left\{\frac{m_l}{m_l + m_r}\right\} \frac{2m_l}{(m_l + m_r)} \frac{T_{exc}}{Q_l}}, \quad (18)$$

where: the last expression for the coefficient (K_d) characterises the dynamic loading of the load-gripping device, provided that the lifting begins when the weight of the load affects the ropes Q_l [10-12].

To establish the dependence K_d (t) the following relation is used:

$$K_d(t) = 1 + \frac{m_l}{(m_l + m_r)} \cdot \frac{T_{exc}}{Q_l} \cdot (1 - \cos \Omega t), \quad (19)$$

or:

$$K_d(t) = 1 + \frac{m_l}{(m_l + m_r)} \cdot \frac{T_{exc}}{Q_l} \cdot 2 \cdot \sin^2\left(\frac{\Omega t}{2}\right). \quad (20)$$

At $\frac{\Omega t}{2} = (2n_1 - 1) \cdot \left(\frac{\pi}{2}\right), n_1 \in N$, that is, at:

$$t** = \frac{(2n_1 - 1) \cdot \pi}{\Omega}, n_1 \in N. \quad (21)$$

K_d (t) assumes the maximum values:

$$K \frac{2 \cdot T_{exc} \cdot m_l}{Q_l \cdot (m_r + m_l)}_{dmax}. \quad (22)$$

When decelerating the descending load P_{dyn} and the coefficient of dynamism K_d are determined by formulas (13)-(18), and K_d (t) – by formulas (19)-(22) [13-15]. Therewith, under the T_{exc} understand the following difference:

$$T_{exc} = T_{dec} - Q_l, \quad (23)$$

where: T_{dec} – deceleration force applied to the load.

As the moment generated by the brakes is usually less than the maximum torque [1; 2] generated by the engine, the dynamic force when braking the load being lowered does not exceed the dynamic force that occurs when lifting the load "from weight". Define further the laws of motion of masses m_p and m_g, based on the equations of the system (4), (6), (7) and the law $\xi(t)$ (11), (12) under the following (non-zero) initial conditions (when lifting the load "from weight"):

$$x_l|_{t=0} = 0; \ \dot{x}_l|_{t=0} = 0; \ x_r|_{t=0} = x_l|_{t=0} + \frac{Q_l}{C} = \frac{Q_l}{C}; \ \dot{x}_r| = 0. \quad (24)$$

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From system (6) for x_r have:

$$\ddot{x}_r + \frac{C}{m_r} \cdot \xi(t) = \frac{Q_l + T_{exc}}{m_r}. \tag{25}$$

From system (6) for x_l have:

$$\ddot{x}_l + g = \frac{C}{m_l} \cdot \xi(t). \tag{26}$$

Equations (25) and (26) must be solved under the initial conditions (24). Knowing $\xi(t)$ (11) and integrating twice for t each of equations (25) and (26), obtain [16-18]:

$$x_r(t) = \frac{2 m_l^2 \cdot T_{exc}}{C \cdot (m_r + m_l)^2} \cdot \sin^2\left(\frac{\Omega t}{2}\right) + \frac{(Q_l + T_{exc})}{2 m_r} \cdot t^2 + \frac{Q_l}{C}, \tag{27}$$

$$x_l(t) = -\frac{2 m_r \cdot m_l \cdot T_{exc}}{C \cdot (m_r + m_l)^2} \sin^2\left(\frac{\Omega t}{2}\right) + \frac{T_{exc}}{(m_r + m_l)} \cdot \frac{t^2}{2}. \tag{28}$$

From (27) and (28) evidently seen that the mass m_l and the mass m_r have oscillations whose frequency is Ω [19]. Define the conditions and modes of motion $\xi(t)$, $x_l(t)$, $x_r(t)$, under which in the period of acceleration of the system (before the lifting mechanism acquires a steady speed of lifting/lowering the load (V)) there are no oscillatory processes in the mass m_r, and mass m_l. Suppose that the duration of the system start-up, during which the steady-state

mode of lifting the load is established, i.e. the speed of movement (during lifting) becomes a constant value and is V, equal to τ_{II}. Then the motion for which the coordinate $\xi(t)$ satisfies the motion quality criterion:

$$\left\{ \frac{1}{\tau_{II}} \cdot \int_0^{\tau p} (\xi(t))^2 dt \right\}^{1/2} \Rightarrow min, \tag{29}$$

and will be the sought mass m_r and m_l systems. A necessary condition for the implementation of the criterion (29) is the Euler-Poisson equation, which can be obtained by replacing the integral expression (29) $\xi(t)$ with the expression (according to (7)):

$$\xi(t) = \frac{1}{\Omega^2} \cdot \left\{ \frac{Q_l + T_{exc}}{m_r} + \frac{Q_l}{m_l} - \ddot{\xi} \right\}. \tag{30}$$

then the above (Euler-Poisson) equation for (29), (30) becomes as follows:

$$\xi^{(IV)} = 0. \tag{31}$$

Consider the solution of (31) as a spline of the third order in t:

$$\xi(t) = a_1 + a_2 \cdot t^1 + a_3 \cdot t^2 + a_4 \cdot t^3. \tag{32}$$

To find the undefined constants $a_i, i = \overline{(1.4)}$, use the following conditions (terminal):

$$\xi|_{t=0} = \frac{Q_l}{C}; \; \dot{\xi}|_{t=0} = 0; \; \ddot{\xi}|_{t=0} = \left\{ \frac{Q_l + T_{exc}}{m_r} + \frac{Q_l}{m_l} \right\} - \Omega^2 \cdot \xi|_{t=0} = \frac{Q_l + T_{exc}}{m_r} + g*, \tag{33}$$

where: $g* = g - \Omega^2 \cdot \frac{Q_l}{C}$. The last terminal condition is of the form:

$$\dot{\xi}|_{t=\tau_p} = 0. \tag{34}$$

Differentiating by t the appropriate number of times the expression (32) and using the conditions (33), (34), obtain:

$$a_1 = \frac{Q_l}{C}; \; a_2 = 0; \; 2a_3 = \frac{Q_l + T_{exc}}{m_r} + g*; \; a_4 = -\frac{2a_3}{3\tau_p}. \tag{35}$$

Thus, for the traffic quality criterion (29) or (30) to be satisfied, it is necessary that $\xi(t)$ the time varies according to the following law:

$$\xi(t) = \frac{Q_l}{C} + \left\{ \frac{Q_l + T_{exc}}{m_r} + g* \right\} \cdot \frac{t^2}{2} - \left\{ \frac{Q_l + T_{exc}}{m_r} + g* \right\} \cdot \frac{t^3}{3\tau_p}. \tag{36}$$

or:

$$\xi(t) = \frac{Q_l}{C} + \left\{ \frac{Q_l + T_{exc}}{m_r} + g* \right\} \cdot \left\{ \frac{t^2}{2} - \frac{t^3}{3\tau_p} \right\}. \tag{37}$$

