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Efficient Questioning in Teaching Mathematics: Teachers' Attitudes and Practices

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Abstract: The paper is an exposure to the latest practices of questioning in teaching and learning math (TLM) on the basis of psychological-semiotic approach. Emphasis is placed on the degree of semantic support of the expected answer when formulating educational questions. The paper explored: whether teachers are able to distinguish between types of questions and to use them in sync with didactic purpose; what types of questions teachers consider to be the most effective; what factors influence this process. To achieve these goals, survey-based research was conducted among 173 high school mathematics teachers across Ukraine. The research proved that, in their majority, teachers are able to correctly distinguish among the types of the questions offered. According to the teachers, questions with full semantic support for the answer are less useful in TLM. The study showed teachers' lack of ability to identify the goals of the questions. There has been revealed a gap between the teachers' attitude to the expedience of using questions with several possible answers and the practice of their implementation in TLM. The study yielded 35 variables characterizing the current status of the problem. They were optimized to 13 factors. It was stated that questioning should comply with the content of educational material, and the questions formulated with the use of topical vocabulary known to the students are viewed as most cost-effective. The research revealed the significant impact of the number of questions that teachers or students ask. The factor of primary importance appears to be that of time.

Keywords: *Educational questioning in teaching mathematics; functions of questions; psychological-semiotic approach; semantic support in questions' wording.*

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1. Introduction

The successful process of learning Mathematics is inseparable from asking questions and finding answers to them. Questions in teaching Mathematics perform different functions (Kulbyakina, & Zotova, 2004) epistemological, praxeological, evaluative-reflexive, monitoring-and-evaluation functions, the function of organizing the content of education, the function of organizing all stages of the learning cycle (didactic cycle). The epistemological function of questions is implemented in it that questions help students in cognizing the real world they live in, which further extrapolates on the way how they cognize abstractions by which the science of mathematics operates.

The praxeological function of questions is manifested in the fact that with their help, students master different modes of mathematical activities and gain experience in performing them. The evaluative-reflexive function of questions furnishes teacher's analysis of students' educational achievements and results of their educational and cognitive activity and, respectively, tudents' self-analysis and self-evaluation. The monitoring-and-evaluation function of questions is related to providing control over the process of students' progress in learning with the help of questions. This function is related to the function of organizing all stages of the learning cycle, but, in our opinion, it ought to be considered separately.

In teaching mathematics, educators actualize these functions in different ways. Some teachers practice "direct" learning, i.e., provide a complete indicative basis for any method of mathematical activity. In this case, questions implement mainly the monitoring-and-evaluation function and the function of organizing all stages of the learning cycle. Other functions of questions are not fully realized in this model of learning.

In educational practice, we can often find another model of asking questions, when the teacher manages the process of moving students "into the zone of proximal development" (according to L. Vygotsky) with the help of questions. Teacher's questions partly provide students with the method of mathematical activity. In this model questioning is held in the form of dialogue or polylogue. This way of asking questions creates the preconditions for a fuller implementation of their functions discussed above.

The third model of asking questions takes place, when the leading place in learning belongs to students' questions. The indicative basis of mathematical activity in this model of questioning is not provided to students, but they build and master it independently through asking their own questions to the teacher. It takes the mastery of the teacher to ask questions and thus encourage students to formulate their own questions, which will gradually and step by step move them to the desired result – finding a way to act or a way to think in order to solve the problem. However, in this model, the teacher's questions implement all the aforementioned functions.

The paper aims at identifying and analyzing the latest practices of employiong questions in teaching Math to high school students.

To achieve this goal, we seek answers to several research questions: 1) whether teachers are able to distinguish between types of questions according to their didactic purpose; 2) whether they understand how to use different types of questions according to their didactic purpose; what types of questions teachers consider the most effective in teaching; 3) how teachers use questions on the content of educational material in the practice of teaching mathematics to students and what factors influence this process.

2. Literature Review

The problem of questions in teaching mathematics is what scientists and practicing teachers keep their focus on. Clear and balanced questions help teacher establish vibrant educational setting, at the same time promoting students' understanding of mathematics and correcting the mistaken notions (Costa & Kallick, 2000). We share the position of scientists (Kulbyakina, & Zotova, 2004; Lebedeva, 2003) who emphasize that questions are not a universal tool, they are rather a subtle pedagogical tool that has its own specifics, which largely determines their didactic value. One of the main specific features of educational questions is that they always give a stimulus to mental processes which are initiated when the question is asked and keep active for a long time (NCTM, 2000, 2014). At this time, the search for the answer is accompanied by active mental activity (Lebedeva, 2003).

2.1. Impact of Questions in Teaching and Learning Mathematics

Many studies (Black, 2001; Chazan, 2000; Ellis, 1993; Franke et al., 2009; Moberg, 2008; Piccolo et al., 2008; Wood, 1998) focus on the impact of questions in teaching and learning Mathematics (TLM). They include a shot at reexamining instructional questioning through Hans-Georg Gadamer's philosophical hermeneutics (Bingham, 2005); advocating an inquiry based (question asking) instructional model (Sigel & Saunders, 1977); promoting such instructional model in traditional TLM (Davis, 1997), in Self-Regulated Learning (NCTM, 2000; Pape et al., 2003) in the Questioning Understandings to Empowering Student Thinking (Qu:Est) Instructional Strategy model (Dantonio, & Beisenherz, 2001); why or how questions and their role in support students developing their explanations (Ingram et al., 2019).

The consensus is: teaching ought to be mapped being focused on proper particulars. There should be no neglect in sticking to instructional goals, keeping in mind varying goals, and the educator's personal way of asking math-focused questions at the lesson. To cover the full context of a question, the teacher should keep in mind (Fusco, 2012): 1) what relationship the question has to the subject; 2) how the question relates to the main concepts that are under consideration in class; 3) what learning experience the students have to understand the question better; 4) what connection between the discussed questions there exists. Central to the art of posing questions are: choosing proper moments for this activity, and how many times every question sounds in class (Aizikovitsh-Udi & Star, 2011).

The wide range of educational theoretical studies and practices focuses on the notions of "True Questions", "Good Questions", "Good Question-asking Skills", and "Focus Pattern and the Funnel Pattern of Communication in Math Class". Practical implementation of instructional questions was analyzed by Bingham (2005) through the prism of Gadamer' philosophical hermeneutics. Bingham viewed instructional questions and statements from the standpoint of hermeneutics and indicated distinctive and general features of the two. The scholar itemized Hans-Georg Gadamer's concept of the 'true question' and attempted reassessing the philosophy of instructional questioning putting at the forefront three basic demands: such questions ought to be non-superficial, not too simple or excessively schematic. Sigel & Saunders (1977) provided a conceptual base for using an inquiry-based (questioning) instructional model. Frager (1979) examined well-known questioning strategies built on question classification systems (hierarchical, non-context bound; hierarchical, context bound; non-hierarchical, non-context bound; and non-hierarchical, context bound). Aizikovitsh-Udi & Star (2011) came up with the idea of inventing some other variations of questions which would motivate the educators to make use of a wide spectrum of educational approaches. The authors named the question aimed at receiving a mathematical answer without emphasizing the student's individuality as "technical" one. By contrast, the questions requiring more explanation and argumentation were named as "investigative". The math field-workers affirm that the latter may be put without inferring any investigation activities.

Discussing the types of questions, the authors (Costa & Kallick, 2000; Schuster, & Anderson, 2005; Sullivan, & Lilburn, 2002) use the term "Good Question". The authors consider "Good questions" to be open-ended ones. The latter are open-ended concerning either the possible response or the approach itself. The answers to such questions are to be comprehensive and information-rich. Mere facts and operations are not enough, the questions of this type should invite establishing connections and synthesizing the information. Open-ended questions call for full answer inspiring students for reasoning and reflecting. Also, they eliminate the appearing of banal and predictable answers and even contain fresh and sometimes surprising ideas, which helps teachers and students build a report of mutual understanding and cooperation in knowledge acquisition. Teachers can mine their students' answers for the grains of gold, while students learn about their own mental potential. The authors discuss the idea that the resourse of these questions lies in their power to incentivize students to ponder on the topic and then, maybe, make up their own questions being driven by a new interest. Open-ended questions help students trace the ways of thinking and understand these ways, they trigger mathematical curiosity. Studens go deep into the contents, based on their new expertize they start making predictions and are able to explain what was driving them in their arriving at some answers and conclusions. Such questions aid in building bridges of understanding and fruitful mutual cooperation between teachers and their students. Examples of such questions for teaching and learning calculus are presented in http://math.colorado.edu/activecalc/

Kress (2017) explicates 6 educational questions that are basic when dealing with complex math problems (NCTM, 2014; Brahier et al., 2014), and are in sync with active learning principles (Webb, 2016). These 6 questions are different from those designed by Polya (1945), but they adhere to the same approach in solving problems. In other words, they can be made use of while dealing with a broad spectrum of problems without sticking to specifically math steps when solving specific problems. The questions we speak about are: What do you observe (notice)? What further details would you ask for? What can you make out of it? How can you check whether what you have done and/or your answer are/is relevant and correct? Could there be another way of dealing with the problem? What commentary could you give on the problem and what additional knowledge/info would you like to possess? We fully agree with this approach. Such a sequence of questions allows teacher to organize and guide the cognitive process, to give students a foothold in educational and cognitive activities. However, there is a lack of issues in this sequence of questions that update and summarize the available mathematical experience of students. They are necessary for the teacher to be able to identify quantitative and qualitative characteristics of student experience (purely mathematical or from life). Such questions can be called "starting" or "preparatory" (Kulbyakina & Zotova, 2004). For example, when studying the material "Units of measurement" such questions include: "What units of length do you know?", "What relationship between them do you know?", "What can you measure using these units?".

However, even the most perfect system of question classification cannot function without leading questions in the mathematics classroom. More often than not, asking and answering questions is interconnected with the train of students' thoughts and their insight into the problem. Clear-cut and well thought-out questions assist in assessing and developing students' reasoning capabilities, they help learners grasp math ideas and establish relationships between math concepts (Brahier et al., 2014; NCTM, 2014). It is necessary for the teachers to understand the importance of "Good Question-asking Skills" in mathematics lessons (Sullivan, & Lilburn, 2002). As we have already pointed out, the way the teacher asks questions characterizes the teacher themselves. Inquiry-based (question asking) instructional model reveals two categories of teachers – the Conserving Teacher and the Leveraging Teacher (Aizikovitsh-Udi & Star, 2011). The Conserving Teacher is the representative of traditional patterns of teaching, in which they are the central figure monitoring all learning processes. The students get well-formulated and precise instructions, the teacher providing them with necessary details. The answers are assessed by the teacher, the teacher being the main authority in the classroom. No discussion is encouraged. The majority of the questions rotate around math notions and concepts without expanding math problems on any other spheres of human life and activities and are meant to get direct and correct answers and not give the teacher any idea about the ways how their students arrived at such answers. Both, teacher and students, do not expand on their questions and answers. No creativity is found. As for the Leveraging Teacher, the situation is different. The teacher's role during the lesson is central, still they do not domineer, they guide the students through explaining rules and laws and pushing their students to seeking their own ways of solving math problems. Debate is encouraged. Asking questions is aimed at assessing the level of flexibility of students' reasoning. The prevailing motives of teacher-student cooperation in such class are creativity and thirst for knowledge. The pattern when the teacher asks leading and clarifying questions which help students spot and correct their mistakes is termed by Wood, 1998 'the focused pattern' (the responsibility for finding solutions is laid in the students with the teacher assisting them by focusing on meaningful aspects of the problem. Besides, Wood puts a finger on the 'funnel' pattern students arrive at expected and predetermined answers by the teacher who directs and leads them to the answer).

The researchers (Davydov, 2019; Kulbyakina & Zotova, 2004; Lebedeva, 2003) analyze questions according to their two characteristics: 1) the degree of their subjective difficulty for students, and 2) the objective complexity of the question itself. The degree of subjective difficulty of the question for students is connected with the "white spots" in the student's thesaurus, gaps in their knowledge. Researchers associate the degree of objective complexity of the question itself with the number of logical steps to obtain the correct answer. However, the text of the question itself can serve as an auxiliary or inhibitory factor. The question is clear to students if it is concise and put correctly, has one focus and is not rhetorical. They do not spend extra mental effort to understand or remember it. Such aspects of the questions are considered by the semiotic approach. Our study will be based on the principles of this approach.

2.2. The Texts of the Questions

Textual forms of the questions are significant symbolic means for students' and teacher's cognitive activity. Decoding the content covered by questions' text shells correlates with the characteristics of the semantic field within which the question is built. This semantic field ought to sufficiently intersect with the students' thesaurus, not to cause misunderstanding and not to bring the student to a deadlock.

In this context, we propose to group the questions as follows:

1) questions, that have full semantic support for the answer (e.g., "Is it correct that the *n*th power of the product is equal to the product of the *n*th power of factors?");

2) questions that have incomplete semantic support for the answer (e.g., "The product of which powers of factors is equal to the *n*th degree of the product?");

3) questions that do not have semantic support for the answer (e.g., "How is the property of the *n*th degree of the product of several factors formulated?").