Then P_{LC} takes the form:

$$P_{LC} = C \cdot \xi(t) = Q_l + C \cdot \left\{ \frac{Q_l + T_{exc}}{m_r} + g* \right\} \cdot \left(\frac{t^2}{2} - \frac{t^3}{3\tau_p} \right). \tag{38}$$

For $K_d(t)$ have:

$$K_d(t) = \frac{P_{LC}(t)}{Q_l} = 1 + C \cdot \left\{ \frac{1 + T_{exc}/Q_l}{m_r} + \frac{g*}{m_l \cdot g} \right\} \cdot \left\{ \frac{t^2}{2} - \frac{t^3}{3\tau_p} \right\}. \tag{39}$$

The maximum values $P_{LC}(t)$ of (38) and $K_d(t)$ (39) are obtained at the end of the transient process (start-up) at $t = \tau_p$:

$$Pl\left\{\frac{Q_l + T\,exc}{m_r} + g *\right\}\frac{\tau_p^2}{6}\bigg|_{LCmax}, \qquad (40)$$

$$K\left\{\frac{1 + T\,exc/Q_l}{m_r} + \frac{g*}{m_l \cdot g}\right\}\frac{\tau_p^2}{6}\bigg|_{dmax}. \qquad (41)$$

Further find the laws of motion $x_l(t)$ and $x_r(t)$, which have masses m_l and m_r at $\xi(t)$ (36), and satisfy the terminal conditions (24):

$$x_r(t) = \left(-\frac{C}{m_r}\right) \cdot \left\{\frac{Q_l}{C} \cdot \frac{t^2}{2} + \left(\frac{Q_l + T\,exc}{m_r} + g *\right) \cdot \left(\frac{t^4}{24} - \frac{t^5}{60\tau_p}\right)\right\} + \left(\frac{Q_l + T\,exc}{m_r}\right) \cdot \frac{t^2}{2} + \frac{Q_l}{C}, \qquad (42)$$

$$x_l(t) = \left(\frac{C}{m_l}\right) \cdot \left\{\frac{Q_l}{C} \cdot \frac{t^2}{2} + \left(\frac{Q_l + T\,exc}{m_r} + g *\right) \cdot \left(\frac{t^4}{24} - \frac{t^5}{60\tau_p}\right)\right\} - \frac{gt^2}{2}. \qquad (43)$$

Using the dependencies $\dot{x}_r(t)$ and $\dot{x}_l(t)$ and their values at the moment $t = \tau_p$, namely: $\dot{x}_r|_{t=\tau_p} = \dot{x}_l|_{t=\tau_p} = V$, can be found the value of τ_{II} and V:

$$\frac{\left(-\frac{C}{m_r}\right) \cdot \left\{\frac{Q_l}{C}\tau_p + \left(\frac{Q_l + T\,exc}{m_r} + g*\right) \cdot \left(\frac{\tau_{II}^3}{12}\right)\right\} + \left(\frac{Q_l + T\,exc}{m_r}\right) \cdot \tau_p}{\left(\frac{C}{m_l}\right) \cdot \left\{\frac{Q_l}{C}\tau_p + \left(\frac{Q_l + T\,exc}{m_r} + g*\right) \cdot \left(\frac{\tau_p^3}{12}\right)\right\} - g \cdot \tau_p} = 1, \quad (44)$$

$$\tau_p = \left\{\frac{12 \cdot \left(\frac{T\,exc}{m_r}\right)}{\Omega^2 \left(\frac{Q_l + T\,exc}{m_r} + g*\right)}\right\}^{1/2}. \qquad (45)$$

Knowing from the relation (45) τ_{II} it is easy to find V as follows (this value must satisfy the provisions, standards that are accepted for crane structures and, obviously, not exceed them for specific types of crane loads and specific types of work performed by them on cargo handling) [20]:

Thus, have:

$$V = \frac{1}{2} \cdot \left\{C \cdot \left(\frac{1}{m_l} - \frac{1}{m_r}\right) \cdot \left[\frac{Q_l}{C} \cdot \tau_p + \left(\frac{Q_l + T\,exc}{m_r} + g *\right) \cdot \left(\frac{\tau_p^3}{12}\right)\right] + \left(\frac{Q_l + T\,exc}{m_r} - g\right) \cdot \tau_p\right\}. \qquad (46)$$

For the case of lifting the load "from the base"/"with pickup", the following initial conditions should be used to solve (7):

$$\xi|_{t=0} = 0; \quad \dot{\xi}\big|_{t=0} = v_0, \qquad (47)$$

where: v_0 – initial speed of steady-state movement of the system in the process of lifting the load.

The following considerations v_0 can be used to determine the value. When the weight of the load $Q_l = m_l \cdot g$ is on the base, it deforms the latter and the magnitude of the displacement $\tilde{x}_{II}$ of the base due to the force on it Q_l is $\tilde{x}_{II} = Q_l/C_{base}$, where C_{base} – is the stiffness coefficient of the base [18; 21]. When the load is detached from the base in this method of lifting it with a crane, all the potential energy that the load has as a result of interaction with this base and its deformation is transferred (due to the presence of the law of conservation of energy in mechanics) to the kinetic energy of the load movement with the initial speed v_0. Thus, there is the following correlation:

$$\frac{C_{base} \cdot \tilde{x}_p^2}{2} = \frac{m_l \cdot v_0^2}{2} \Rightarrow v_0 = \sqrt{\frac{m_l \cdot g^2}{C_{base}}} = g \cdot \sqrt{\frac{m_l}{C_{base}}} = \sqrt{\frac{Q_l \cdot g}{C_{base}}}. \qquad (48)$$

The general solution of (7) is again sought in the form of (9), but now considering the conditions (47), obtained:

$$A_1 = \left(-\frac{1}{\Omega^2}\right) \cdot \left(\frac{Q_l + T\,exc}{m_r} + \frac{Q_l}{m_l}\right); \quad A_2 = \frac{v_0}{\Omega}. \qquad (49)$$

Then the solution of $\xi(t)$ (9) is given as follows:

$$\xi(t) = \frac{2}{\Omega^2} \cdot \left\{\frac{Q_l + T\,exc}{m_r} + \frac{Q_l}{m_l}\right\} \cdot sin^2\left(\frac{\Omega t}{2}\right) + \frac{v_0}{\Omega} \cdot sin\,\Omega\,t. \qquad (50)$$