We distinguish the following three groups of the questions according to their dominant function:

1) questions aimed at the organization of knowledge consolidation (reproduction of the studied material, the initial systematization of concepts and facts, the formation of skills);

2) questions that contribute to mastering the techniques of logical thinking and experience of creative activity (independent work on analysis, synthesis, comparison, generalization; the formation of assessments, conclusions; deepening of the knowledge system: classification, specification, systematization);

3) questions that require the application of the acquired knowledge (performing independent works, mastering skills and abilities).

The texts of such questions differ significantly from each other. The teacher must be able to distinguish and recognize these types of questions in order to use them in a didactically balanced way in TLM. Therefore, it is important to clarify whether teachers are able to distinguish between these types of questions and make full use of their potential in teaching.

3. Methodology

To achieve the goal set, a survey of Mathematics teachers was conducted on the problem of using questions in TLM.

3.1. Study Design

The research was carried out in Ukraine and Ukrainian teachers representing different geographical regions of Ukraine participated in it. 173 Mathematics teachers from the cities of Cherkasy, Dnipro, Kropyvnytskyi, and other regions of Ukraine constitute its total coverage sample. 57.8% of respondents work in urban schools and 42.2% in rural areas. Work experience as a Mathematics teacher in the survey participants is as follows: 1-5 years (9.8%), 6-10 years (8.8%), 11-15 and 16-20 years (12.1% each), 21-25 years (10.4%), 25 years and more (46.8%). The respondents are approximately evenly distributed by experience in middle and secondary school: 85% have experience in grades 5-6, 87.3% – in grades 7-9, 85.5% – in grades 10-11.

3.2. Data Collection

To gauge the teachers' views of and attitudes to such math class activity as asking questions, the researches developed the literature-backed standard closed ended questionnaire. It took one month to collect the data which was then processed and analyzed manually by simple statistical methods with the application of the SPSS 23.0 software package. The data is presented in tables, graphs and figures.

The study did not require ethics committee approval. The survey was conducted anonymously, randomly, no personal data were collected or used.

Answers to the questionnaire were provided only by those who wished to participate in the survey.

4. Results

4.1. Overall results

In general, 100% of respondents use questions in TLM. Table 1 shows the respondents' answers regarding the intended purpose of questions (Table 1).

Table 1. Intended purpose of questions in TLM

Description of questions Questions in teaching Mathematics …	Column A (*f*)
… require students not only to apply the acquired knowledge, but also encourage generalizations and connections (interdisciplinary and intradisciplinary)	78%
… help students understand the essence of mathematics science	65.9%
… create conditions for students to understand and realize their gaps or inaccuracies and mistakes in knowledge	64.7%

Source: Author's own conception

Thus, we record the focus of teachers on the implementation of epistemological, praxeological, evaluative-reflexive function of questions in practice of teaching Mathematics. On average, according to respondents, 42.2% of questions during the lesson arise on the content of educational material. Teachers ask most questions (from 6 and more) during the consolidation and application of new educational information (60.7%), during the repetition of theoretical information (56.6%) and in the process of updating the basic skills of students (46.8%). Teachers ask the least questions (up to 5) during checking homework (76.5%) and during reflection and summarizing the lesson (56.1%).

Histograms (Fig. 1-4) show the frequency of respondents' answers to the question: "How many questions do you use on average when updating basic knowledge?" (Fig. 1), "How many questions on average do you use when repeating theoretical information?" (Fig. 2), "How many questions on average do you use when explaining new material?" (Fig. 3), "How many questions on average do you use when consolidating new concepts, facts, methods of activity?" (Fig. 4).

Thus, the survey confirmed the implementation of the organizational function of questions in the practice of teaching Mathematics to students by teachers. One of the focuses of the survey was to record the types of questions that teachers consider most effective in teaching. According to 50.9% of the teachers surveyed, questions should be formulated from words and terms that are understandable to

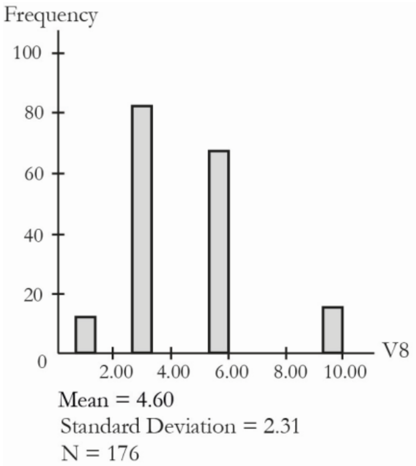

Fig. 1. The frequency of respondents' answers to the question: "How many questions do you use on average when updating basic knowledge?"
Source: Author's own conception

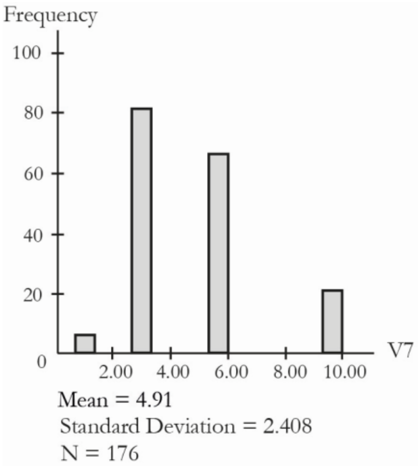

Fig. 2. The frequency of respondents' answers to the question: "How many questions on average do you use when repeating theoretical information?"
Source: Author's own conception

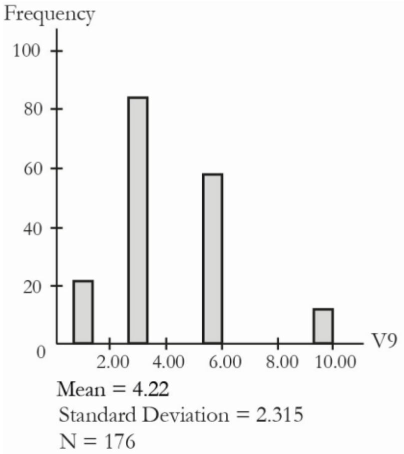

Fig. 3. The frequency of respondents' answers to the question: "How many questions on average do you use when explaining new material?"
Source: Author's own conception

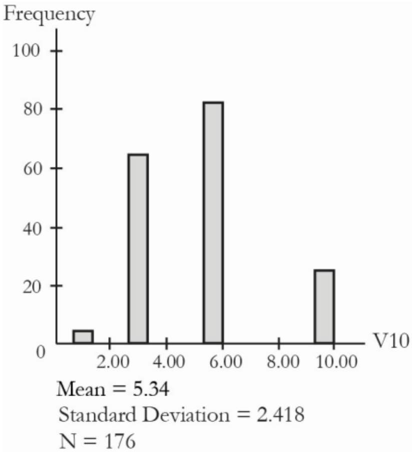

Fig. 4. The frequency of respondents' answers to the question: "How many questions on average do you use when consolidating new concepts, facts, methods of activity?"
Source: Author's own conception

the student; 24.3% of respondents emphasize the effectiveness of questions that involve several different ways to get an answer, but the use of questions that involve several answers, is appropriate only in the opinion of 9.8% of the teachers surveyed.