The force in the elastic element/rope, which can now be considered as the impact of the load on the load-carrying devices, will be:

$$P_{LC} = C \cdot \xi(t) = \frac{2C}{\Omega^2}\left\{\frac{Q_l + T\,exc}{m_r} + \frac{Q_l}{m_l}\right\} \cdot sin^2\left(\frac{\Omega t}{2}\right) + \frac{C \cdot v_0}{\Omega} \cdot sin\,\Omega\,t. \qquad (51)$$

Using the relations of elementary trigonometry, (51) can be represented as follows:

$$P_{LC}(t) = \frac{2C}{\Omega} \cdot sin\left(\frac{\Omega t}{2}\right) \cdot \left\{\frac{\left(\frac{Q_l + T\,exc}{m_r} + \frac{Q_l}{m_l}\right)}{\Omega} + v_0 \cdot cos\left(\frac{\Omega t}{2}\right)\right\}. \qquad (52)$$

Analysis of the relations (50)-(52) demonstrates that in this variant of lifting the load ("with pickup") at the time moments t^* (15) the value P_{LCmax} is even greater than in the case of lifting the load "from the weight", since now:

$$P_{LCmax}|_{t=t^*}l\frac{2m_l}{(m_r+m_l)}\bigg|_{exc}.\qquad(53)$$

Considering the arguments given above for (17), (18), now have that the dynamism coefficient takes the form:

$$K_d(t)|_{t=t^*}=2+\frac{2m_l}{(m_r+m_l)}\cdot\frac{T_{exc}}{Q_l}.\qquad(54)$$

Consequently, when lifting the load "from the base"/"with pickup", the value K_d at specific moments of time (t^*) acquires even greater values than when lifting "from the weight". Now define the law of motion $\xi(t)$ that satisfies the quality criterion (29), but with the lifting method "from the base"/"with pickup".

$$P_{LC}(t)=C\cdot\xi(t)=C\cdot v_0\cdot t+C\cdot\left\{\frac{Q_l+T_{exc}}{m_r}+g\right\}\cdot\left\{\frac{t^2}{2}-\frac{t^3}{3\tau_p}\right\}.\qquad(58)$$

For $K_d(t)$ have:

$$K_d(t)=\frac{P_{LC}^{(total)}}{Q_l}=\frac{Q_l+P_{LC}(t)}{Q_l}=1+\frac{P_{LC}(t)}{Q_l},\qquad(59)$$

where $P_{LC}^{(total)}$ – is the total force load of the load-carrying device: $P_{LC}^{(total)}=Q_l+P_{LC}(t)$, or:

$$K_d(t)|_{t=\tau_p}=K_d(\tau_p)=K\frac{C\cdot v_0\cdot\tau_p}{Q_l}\frac{C}{Q_l}\left\{\frac{Q_l+T_{exc}}{m_r}+g\right\}\frac{\tau_p^2}{6}_{dmax}.\qquad(61)$$

From (60) it is obvious that in this case $K_d(t)$ it is less than in the law of motion $\xi(t)$ (50). The latter, moreover, results in undesirable oscillations of the load on the rope. For $\xi(t)$ (57) there are no such fluctuations [23]. Define further the laws of motion $x_r(t)$, $x_l(t)$. For this purpose again using the laws and

Repeating the above algorithm for the method of lifting the load "from the weight" have the following results. Again $\xi(t)$ search in the form (32), since $\xi(t)$ must satisfy (31). But now the initial and final conditions change, i.e. instead of (33) have [22]:

$$K_d(t)|_{t=t^*}=2+\frac{2m_l}{(m_r+m_l)}\cdot\frac{T_{exc}}{Q_l}.\qquad(55)$$

Then for the coefficients of the spline by t function $\xi(t)$ (32) have:

$$a_1=0;\ a_2=v_0;\ 2a_3=\frac{Q_l+T_{exc}}{m_r}+g;\ a_4=-\frac{2a_3}{3\tau_p}.\ (56)$$

Thus, for the traffic quality criterion (29) or (30) to be satisfied, it is necessary that $\xi(t)$ the following law of change over time be satisfied t:

$$\xi(t)=v_0\cdot t+\left\{\frac{Q_l+T_{exc}}{m_r}+g\right\}\cdot\left\{\frac{t^2}{2}-\frac{t^3}{3\tau_p}\right\}.\qquad(57)$$

Then P_{LC} takes the form:

$$K_d(t)=1+\frac{C\cdot v_0\cdot t}{Q_l}+\frac{C}{Q_l}\cdot\left\{\frac{Q_l+T_{exc}}{m_r}+g\right\}\cdot\left\{\frac{t^2}{2}-\frac{t^3}{3\tau_p}\right\}.\ (60)$$

This coefficient reaches its maximum value at the moment $t=\tau_p$:

equations (25), (26) and initial conditions:

$$x_l(t)|_{t=0},\dot x_l(t)|_{t=0};\ \dot x_r(t)|_{t=0}=v_0;\ x_r(t)|_{t=0}=0.\ (62)$$

For $\ddot x_r(t)$ have equation (25), which must be solved under the conditions (62) at $\xi(t)$ (57). Have:

$$x_r(t)=\left(-\frac{C}{m_r}\right)\cdot\left\{\frac{v_0\cdot t^3}{6}+\left(\frac{Q_l+T_{exc}}{m_r}+g\right)\cdot\left(\frac{t^4}{24}-\frac{t^5}{60\cdot\tau_p}\right)\right\}+\left(\frac{Q_l+T_{exc}}{m_r}\right)\cdot\frac{t^2}{2}+v_0\cdot t.\qquad(63)$$

$$x_l(t)=\frac{(-gt^2)}{2}\cdot\left(\frac{C}{m_l}\right)\cdot\left\{\frac{v_0\cdot t^3}{6}+\left(\frac{Q_l+T_{exc}}{m_r}+g\right)\cdot\left(\frac{t^4}{24}-\frac{t^5}{60\tau_p}\right)\right\}.\qquad(64)$$

Since at where V – steady speed of lifting the load, it can be used to determine this circumstance τ_{Π}. Generally, the value of V is set by the operating standards of crane structures of a specific type and load

lifting mechanisms that are used in this case. Thus, to find τ_p can be used, for example, expression $x_l(t)$ (64), differentiate it in time t and find the value of this last expression at $t=\tau_p$. Then have:

$$\dot x_l(t)|_{t=\tau_p}=V\Leftrightarrow(-g\tau_p)+\left(\frac{C}{m_l}\right)\cdot\left\{\frac{v_0\cdot\tau_p^2}{2}+\left(\frac{Q_l+T_{exc}}{m_r}+g\right)\cdot\frac{\tau_p^3}{12}\right\}=V.\qquad(65)$$

The cubic equation (65) is obtained with regard to τ_{IP} is solved using the Cardano formulas. In contrast, if the value τ_{II} is given, using (65), it is easy explicitly to calculate the value V for a particular variant

of lifting the load "from the base"/"with pickup". In calculation in the case of dynamic loading of the crane loading device when lifting the load "with pickup" ("from the base"), other approaches can be used [1].

In particular, it allows neglecting the stiffness of one of the elements (ropes, since the elasticity of the metal structure of the crane is much higher than that of the ropes themselves, and the oscillations of the latter quickly damped) and consider only the elasticity of the second element of stiffness – the crane structure, that is, the mass of the crane $m_к$ and the load m_l are considered as one mass m (Fig. 2).
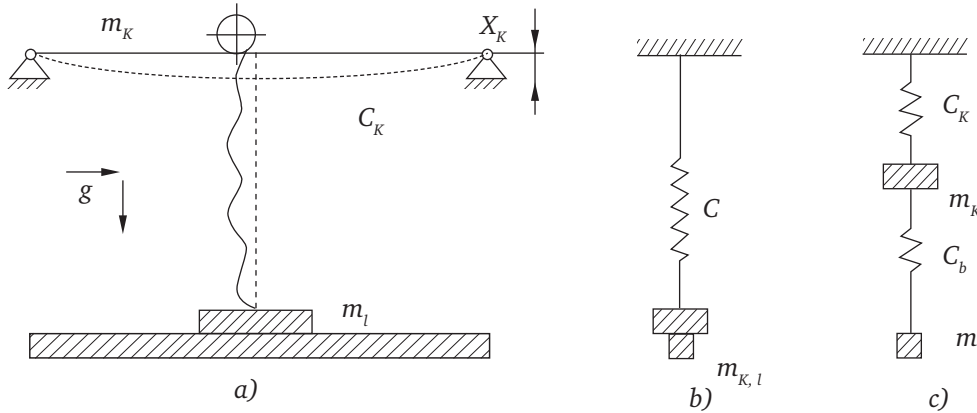

Figure 2. Scheme of dynamic loading of the load device when lifting the load "with pickup"
Note: a) on overhead cranes; b) and c) design schemes of single- and double-mass systems
Source: [1; 2]

Under the assumptions made, it can be considered that the load is lifted as follows. In the first stage, after turning on the engine. The rope slack is selected, in the second stage – elastic deformation of all structural elements (Fig. 2). The second stage continues until the force P_0 on the load-gripping devices, increasing from zero, becomes equal $Q_l = m_l \cdot g$. Only after that, in the third stage, the lifting of the load begins [18]. When moving $x_к$ the mass of the crane $m_к$ with rigidity $C_к$ (more precisely, the crane beam as part of the metal structure of the crane) kinetic energy:

$$W = \widetilde{m}_к \cdot \frac{\dot{x}_к{}^2}{2}, \widetilde{m}_к = m_l + m_к, \qquad (66)$$

but potential energy:

$$U = C_к \cdot x_к{}^2/2. \qquad (67)$$

The driving force here P, varies for different stages of lifting the load. The equation of motion of the system, which arises from the Lagrange equation of the second kind, has the form:

$$\widetilde{m}_к \cdot \ddot{x}_к + C_к \cdot x_к = P, \qquad (68)$$

its solution is as follows:

$$x_к = y_{ST} + A \cdot \sin r\, t + B \cdot \cos r\, t, r = \sqrt{\frac{C_к}{(m_к + m_l)}}, (69)$$

provided that at the beginning of the movement the following relations are satisfied for $x_к(t)$ and $\dot{x}_к(t)$:

$$x_к(t)|_{t=0} = y_{ST} = \frac{(m_l + m_к) \cdot g}{C_к}; \ \dot{x}_к|_{t=0} = v_0, \qquad (70)$$

where v_0 – the initial speed of lifting the load, i.e. the initial speed of movement of the rope when it is fully selected (its slack is eliminated); y_{ST} – deflection of the structure from a static load.

After setting the values of the coefficients A and L have:

$$\begin{cases} x_к = y_{ST} + y_{ST} \cdot \cos r\, t + \frac{v_0}{r} \cdot \sin r\, t; \\ \dot{x}_к = -r \cdot y_{ST} \cdot \sin r\, t + v_0 \cdot \cos r\, t; \\ \ddot{x}_к = -r^2 \cdot y_{ST} \cdot \cos r\, t - v_0 \cdot r \cdot \sin r\, t. \end{cases} \qquad (71)$$

P_{dyn} – the dynamic load applied to the load-carrying device takes the form:

$$P_{dyn} = m_l \cdot \ddot{x}_к = \left(\frac{Q_l}{g}\right) \cdot \{-r^2 \cdot y_{ST} \cdot \cos r\, t - v_0 \cdot r \cdot \sin r\, t\}. \qquad (72)$$

If the condition holds: $v_0 >> r \cdot y_{ST}$, then from (71) have:

$$P_{dyn} = -\left(\frac{Q_l}{g}\right) \cdot v_0 \cdot r \cdot \sin r\, t. \qquad (73)$$

The maximum value P_{dyn} in this case is obtained under the condition:

$$\sin r\, t = -1. \qquad (74)$$

This mode and condition are implemented at high initial load lifting speed (v_0). If the ratio: $v_0 << r \cdot y_{ST}$, then from (72) have:

$$P_{dyn} = -\left(\frac{Q_l}{g}\right) \cdot r^2 \cdot y_{ST} \cdot \cos r\, t. \qquad (75)$$

The maximum value P_{dyn} in this case is obtained under the condition:

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$$cos\, r\, t = -1. \qquad (76)$$

This mode and condition are implemented at

$$P_{dyn} = \left(-\frac{Q_l}{g}\right) \cdot \left\{\sqrt{v_0{}^2 \cdot r^2 + r^4 \cdot y_{ST}{}^2} \cdot sin(rt + \alpha)\right\}, \alpha = arctg\left\{\frac{r \cdot y_{ST}}{v_0}\right\}. \qquad (77)$$