The survey explicated that math teachers tend to undervalue the positive power of the questions with several possible answers or those that offer several reasoning patterns for arriving at a correct answer.

One of the focuses of the survey was to find out whether teachers are able to differentiate questions according to the representation degree of semantic support in their formulation in order to answer them. To this purpose, the questionnaire provided relevant examples of questions (Q_{13}, Q_{14}, Q_{15}). The results are presented in Table 2.

Table 2. Survey results on teachers' ability to distinguish between types of questions

	Q_{13}	Q_{14}	Q_{15}
Frequency of incorrect answer	63	60	84
Frequency of correct	113	116	92

answer			
Average	0.66	0.68	0.53
Median	1.00	1.00	1.00
Dispersion	0.231	0.226	0.251
Standard deviation	0.481	0.475	0.501

Source: Author's own conception

Questions with full semantic support for the answer (Q_{13}) were correctly indicated by 63.3% of respondents; questions with incomplete semantic support for the answer (Q_{14}), – 65.3%; questions without semantic support for the answer (Q_{15}), – 51.4% of the teachers surveyed.

According to 22.5% of the teachers surveyed, questions should always contain semantic support for the answer; the rest of the respondents (75.5%) indicate that such questions are better to use unsystematically, from time to time.

In the questionnaire, it was offered to indicate the intended purpose (according to the accepted classification) of the following options for formulating questions related to the concept of distance from a point to a line: 1) "What is the distance from a point to a line?" ($Q_{16\text{-}1}$); 2) "How is the definition of the distance from a point to a line formulated?" ($Q_{16\text{-}2}$); 3) "Is it true that the distance from a point to a line is the length of the perpendicular drawn from a given point to a given line?" ($Q_{16\text{-}3}$); 4) "What is the difference between the concepts of distance from a point to a line and a perpendicular drawn from a point to a line?" ($Q_{16\text{-}4}$); 5) "How to find the distance from a point to a line?" ($Q_{16\text{-}5}$). Table 3 presents the statistical data.

Table 3. The results of the questionnaire on teachers' ability to distinguish the intended purpose of questions

	$Q_{16\text{-}1}$	$Q_{16\text{-}2}$	$Q_{16\text{-}3}$	$Q_{16\text{-}4}$	$Q_{16\text{-}5}$
Frequency of incorrect answer	72	98	92	87	119
Frequency of correct answer	104	78	84	89	57
Average	0.60	0.44	0.47	0.51	0.30
Median	1.00	0.00	0.00	1.00	0.00
Dispersion	0.243	0.248	0.251	0.251	0.220
Standard deviation	0.493	0.498	0.501	0.501	0.469

Source: Author's own conception

We consider the ways of reasoning to answer the question in details. The first two questions (Q_{16-1}, Q_{16-2}) direct students to reproduce the definition of the distance from a point to a line. Thus, according to the above classification, they are questions aimed at organizing the consolidation of knowledge (reproduction of the studied material, the initial systematization of concepts and facts, the formation of skills). However, from the standpoint of the difficulty of constructing the answer, they are not equivalent.

The first question (Q_{16-1}) gives the students some clue, at least, allows them to use a certain part of the text of the question to build the answer: "The distance from a point to a line is called ...". During the survey, 58.4% of respondents correctly indicated the intended purpose of this question.

The second formulation (Q_{16-2}) requires additional mental work of the student. Before formulating a definition, it is necessary to decode the term content of "definition", "formulation of a definition", to remember how to construct a definition and its characteristic verbal constructions so that, at least, start responding. In addition, if the first question allows students to answer "in their own words", the second question requires the formulation of a strict definition. All this is the evidence indicating that that the first question (Q_{16-1}) should be considered easier for students than the second one (Q_{16-2}). 43.4% of the teachers surveyed correctly indicated the intended purpose of these questions.

In order to answer the third (Q_{16-3}) and the fourth questions (Q_{16-4}), it is necessary not only to remember the definition of the distance from a point to a line, but also to compare it with the text of the relevant part of the question, to analyze not only the shells of both texts, but also, to compare them in content. Thus, these two questions should be categorized as the ones helping in mastering the techniques of logical thinking and creative experience (independent work on analysis, synthesis, comparison, generalization; formation of assessments, conclusions; deepening of the knowledge system: clarification, specification, systematization). Moreover, they are not the same in terms of the difficulty of decoding their content, and the difficulty of finding the answer. The first (Q_{16-3}) of them is easier than the second one (Q_{16-4}). The intended purpose of the first of these questions (Q_{16-3}) was correctly indicated by 31.8% of respondents, and the second (Q_{16-4}) – 58.3% of the teachers surveyed.

The fifth question (Q_{16-5}) refers to the type of questions that direct students to apply knowledge, including the concept of distance from a point to a line. This question involves a semantic reorganization of the definition into a rule for finding the distance from a point to a line. The student's ability to correctly answer such questions can be considered evidence of the formed notion of distance from a point to a line. 32.4% of respondents agree with this intended purpose of the question. So, the survey discovered the inconclusive level of high school teachers' aptitude to assess the alleged goals of the questions (the incidence of wrong answers to questions Q_{15}, Q_{16-1}, Q_{16-2}, Q_{16-3}, Q_{16-4}, Q_{16-5} is rather high (See Table 3).