The maximum value P_{dyn}, which is equal to:

$$P_{dyn}^{(max)} = \frac{Q_l}{g} \cdot \sqrt{v_0{}^2 \cdot r^2 + r^4 \cdot y_{ST}{}^2}, \qquad (78)$$

acquires at times $t\,*$,determined from the ratio:

low initial load lifting speed (v_0). In the general case (for arbitrary values of v_0) have:

$$r \cdot t_n * + arctg\left\{\frac{r \cdot y_{ST}}{v_0}\right\} = \frac{\pi}{2} \cdot (4n - 1), n \in N. \qquad (79)$$

The full load applied to the load-gripping device is as follows:

$$P_{LC} = Q_l + P_{dyn}^{(max)} = Q_l \cdot \left\{1 + \frac{1}{g} \cdot \sqrt{v_0{}^2 \cdot r^2 + r^4 \cdot y_{ST}{}^2}\right\},$$
$$K_d{}^{(max)} = 1 + \frac{1}{g} \cdot \sqrt{v_0{}^2 \cdot r^2 + r^4 \cdot y_{ST}{}^2} \qquad (80)$$

and when performing the relation that results in (73), (74), have:

$$P_{LC} = Q_l \cdot \left(1 + \frac{v_0}{g} \cdot r\right) = Q_l \cdot \left\{1 + \frac{v_0}{g} \cdot \sqrt{\frac{C_\kappa}{(m_\kappa + m_l)}}\right\}. \qquad (81)$$

Since $C_\kappa = Q_l/y_{ST} = \frac{m_l \cdot g}{y_{ST}}$, then from (81) can be obtained:

$$K_d{}^{(max)} = 1 + \frac{v_0}{g} \cdot \sqrt{\frac{C_\kappa}{(m_\kappa + m_l)}} = 1 + v_0 \cdot \sqrt{\frac{1}{g \cdot y_{ST}} \cdot \left\{\frac{m_l}{(m_\kappa + m_l)}\right\}}. \qquad (82)$$

Define further the law of motion $x_\kappa(t)$, for which there are no oscillations in the considered system and the initial conditions (70) and the criterion of quality of type motion are performed:

$$\left\{\frac{1}{\tau_p} \cdot \int_0^{\tau_p} (x_\kappa(t))^2 dt\right\}^{1/2} \Rightarrow min. \qquad (83)$$

Considering that in (68) $P = const$, condition (83) is performed at (Euler-Poisson equation):

$$x_\kappa^{(IV)} = 0. \qquad (84)$$

The solution of (84) is sought under the following terminal (initial and final) conditions of lifting the load "with pickup":

$$x_\kappa(t)|_{t=0} = y_{ST}; \ \dot{x}_\kappa(t)|_{t=0} = v_0; \ \ddot{x}_\kappa(t)|_{t=0} = \{(m_l + m_\kappa) \cdot g - C_\kappa \cdot y_{ST}\} = \frac{m_\kappa \cdot g}{(m_l + m_\kappa)}; \ \dot{x}_\kappa(t)|_{t=\tau_{II}} = V, \qquad (85)$$

where: V – steady-state speed of lifting the load after the transition process $(t \geq \tau_p)$.

Search for $x_\kappa(t)$, which satisfies equation (84) and conditions (85) in the form of a spline on t:

$$x_\kappa(t) = b_0 + b_1 \cdot t + b_2 \cdot t^2 + b_3 \cdot t^3. \qquad (86)$$

Then from (85) have:

$$b_0 = y_{ST}; \ b_1 = v_0; \ 2b_2 = \frac{m_\kappa \cdot g}{(m_l + m_\kappa)}; \ b_3 = -\frac{2b_2}{3\tau_p}, \qquad (87)$$

or:

$$x_\kappa(t) = y_{ST} + v_0 \cdot t + \frac{m_\kappa \cdot g}{2 \cdot (m_\kappa + m_l)} \cdot t^2 - \frac{m_\kappa \cdot g}{3\tau_{II} \cdot (m_l + m_\kappa)} \cdot t^3. \qquad (88)$$

The dynamism coefficient $K_d(t)$ has no oscillating character in the driving mode (88):

$$K_d(t) = \frac{Q_l + m_l \cdot \ddot{x}_\kappa}{Q_l} = 1 + \frac{\ddot{x}_\kappa}{g} = 1 + \frac{m_\kappa}{(m_\kappa + m_l)} \cdot \left(1 - \frac{2t}{\tau_p}\right). \quad (89)$$

Note that its maximum value K_d reaches at the initial moment of time $t = 0$:

$$K_d^{(max)}(t)\Big|_{t=0} = 1 + \frac{m_\kappa}{(m_\kappa + m_l)}. \qquad (90)$$

The obtained formulas (in the approximation of a single-mass model of lifting a load "with a pickup")

are quite simple and can be used in practical calculations, although, they do not consider the influence of the second stiffness element that exists in the system under consideration (Fig. 2). Accounting for it, the system should be considered as a biaxial system with two elastic couplings and, accordingly, as having two degrees of freedom of motion, with the corresponding superposition of oscillations at each of the frequencies and finding the maximum during several periods of oscillations. Using the Lagrange function for this problem allows writing in this case the following system of equations for x_l and x_κ:

$$\begin{cases} m_\kappa \cdot \ddot{x}_\kappa + C_\kappa \cdot x_\kappa + C_l \cdot (x_\kappa - x_l) = P_r - Q_l + Q_l = T_{exc} + Q_l; \; T_{exc} = P_r - Q_l; \; Q_l = m_l \cdot g; \\ m_l \cdot \ddot{x}_l - C_l \cdot x_\kappa + C_l \cdot x_l = - Q_l \end{cases} \quad (91)$$

where: P_r – the driving force of the lifting mechanism drive. Using the second equation of the system (91) can establish that:

$$x_\kappa = \frac{1}{C_l} \cdot (m_l \cdot \ddot{x}_l + C_l \cdot x_l + Q_l). \quad (92)$$

Then, from (92) for $\ddot{x}_\kappa$ have:

$$\ddot{x}_\kappa = \frac{1}{C_l} \cdot \left(m_l \cdot x_l^{(IV)} + C_l \cdot \ddot{x}_l \right). \quad (93)$$