In general, the survey finds such independent variables that, according to teachers, affect the process and effectiveness of using questions in TLM: V_1 – school location; V_2 – work experience as a teacher; V_3 – work experience as a teacher in grades 5-6; V_4 – work experience as a teacher in grades 7-9; V_5 – work experience as a teacher in grades 10-11; V_6 – teacher believes most questions are used when checking homework; V_7 – teacher believes most questions are used when repeating theoretical information; V_8 – teacher believes most questions are used when updating students' basic knowledge and skills; V_9 – teacher believes most questions are used when explaining new material; V_{10} – teacher believes most questions are used when consolidating new concepts, facts and types of activity; V_{11} – teacher believes most questions are used when summarizing and reflecting; V_{12} – teacher believes most time should be given to a student to think about the answer to questions when checking homework; V_{13} – teacher believes most time should be given to a student to think about the answer to questions when repeating theoretical information; V_{14} – teacher believes most time should be given to a student to think about the answer to questions when updating students' basic knowledge and skills; V_{15} – teacher believes most time should be given to a student to think about the answer to questions when explaining new material; V_{16} – teacher believes most time should be given to a student to think about the answer to questions when consolidating new concepts, facts and types of activity; V_{17} – teacher believes most time should be given to a student to think about the answer to questions when summarizing and reflecting; V_{18} – questions on the content of the new educational material predominate among teacher's questions; V_{19} – teacher spends time on asking questions on the content, and on students' answers to them; V_{20} – teacher believes students should ask questions about the content of new learning material in class; V_{21} – teacher has a certain strategy of using questions, which is mostly followed; V_{22} – teacher follows a strategy of "bringing" students to the correct answer with the help of a sequence of questions; V_{23} – teacher considers the worldview function to be the priority function of questions; V_{24} – teacher believes questions should be exclusively open-ended in teaching Mathematics; V_{25} – teacher believes questions should include several possible answers in teaching Mathematics; V_{26} – teacher believes questions should provide several different ways to get an answer in teaching Mathematics; V_{27} – teacher considers the corrective-evaluation function to be the priority function of questions; V_{28} – teacher believes questions encourage not only the application of the acquired knowledge, but also generalizations and the establishment of interdisciplinary and intradisciplinary links in teaching Mathematics; V_{29} – teacher believes questions should be formulated from words and terms understandable to the students in teaching Mathematics; V_{30} – teacher believes questions should contain semantic support for the answer; V_{31} – teacher believes it is advisable to use the following types of questions (aimed at organizing the reproduction of the studied material / initial systematizing of concepts and facts / initial consolidating of skills and abilities) in teaching Mathematics; V_{32} – teacher believes it is advisable to use questions that contribute to mastering the techniques of logical thinking and experience of creative activity; V_{33} – teacher believes it is advisable to use questions requiring students to apply the acquired knowledge in changed unusual conditions; V_{34} – teacher uses alternative questions in teaching; V_{35} – teacher encourages students to ask questions about the content of the material.

Thus, the study finds 35 independent variables (V_1 – V_{35}), which characterize the current state of using questions in practice of TLM. A large number of independent variables makes it difficult to study this problem. Hence, there arises the question concerning narrowing the dimension of the pedagogical phenomenon under consideration through factor analysis that will combine notably correlated variables(correlation coefficient is more than 0.5). On the basis of factor analysis, it is necessary to identify influential factors, perhaps not so obvious and predictable, as well as to make appropriate generalizations.

4.2. The Factor Analysis

In processing the study results, the factor analysis was employed (Agresti, 1996; Harrington, 2009). It came handy in: 1) exploring the interactions of the input variables (the determining factor of each set of variables is their utmost efficiency); 2) identifying the factors that make input variables dependent interdependent; 3) calculating numerical values of factors as new integral variables. The data were processed and presented in tables, graphs, and figures. Factor analysis follows the sequence (Arbuckle, 2006): 1) there was calculated the correlation matrix for all variables (based on the data obtained from the teachers participating in the survey); 2) factors were categorized via the principal components analysis method; 3) to simplify the structure, factors were rotated (Varimax rotation with Kaiser normalization was used); 4) with SPSS 23.0 software package factors were interpreted (Nasledov, 2013).

The numerical value obtained (0.595) of the Kaiser-Meyer-Olkin sampling adequacy demonstrates a high sample correlation for the factor analysis. The Bartlett spherical criterion indicated a statistically significant result, since correlations between variables differed significantly from zero (Table 4).

Table 4. Measure of sampling adequacy and Bartlett's criterion

Kaiser-Meyer-Olkin measure of sampling adequacy		.595
Bartlett's test of sphericity	Approx. χ^2	1352.036
	df	595
	Sig	0

Source: Author's own conception

Table 5 lists the names of variables and grouping results (community).

Table 5. Variables and grouping results (community)

Names of variables	Input	Output	Names of variables	Input	Output	Names of variables	Input	Output
V_1	1	.652	V_{12}	1	.663	V_{24}	1	.656
V_2	1	.633	V_{13}	1	.602	V_{25}	1	.671
V_3	1	.79	V_{14}	1	.667	V_{26}	1	.675
V_4	1	.742	V_{15}	1	.716	V_{27}	1	.562
V_5	1	.677	V_{16}	1	.614	V_{28}	1	.598
V_6	1	.655	V_{17}	1	.571	V_{29}	1	.582
V_7	1	.652	V_{18}	1	.633	V_{30}	1	.607

V_8	1	.584	V_{19}	1	.607	V_{31}	1	.671
V_9	1	.641	V_{20}	1	.723	V_{32}	1	.621
V_{10}	1	.622	V_{21}	1	.659	V_{33}	1	.735
V_{11}	1	.536	V_{22}	1	.634	V_{34}	1	.649
			V_{23}	1	.629	V_{35}	1	.77

Source: Author's own conception

Table 6 presents the features of the separate factors: the number, the sum of the squared loadings, the percentage of the joint dispersion caused by the factor, the corresponding cumulative percentage before and after loading.

Table 6. Total Variance Explained

Component	Initial Eigenvalues			Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.719	1.624	1.624	3.719	1.624	1.624	3.35	9.571	9.571
2	2.799	7.996	18.621	2.799	7.996	18.621	2.339	6.684	16.255
3	2.108	6.022	24.643	2.108	6.022	24.643	1.943	5.551	21.806
4	2.014	5.755	3.398	2.014	5.755	3.398	1.746	4.988	26.794
5	1.832	5.233	35.631	1.832	5.233	35.631	1.713	4.895	31.689
6	1.597	4.562	4.192	1.597	4.562	4.192	1.605	4.586	36.275
7	1.535	4.385	44.577	1.535	4.385	44.577	1.6	4.572	4.847
8	1.435	4.101	48.679	1.435	4.101	48.679	1.55	4.43	45.277
9	1.285	3.671	52.349	1.285	3.671	52.349	1.467	4.192	49.469
10	1.165	3.328	55.677	1.165	3.328	55.677	1.377	3.936	53.405
11	1.124	3.211	58.888	1.124	3.211	58.888	1.373	3.922	57.327
12	1.07	3.057	61.946	1.07	3.057	61.946	1.372	3.919	61.246
13	1.016	2.903	64.849	1.016	2.903	64.849	1.261	3.602	64.849
14	.94	2.686	67.535						
15	.938	2.68	7.215						
16	.82	2.342	72.557						
17	.799	2.283	74.84						
18	.775	2.215	77.055						
19	.727	2.076	79.131						
20	.682	1.947	81.079						
21	.675	1.929	83.008						
22	.613	1.751	84.759						
23	.591	1.689	86.448						

24	.54	1.543	87.99
25	.525	1.501	89.492
26	.505	1.443	9.935
27	.474	1.353	92.288
28	.455	1.301	93.59
29	.424	1.211	94.801
30	.409	1.169	95.97
31	.368	1.052	97.021
32	.316	.903	97.924
33	.287	.821	98.745
34	.23	.656	99.401
35	.21	.599	100