Using the relations (92), (93), the first equation of the system (93) can be reduced to the equation for x_l the fourth order, which is a linear inhomogeneous differential equation:

$$x_l^{(IV)} + \ddot{x}_l \cdot \left\{ \frac{C_l}{m_l} + \frac{C_l + C_\kappa}{m_\kappa} \right\} + \frac{C_\kappa}{m_\kappa} \cdot \frac{C_l}{m_l} = \frac{C_l}{m_l \cdot m_\kappa} \cdot \left(T_{exc} - \frac{C_\kappa}{C_l} \cdot Q_l \right), \quad (94)$$

or:

$$x_l^{(IV)} + \ddot{x}_l \cdot \left\{ \Omega_l^2 + \Omega_{red}^2 \right\} + \Omega_\kappa^2 \cdot \Omega_l^2 \cdot x_l = \Omega_l^2 \cdot \frac{1}{m_\kappa} \cdot \left(T_{exc} - \frac{C_\kappa}{C_l} \cdot Q_l \right), \quad (95)$$

where: $\Omega_l^2 = \frac{C_l}{m_l}, \Omega_{red}^2 = \frac{C_l + C_\kappa}{m_\kappa}, \Omega_\kappa^2 = \frac{C_\kappa}{m_\kappa}$. Here introduced the following designations: C_l – stiffness of hoisting ropes and reduced to them the stiffness of the drive elements, N/m; $\Omega_{red} = \sqrt{\frac{C_l + C_\kappa}{m_\kappa}}$ reduced frequency, 1/s; C_κ – stiffness of the metal structure of the crane, N/m; m_κ – reduced weight of the metal structure of the crane, kg; m_l – weight of the load, kg [8; 9].

The partial frequencies of this system (in s⁻¹) $r_{1,2} \equiv \Omega_{1,2}$ can be obtained from the following relations:

$$\Omega_{1,2} = \left\{ \frac{\Omega_l^2 + \Omega_{red}^2}{2} \pm \sqrt{\frac{\left(\Omega_l^2 + \Omega_{red}^2\right)^2}{4} - \Omega_\kappa^2 \cdot \Omega_l^2} \right\}^{1/2}. \quad (96)$$

The duration of the time corresponding to the moment of the detachment of the load from the support (when lifting the load by the "with pickup" method) is obtained from the following transcendental equation [1]:

$$\tau_0 + \left(\frac{C_l}{C_\kappa \cdot \Omega_{red}} \right) \cdot sin(\Omega_{red}\tau_0) = g \cdot \Omega_{red}^2 / \left(v_0 \cdot \Omega_\kappa^2 \cdot \Omega_l^2 \right), \quad (97)$$

where: v_0 – speed of lifting the load after its separation from the surface of the support.

Quite often v_0 is identified with V – is the steady-state lifting speed of the load [1; 3], i.e. ($v_0 \approx V$) [3]. Although in the exact analysis of the process of lifting the load by the method "with pickup" v_0 is from (48). Notably, neglect the costs of thermal processes that occur in the

deformed foundation during its deformation and detachment of the load from it, i.e. neglect the heat losses that exist in the foundation during the restoration of its original, (almost) undeformed state. At $t > \tau_0$ starts lifting the load by the method "from the weight", which is defined above as a system of equations (4), where $m_r \equiv m_\kappa$. The solution of equation (95) can be given as follows:

$$\begin{cases} x_l(t) = A_1 \cdot sin \, \Omega_1 \, t + A_2 \cdot cos \, \Omega_1 \, t + A_3 \cdot sin \, \Omega_2 \, t + A_4 \cdot cos \, \Omega_2 \, t + X *, \\ X *= \frac{1}{C_\kappa} \cdot \left(T_{exc} - \frac{C_\kappa}{C_l} \cdot Q_l \right), \end{cases} \quad (98)$$

and the constants $A_i, i = \overline{(1,4)}$, are obtained from the following terminal (initial and final) conditions of lifting

the load by the "with pickup" method:

$$x_l(t)|_{t=0} = 0; \dot{x}_l(t)|_{t=0} = 0; \ddot{x}_l(t)|_{t=\tau_0} = -g; \dot{x}_l(t)|_{t=\tau_0} = v_0. \quad (99)$$

From (98), evidently, the load, which is lifted by the method "with a pickup", is inherent in oscillations with partial frequencies, $\Omega_1, \Omega_2,$ which causes inconvenience and dynamic overloading of the crane rope system during the implementation of the last series of loading and unloading operations [1]. Define what mode of movement x_l (t) can eliminate the above inconveniences. For this, it is required to consider the

system of equations (4), (6), which is reduced to equation (7), but for $t > \tau_0$ (when the load has already been detached from the support), with the following terminal (initial and final) conditions of motion (they are valid at the moments of time $t \in (\tau_0, \tau_p)$, where τ_p – the moment of time, when the speed of lifting the load reaches a steady state V, value at which a uniform movement (in particular, lifting) of the load is performed:

$$\xi(t)|_{t=\tau_0} = \frac{Q_l}{C}; \dot{\xi}(t)\big|_{t=\tau_0} = v_0; \ddot{\xi}(t)\big|_{t=\tau_0} = \frac{T_{exc}}{m_r}; \dot{\xi}(t)\big|_{t=\tau_p} = V. \tag{100}$$

From equation (7) it can be determined easily $\xi(t)$:

$$\xi(t) = \frac{1}{\Omega^2} \cdot \left\{ \left(\frac{Q_l + T_{exc}}{m_r} + \frac{Q_l}{m_l} \right) - \ddot{\xi} \right\}. \tag{101}$$

The following quality criterion (of this movement)

will be searched for:

$$\int_{\tau_0}^{\tau_p} \{\xi(t)\}^2 dt \Rightarrow min. \tag{102}$$

If you enter a new variable $t* = t - \tau_0$, then the criterion (102) and conditions (100) will be as follows:

$$\begin{cases} \int_0^{\tau_p - \tau_0} \{\xi(t* + \tau_0)\}^2 dt* \Rightarrow min; \\ \xi(t*)|_{t*=0} = \frac{Q_l}{C}; \ \dot{\xi}(t*)\big|_{t*=0} = v_0; \ \ddot{\xi}(t*)\big|_{t*=0} = \frac{T_{exc}}{m_r}; \ \dot{\xi}(t*)\big|_{t*=\tau_p - \tau_0} = V. \end{cases} \tag{103}$$

Using the substitution $\xi(t* + \tau_0)$ (101) and substituting it into the motion quality criterion (102), obtain the necessary condition for the realisation of this criterion (Euler-Poisson equation) of the following form:

$$\xi^{(IV)}(t* + \tau_0) = 0. \tag{104}$$

Consider the solution of this equation in the form of a spline of the third order by the argument

$t* + \tau_0 = t$. The same will be for $t* \geq 0$ or $t \geq \tau_0$ to find the solution of the equations accordingly:

$$\xi^{(IV)}(t*) = 0 \tag{105}$$

and:

$$\xi^{(IV)}(t) = 0. \tag{106}$$

Thus, for $\xi(t*)$ have:

$$\xi(t*) = d_0 + d_1 \cdot (t*)^1 + d_2 \cdot (t*)^2 + d_3 \cdot (t*)^3, t* \geq 0. \tag{107}$$

Considering the conditions (103) for $\xi(t*)$ (107), have:

$$\xi(t*) = \frac{Q_l}{C} + v_0 \cdot t* + \frac{T_{exc}}{2m_r} \cdot (t*)^2 + \left\{ \frac{(V - v_0) - \frac{T_{exc}}{m_r} \cdot (\tau_{II} - \tau_0)}{3 \cdot (\tau_p - \tau_0)^2} \right\} \cdot (t*)^3. \tag{108}$$

The dynamic component of the force that occurs in the rope system when lifting a load "with a

pickup" has the following form:

$$P_{LC}(t*) = C \cdot \xi(t*) = Q_l + C \cdot v_0 \cdot t* + \frac{C \cdot T_{exc}}{2m_r} \cdot (t*)^2 + C \cdot \left\{ \frac{(V - v_0) - \frac{T_{exc}}{m_r} \cdot (\tau_p - \tau_0)}{3 \cdot (\tau_p - \tau_0)^2} \right\} \cdot (t*)^3. \tag{109}$$

The dynamism coefficient $K_d(t*)$ has the following form:

$$K_d(t*) = \frac{P_{LC}(t*)}{m_l \cdot g} = 1 + \frac{C \cdot v_0 \cdot (t*)}{m_l \cdot g} + \frac{C \cdot T_{exc}}{2m_r \cdot m_l \cdot g} \cdot (t*)^2 + \frac{C \cdot \left\{ (V - v_0) - \frac{T_{exc}}{m_r} \cdot (\tau_p - \tau_0) \right\}}{m_l \cdot g \cdot 3 \cdot (\tau_p - \tau_0)^2} \cdot (t*)^3. \tag{110}$$

From (110) evident that $K_d(t*)$ it has no oscillatory character. The maximum value this value ac-

quires at $t* = \tau_p - \tau_0$, namely:

$$K_d^{(max)}(t*) = K_d(t*)|_{t*=\tau_p - \tau_0} = 1 + \frac{2C \cdot v_0 \cdot (\tau_p - \tau_0)}{3 \cdot m_l \cdot g} + \frac{C \cdot V \cdot (\tau_p - \tau_0)}{3 \cdot m_l \cdot g} + \frac{C \cdot T_{exc} \cdot (\tau_p - \tau_0)^2}{6 m_r \cdot m_l \cdot g}. \tag{111}$$

As a rule: $\tau_p \gg \tau_0$, $V \approx v_0$, thus from (111) have:

$$K_d^{(max)}(t*)\big|_{t*=\tau_p} = 1 + \frac{C \cdot V \cdot \tau_p}{m_l \cdot g} + \frac{C \cdot T_{exc} \cdot \tau_p^2}{6 m_r \cdot m_l \cdot g}. \tag{112}$$

Due to the small values of the terms included in (112), compared to unity, practically the value of the dynamism coefficient for this method of lifting the load (as, incidentally, for the case of lifting the load "from weight") differs little from unity (not more than 10%, exceeding unity).

Discussion

The quality of operation and functioning of the lifting mechanisms of bridge cranes largely depends on the proper optimisation of the dynamic loads of rope systems due to the handling of goods, and their efficiency is one of the most pressing issues of our time, and some problems require immediate solutions. For example, the mechanisms with which it can be performed installation and transporting works are quite dangerous and cumbersome during operation, this is

a significant problem in using these mechanisms, and it is essential to increase the efficiency of exploring these problems to solve them as soon as possible. Most of the lifting mechanisms at the moment are outdated both technically and morally, and the significant amount of resource cranes has greatly decreased.

According to the results of recent studies by Q. Guo et al. [24], the constant use of lifting machines in such a technically worn-out condition can result in their destruction and breakdowns, which causes the termination of the cargo flows used by them. On the one hand, the provision of high productivity of shifting loads in the river and sea warehouses, ports, construction sites, and production workshops is combined with increased dynamic loads in the details of cranes, they are one of the main reasons for their transition to the limiting environment, after which the efficient and high-quality operation of the mechanism is unrealistic. The most urgent issue that must be considered during the development and operation of the cargo lifting facility is the energy efficiency of this mechanism.

The entire mechanism of lifting machines was analysed, and as a result, it was decided that for the application of various constructions, especially theoretical ones, it is necessary to have a basic knowledge of these objects to indicate the physical properties of this device and their number, which will allow understanding the process of operation of lifting mechanisms under the given conditions of their use. The problem of energy efficiency of lifting mechanisms was almost indecisive before, as there were not enough available means of energy saving in the crane drive, and they appeared recently. Complex vibrations have an adverse impact on the stability of the elements of lifting machines, which results in numerous destructions.

According to the definition of H.M. Omar et al. [25], gantry cranes are most often used for the construction of civil and industrial facilities. The features of operation of these lifting mechanisms are that a large part of the work process is represented by transient modes, namely starting and deceleration, machines for changing the departure and lifting loads. Thus, the productivity of the crane is connected with the duration of the transitional processes of the machines for changing the outreach and lifting loads.

It indicates that in the design and modelling of gantry cranes used for the construction of various structures, it is required to consider all the features of the mechanism and its modes that affect the quality of operation and ensure proper performance. Efforts to reduce the duration of transients in the departure change machine are not successful due to the necessity to reduce the load fluctuations that occur during transients. All these damages the operational performance and reliability of gourd cranes, and, in general, lifting and transporting mechanisms.

Researcher Y. Liu [26] determined that there are almost no systems for monitoring the energy consumption of lifting mechanisms in Ukraine, although lifting machines, compared to other industrial equipment and mechanisms, have low efficiency and are one of the most energy-efficient mechanisms. Very large energy losses in crane electric drives occur through the outdated control systems of drives, inefficient cycles of operation of machines, and their worn-out technical condition. In general, this applies to 80% of cranes that have reached the end of their regulatory service life.

But when modelling lifting mechanisms and other industrial equipment, using this class of energy consumption monitoring systems, allows for improving the work of these machines, due to the massive number of receiving elements and the large number of parts that are involved in the process, thus, it is very necessary to consider the specific features of using this type of mechanisms and systems, timely investigation of data and possible causes of problems, for further prospective development of using lifting machines and monitoring systems in Ukraine. It is essential to draw attention to the development of systems for monitoring the energy consumption of cranes, considering the regulatory requirements for their safe operation. To improve the reliability of the operation of lifting mechanisms, it is required to reduce the dynamic loads on their body and drive parts.