Separating Factors: Main Component Method
Source: Author's own conception

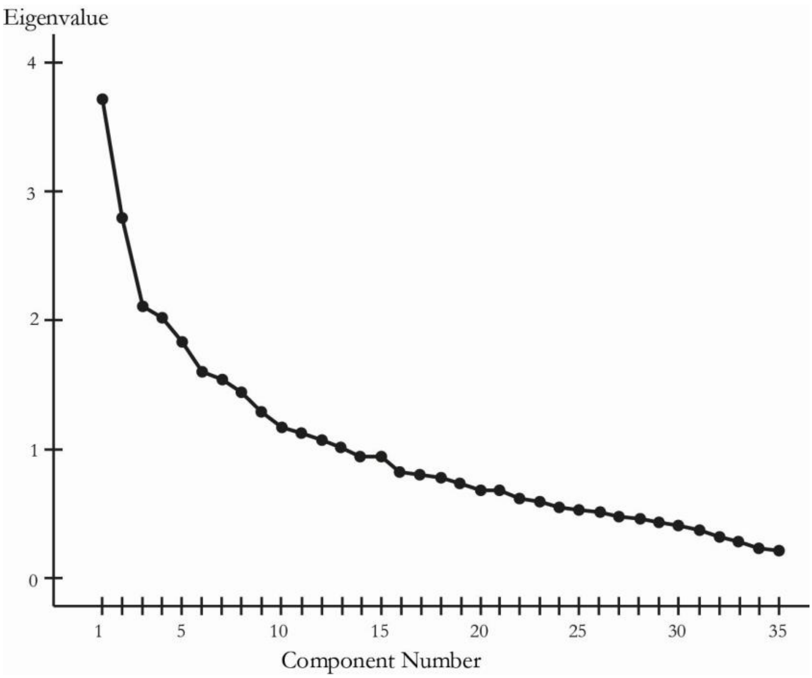

Fig. 5 Eigenvalue graph
Source: Author's own conception

Subsequently, the factors were rotated (Table 7) to create a simplified structure (using Rotation Method: Varimax with Kaiser Normalization was used).

Table 7. Rotated Component Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13
V_{1}													
V_{2}							.535						
V_{3}			.865										
V_{4}			.822										
V_{5}							.790						
V_{6}						.738							
V_{7}		.546				.502							
V_{8}		.666											
V_{9}		.709											
V_{10}		.730											
V_{11}		.530											
V_{12}	.675												
V_{13}	.685												
V_{14}	.731												
V_{15}	.742												
V_{16}	.755												
V_{17}	.685												
V_{18}								.742					
V_{19}								.634					
V_{20}													
V_{21}					–.559								
V_{22}						-.516							
V_{23}										.736			
V_{24}													
V_{25}													.782
V_{26}												.790	
V_{27}										.510			
V_{28}					.632								
V_{29}					.601								
V_{30}													
V_{31}													
V_{32}				.751									
V_{33}				.803									
V_{34}											.718		
V_{35}									.841				

Source: Author's own conception

5. Limits and discussion

Interpretation of factors. As a result of rotation, the factors which, according to teachers, affect the process and effectiveness of the questioning in teaching mathematics, have been identified (Table 8-20).

Factor 1 (factor of question correspondence to the content of educational material), and *Factor 2 (factor of question significance on the content of educational material)* indicate that during a lesson, teachers prefer questions on the content of education. Such questions are asked by both teacher (Table 8) and students (Table 9). Teachers believe that it is not advisable to limit the time allotted for questions. It is rather unexpectedly that the variables that make up these factors did not show any correlations.

Table 8. Factor 1 (factor of question correspondence to the content of educational material)

Names of variables	Variable Description	Value after rotation
V_{18}	Teacher spends time on asking questions on the content, and on students' answers to them	.742
V_{19}	Teacher believes students should ask questions about the content of new learning material in class	.634

Source: Author's own conception

Table 9. Factor 2 (factor of question significance on the content of educational material)

Names of variables	Variable Description	Value after rotation
V_{35}	Teacher encourages students to ask questions about the content of the material	.841

Source: Author's own conception

Factor 3 (factor of importance of the epistemological function of questions).

The survey revealed a significant influence of teachers' beliefs on the priority functions of questions in teaching Mathematics. Teachers consider worldview and corrective and evaluative functions of questions to be the most essential (Table 10).

Table 10. Factor 3 (factor of importance of the epistemological function of questions)

Names of variables	Variable Description	Value after rotation
V_{23}	Teacher considers the worldview function to be the priority function of questions	.736
V_{27}	Teacher considers the corrective-evaluation function to be the priority function of questions	.510

Source: Author's own conception

Factor 4 (factor of the number of questions at a certain stage of the lesson).

The survey revealed the significant impact of the number of questions that teachers or students ask at a certain stage of the math class (Table 11). The teacher and students ask the largest number of questions when explaining or consolidating new information. This indicates that teachers mostly use the method of problem statement, heuristic conversation, the creation of problem situations explaining new material. During the consolidation, teachers organize learning so that students learn a new way of activities, not just reproducing the action on the model. After giving the student a complete and correct indicative basis of action (its plan and algorithm) when explaining new material, teachers, with the help of questions, guide students to perform the method of mathematical activities (calculation, simplification of expression, solving equations, etc.) expanded, step by step with fixing all its stages. After that, the teacher's questions no longer perform an organizational-guiding, but an organizational-auxiliary function. Using questions, teacher gives indirect advice, instructions, memos, "support points" for the student to perform the action fully in details in the form of external speech. At the next stage, when the student begins to skip auxiliary operations, naming aloud only the main stages of the method of mathematical activity, and the action turns into a form of speech to himself, then the questions become more encouraging, stimulating students to self-assessment. Finally, when the action is minimized and automated, performed quickly and mentally, then the questions can be a "provocation", so that students pay attention to the "weak points" that need to be considered in solving problems, apply knowledge in a changed situation.

Table 11. Factor 4 (factor of the number of questions at a certain stage of the lesson)

Names of variables	Variable Description	Value after rotation
V_{10}	Teacher believes most questions are used when consolidating new concepts, facts and types of activity	.730
V_9	Teacher believes most questions are used when explaining new material	.709
V_8	Teacher believes most questions are used when updating students' basic knowledge and skills	.666
V_7	Teacher believes most questions are used when repeating theoretical information	.546
V_{11}	Teacher believes most questions are used when summarizing and reflecting	.530

Source: Author's own conception

Factors 5 (factor of importance of alternative questions), 6 (factor of importance of questions that can be answered in different ways), 7 (factor of importance of questions that may involve several answer options) (Table 12-14) indicate the importance of diverse questions in teaching Mathematics. It is rather unexpectedly that these factors were isolated. Factor analysis does not reveal any relationships between the variables that make up these factors. The survey finds that Mathematics teachers prefer alternative questions in teaching process. By preferring alternative questions, teachers, on the one hand, save time in class. However, this way of organizing education process does not encourage students to develop a way of thinking when answering questions, creates obstacles for the teacher to find students' misconceptions about the content of mathematical concepts, or their shortcomings and mistakes in implementing certain steps in mathematical activities. In addition, factors 6 and 7 are quite indicative showing that, in the opinion of teachers, the acquisition of mathematical knowledge is facilitated by questions that may involve several possible answers, or those that can be answered in different ways. Naturally, finding answers to such questions takes more time, but the benefits of them are much greater for students' mental development. Therefore, we state that the survey revealed a gap between the values of teachers' attitude to the feasibility of using these types of questions in the educational process in Mathematics and the practice of their implementation.

Table 12. Factor 5 (factor of importance of alternative questions)

Names of variables	Variable Description	Value after rotation
V_{34}	Teacher uses alternative questions in teaching	.718

Source: Author's own conception

Table 13. Factor 6 (factor of importance of questions that can be answered in different ways)

Names of variables	Variable Description	Value after rotation
V_{26}	Teacher believes questions should provide several different ways to get an answer in teaching Mathematics	.790

Source: Author's own conception

Table 14. Factor 7 (factor of importance of questions that may involve several answer options)

Names of variables	Variable Description	Value after rotation
V_{25}	Teacher believes questions should include several possible answers in teaching Mathematics	.782

Source: Author's own conception

Factor 8 (factor of non-template questions) (Table 15) shows that teachers consider it necessary and important to use questions to encourage students to be creative in solving problems, to master the techniques of logical thinking. The questions should be formulated both in the usual and unusual form, immerse students in an unfamiliar (unusual) context, encourage the near or far transfer of knowledge. At the same time, it is important that the questions are formulated from words and terms understandable to the student (factor 9).

Table 15. Factor 8 (factor of non-template questions)

Names of variables	Variable Description	Value after rotation
V_{33}	Teacher believes it is advisable to use questions requiring students to apply the acquired knowledge in changed unusual conditions	.803
V_{32}	Teacher believes it is advisable to use questions that contribute to mastering the techniques of logical thinking and experience of creative activity	.751

Source: Author's own conception

Factors 9 (question strategy) and 10 (quantitative-strategic factor) relate to question strategy (Tables 16-17). They point out that teachers think that questions are important for generalizing or systematizing students' knowledge, for identifying and establishing interdisciplinary or intradisciplinary links. However, a specific strategy in the use of questions, including leading questions should not be followed. We conclude that the strategy of asking questions is not fixed for the teacher, it should change depending on the specific learning environment. Teachers demonstrate the greatest variability of approaches to asking questions when they check homework, repeat and update basic knowledge. The greater the number of questions the teacher asks at these stages of the lesson, the less the teacher follows a certain strategy when asking questions.

Table 16. Factor 9 (question strategy)

Names of variables	Variable Description	Value after rotation
V_{28}	Teacher believes questions encourage not only the application of the acquired knowledge, but also generalizations and the establishment of interdisciplinary and intradisciplinary links in teaching Mathematics	.632
V_{29}	Teacher believes questions should be formulated from words and terms understandable to the students in teaching Mathematics	.601
V_{21}	Teacher has a certain strategy of using questions, which is mostly followed	–.559

Source: Author's own conception

Table 17. Factor 10 (quantitative-strategic factor)

Names of variables	Variable Description	Value after rotation
V_6	Teacher believes most questions are used when checking homework	.738
V_7	Teacher believes most questions are used when repeating theoretical information	.502
V_{22}	Teacher follows a strategy of "bringing" students to the correct answer with the help of a sequence of questions	–.516

Source: Author's own conception

Factor 11 (factor of time for considering the answer) combines the largest number of variables (Table 18). According to mathematics teachers, it is important to give students enough time to think about the answer, especially when repeating and updating basic knowledge and skills, explaining new material and consolidating new concepts, facts and methods of mathematical activities for students. You do not need to "push" students to the correct answer, saving time. However, the stages of reflection and homework check should be carried out at a fast pace.

Table 18. Factor 11 (factor of time for considering the answer)

Names of variables	Variable Description	Value after rotation
V_{16}	Teacher believes most time should be given to a student to think about the answer to questions when consolidating new concepts, facts and types of activity	.755
V_{15}	Teacher believes most time should be given to a student to think about the answer to questions when explaining new material	.742
V_{14}	Teacher believes most time should be given to a student to think about the answer to questions when updating students' basic knowledge and skills	.731
V_{13}	Teacher believes most time should be given to a student to think about the answer to questions when repeating theoretical information	.685
V_{17}	Teacher believes most time should be given to a student to think about the answer to questions when summarizing and reflecting	.685
V_{12}	Teacher believes most time should be given to a student to think about the answer to questions when checking homework	.675

Source: Author's own conception

Naturally, to navigate the selection of questions according to the lesson stage quickly and effectively, the teacher should have not only theoretical training but also practical experience in teaching students. This is evidenced as expected by *factors 12 (teacher's experience) and 13 (praxeological factor)* (Table 19-20). In addition, the survey finds out that in the opinion of teachers, in order to effectively use questions in teaching, it is important for them to have experience in different classes (K5-K11).

Table 19. Factor 12 (teacher's experience)

Names of variables	Variable Description	Value after rotation
V_3	Work experience as a teacher in grades 5-6	.865
V_4	Work experience as a teacher in grades 7-9	.822

Source: Author's own conception

Table 20. Factor 13 (praxeological factor)

Names of variables	Variable Description	Value after rotation
V_2	Work experience as a teacher	.535
V_5	Work experience as a teacher in grades 10-11	.790

Source: Author's own conception

Conclusions

The study performed is a component of a multi-vector research of the aspects of high-end teaching practices in terms of mathematical skills, problem-solved skills, cognitive demand, the involvement of all students in classroom activities, educational demands, productive output requirements and student-centered and teacher-orchestrated environments (Boychuk et al., 2022) and practices (Cao, 2018; Ryve et al., 2016). Century & Cassata's (2016) educational research displays such, essential to TLM, contextual factors as teachers' personal features (expertise, values, and beliefs) and institutional features (the number of students, classroom environment characteristics, school management, and the mind-set of the subjects of the educational process). Other contectual factors which are also of importance in improving math teaching are introduced by Ryve and Hemmi (2019). They are: Teacher's role in the educational process, teacher's place in the classroom and traditional explicit and hidden pedagogical nuances (Hemmi et al., 2017). The research is aimed at strengthening such aspects of TLM as teachers' engagement in enhancing the qualitative capacities of math class through inventing and employing proper math-focused questions.