C.M. Niu et al. [27] identified that the cost of energy resources is constantly increasing, which optimises research to reduce energy consumption when lifting or lowering loads. Suboptimal and unreliable choices of cycles of operation of the lifting machine, even in the presence of innovative hardware, can cause energy overruns during the unloading and loading processes. The investigation and resolution of these contradictions can be promising only using the mechatronic approach, through which lifting mechanisms are presented as a synergistic combination of electrical, hydraulic, mechanical and electronic components.

The results of this mechatronic approach to resolving contradictions were analysed and explored more precisely, it can be concluded that the efficiency of using innovative designs of lifting mechanisms and support apparatus, their use allows for keeping energy overrun. The main role in the modelling and design of mechatronic crane systems is the development and calculation of software that allows the implementation of control of some individual mechanisms, devices and parts. During start-up and deceleration of drive devices, i.e. transient modes, dynamic loads dangerous for lifting mechanisms appear.

M. Ziaei et al. [28] demonstrated that such transient modes in cranes have an impact on the energy and dynamic properties and performance of the crane. For a load-lifting machine, the dynamic cycles that occur during the transitional modes of movement, i.e. lifting or lowering the load, have an impact on the magnitude of the moment and force loads in the flexible

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suspension and drive of the machine. The vibrations in the flexible suspension are transferred to the crane boom and load it with additional bending moments.

These modes may have a specific modification, it happens due to the specific features of the cranes that are observed and considered. It is essential to use the tower crane in such a way that the oscillatory dynamic cycles in its mechanisms are reduced during transient modes of movement. Small elimination of dynamic loads during the start-up and deceleration of gourd cranes helped to increase the overhaul time of these cranes many times.

As noted by N. Zhao et al. [29], modern monitoring systems operate on the established mechanism link between the replacement of energy losses during the operation of lifting mechanisms, namely cranes, and their technological condition. At the time of operation of cranes, it changes a lot: the position of the crane runway, flanges and rims of the running wheel, brake linings, brake pulleys, rope block, drum, bearing and various friction units in crane machines, the position of the electric motor winding and the resistance of the separate relay and contact equipment, the electromagnetic pusher coil, the viscosity of the operating emulsion in gearboxes and electrohydraulic brakes. Thus, as a result of these work processes, the energy consumption of lifting mechanisms changes.

To optimise the energy performance of lifting mechanisms and their electric drives, dynamic loads and kinematic parameters of cranes, it is required to develop mathematical models that can consider transient cycles in lifting electric drives, structural vibrations, loosening of loads and represent a set of nonlinear differential equations. It is essential to increase funding and improve the skills of employees, to start introducing new technologies to improve the design and modelling of lifting machines and to reduce errors in the operation of these mechanisms.

ways are substantiated: "from weight" and "with pick-up"/"from the base". The laws of motion of the elements of these models within the one- and two-mass calculation schemes are determined, which allows for avoiding fluctuations of the load and significantly (in some cases, several times) reducing the value of the dynamism coefficients. In the future, it is essential to improve, according to the authors of this work, the investigation of the processes of lifting loads by overhead cranes in various ways by considering the discrete-continuous properties of systems and lifting mechanisms designed for this purpose, and to improve control processes through using modern mechatronic control systems.

The main problems of increasing the efficiency and optimisation of dynamic loads of rope systems of lifting mechanisms of overhead cranes during cargo handling are mediocre training, problems of proper modelling and design of monitoring systems, multiple errors in the preparation for the operation of lifting machines, these problems are and will be relevant and require further investigation. The obtained results demonstrate that it is essential to be able to eliminate uncertainties in the process of functioning lifting machines if a specific scheme and rules are followed when operating the lifting mechanisms of overhead cranes. In this research, the purpose of the study was achieved, namely, the dynamic loads that arise in the rope systems of lifting machines of bridge cranes during start-up and deceleration were analysed, and efficient ways to help reduce these loads were found, the magnitude and nature of changes in dynamic loads in the elements of the lifting mechanisms of bridge cranes were identified, a comprehensive dynamic analysis of the lifting mechanisms of the bridge crane and its elastic elements was performed. To improve these mechanisms in the country, it is essential to increase the use and development of lifting machines, especially overhead cranes. In addition, it is necessary to draw attention to the quality of the elements and introduce newly developed methods for their quality development.

Conclusions

The physical-mechanical models for the analysis of the process of lifting loads by overhead cranes in two

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Оптимізація динамічних навантажень канатних систем вантажопідйомних механізмів мостових кранів при обробці вантажів

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Анотація.

Актуальність. У даному дослідженні приділена значна увага аналізу динамічних навантажень, які виникають у канатних системах вантажопідйомних механізмів мостових кранів під час пуску та гальмування, а також зменшенню цих навантажень.

Мета. Для виявлення величини та характеру зміни динамічних навантажень у елементах вантажопідйомних механізмів мостових кранів проведений всебічний динамічний аналіз саме механізму підйому мостового крану та його пружних елементів (канатних систем).

Методи. Динамічний аналіз вказаних вище механізмів та систем проведено на обґрунтованих математичних моделях кранів мостового типу (одно- та двомасових).

Результати. Аналіз отриманих розрахунків математичних моделей механізму підйому вантажу мостових кранів показав, що динамічні навантаження, які діють на елементи конструкцій та приводних механізмів, мають коливний характер і по величні співставлені зі статичними навантаженнями. Тому для зменшення динамічних навантажень в елементах конструкцій (зокрема, у канатах) під час перехідних процесів у таких вантажопідйомних механізмах мостових кранів запропоновано провести оптимізацію режимів руху їх приводних механізмів. Для мінімізації інтегральних функціоналів використовувались методи класичного варіаційного числення, математичної фізики та диференціальні рівняння, які моделюють динаміку процесів навантаження канатних систем та приводів мостових кранів, а також були враховані термінальні (початкові та кінцеві умови руху подібних систем), що дозволило однозначно розв'язати оптимізаційну задачу. Важливе місце в такій оптимізації займає вибір критерію оптимізації. Серед таких критеріїв використовувались інтегральні критерії оптимізації. У якості таких інтегральних критеріїв оптимізації використовувались середньоквадратичні значення діючих навантажень у пружних елементах (канатах) мостових кранів.

Висновки. Такі інтегральні критерії являють собою інтегральні функціонали, які, як правило, відображають небажані властивості машин та їхніх механізмів, тому вони підлягають мінімізації

Ключові слова: динамічна оптимізація, канатні системи, вантажопідйомні механізми, мостові крани, обробка вантажів, перехідні процеси