The study is based on Tarasenkova's (2002) psychological-semiotic approach based on which the emphasis was placed on it how the questions are formulated (verbalized). The semantic field to which the definite question belongs, the intersection of this semantic field with the student's thesaurus, the volume of this intersection influence the success of decoding the content of the question wrapped in text shells.

In this context, we propose to group the questions according to the availability of the semantic support for the answer in the question wording:

1) questions with full semantic support; 2) questions with incomplete semantic support; 3) questions that do not have semantic support for the answer. We also distinguish the following three groups of questions according to their dominant didactic function:1) questions for the organization of knowledge consolidation; 2) questions that contribute to mastering the techniques of logical thinking and experience of creative activity; 3) questions that require the application of the acquired knowledge. All these kinds of questions implement such functions as: epistemological, praxeological, evaluative-reflexive, monitoring-and-evaluation, the function of organizing the content of education, the function of organizing all stages of the learning-cycle-focused (didactic-cycle-focused) functions of questions.

Within the study we researched the teachers' ability of distinguishing among the question types based on the representation degree of semantic support of the envisaged answers provided in the very texts of the questions. Generally, the majority of the teachers are able to do this. Moreover, teachers indicate that questions with full semantic support for the answer are better to use unsystematically, from time to time. Thus, we found that teachers consider these types of questions to be less effective in TLM. The next issue to study was whether teachers understand how to use different types of questions according to their didactic purpose. The ability to tell the intended purpose of questions proved insufficient.

The study revealed 35 independent variables which characterized the current state of using questions in practice of TLM. They were optimized into 13 factors that influence the educational process. These factors strengthen the priority of worldview and corrective and evaluative functions of questions (teachers consider these questions' functions to be the most essential). Educators support the idea that questioning in TLM should be in sync with the content of educational material, and the questions formulated with the use of topical vocabulary known to the students are viewed as most cost-effective. The research reveals the significant impact of the number of questions that teachers or students ask at a lesson stage. Particularly important factor is the factor of time. According to Mathematics teachers, it is not advisable to limit the time allotted for questions, it is important to give students enough time to think about the answer especially when repeating and updating basic knowledge and skills, explaining new material and consolidating new concepts, facts and methods of mathematical activities for students. Teacher should not "push" students to the correct answer, saving time.

Questions that may involve several possible answers or those that can be answered in different ways are vital and significant in TLM. At the same time, in practice, teachers prefer alternative questions in teaching process. Thus, our study stated a gap between the teachers' attitude to the feasibility of using these types of questions in teaching mathematics and the practice of their implementation. Moreover, teachers believe that a specific strategy in questioning should not be followed. We conclude that the strategy of asking questions is not fixed for the teacher; it should change depending on the specific learning conditions. It contracts with Wood's (1998) approach when "the focused pattern" and "the funnel pattern" in asking questions in math class have been considered. Quite expected that teacher's experience is a meaningful factor. Moreover it is important for teacher to have teaching experience in *different* classes. The survey findings can be supportive in deciding on the role of questions and applying questioning in TLM.

The conducted research has its limitations caused by the territorial factor, as the research is based on a survey of mathematics teachers from Ukraine and takes into account the context of educational reformation processes in the country. At the same time, the research interest is focused on the questions, challenges and problems with which mathematics teachers from all over the world deal, and personal-semiotic approach and the fundamental principles of cognitive psychology, which have international recognition, were chosen as the research methodology. Therefore, in our opinion, the obtained results will be important for the further development of the didactics of mathematics as a science of the regularities of the process of teaching and learning mathematics not only in the local, but also in the international dimension.

References

Agresti, A. (1996). *An Introduction to Categorical Data Analysis*. Wilely.

Aizikovitsh-Udi, E., & Star, J. R. (2011). The skill of asking good questions in mathematics teaching. *Procedia Social and Behavioral Sciences*, *15*, 1354–1358. https://doi.org/10.1016/j.sbspro.2011.03.291

Arbuckle, J. L. (2006). *Amos 7.0 user's guide*. SPSS.

Bingham, C. (2005). The Hermeneutics of Educational Questioning. *Educational Philosophy and Theory*, 37(4), 553-565. https://doi.org/10.1111/j.1469-5812.2005.00140.x

Black, S. (2001). Ask Me a Question: How Teachers Use Inquiry in a Classroom. *American School Board Journal*, *188*(5), 43-45.

Boychuk, Y., Novikova, V., Opanasenko, Y., Olena, K., & Kostina, V. (2022). Pedagogical conditions for the introduction of blended learning technologies in ukrainian higher education institutions. *Revista Romaneasca Pentru Educatie Multidimensionala, 14*(3), 32-50. https://doi.org/10.18662/rrem/14.3/596

Brahier, D., Leinwand, S., & Huinker, D. (2014). Principles to Actions: Mathematics Programs as the Core for Student Learning. *Mathematics Teaching in the Middle School*, *19*(9), 516. https://eric.ed.gov/?id=EJ1044582

Cao, H. T. (2018). Teachers' Capacity of Instruction for Developing Higher – Order Thinking Skills for Upper Secondary Students – A Case Study in Teaching Mathematics in Vietnam. *Revista Romaneasca Pentru Educatie Multidimensionala, 10*(1.Suppl), 8-19. https://doi.org/10.18662/rrem/33

Century, J., & Cassata, A. (2016). Implementation research: Finding common ground on what, how, why, where, and who. *Review of Research in Education*, *40*(1), 169–215. https://doi.org/10.3102/0091732X1666533

Chazan, D. (2000*). Beyond formulas in mathematics and teaching: Dynamics of the high school algebra classroom.* Teachers College Press.

Costa, A. L., & Kallick, B. (Eds.), (2000). *Activating and Engaging Habits of Mind.* Association for Supervision and Curriculum Development.

Dantonio, M., & Beisenherz, P. C. (2001). *Learning to Question, Questioning to Learn: Developing Effective Teacher Questioning Practices*. Allyn and Bacon.

Davydov, A. N. (2019). "The technology of leading questions" as a teaching method for solving geometric problems for proof. *Science And Mathematics Education,* 22(6), 556–565.

Ellis, K. (1993). *Teacher Questioning Behavior and Student Learning: What Research Says to Teachers*. ERIC. https://eric.ed.gov/?id=ED359572

Frager, A. M. (1979). *Questioning Strategies: Implications for Teacher Training*. ERIC. https://eric.ed.gov/?id=ED238845

Franke, M., Webb, N., Chan, A., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education*, *60*(4), 380–392. https://doi.org/10.1177/0022487109339906

Fusco, E. (2012). *Effective Questioning Strategies in the Classroom: A Step-by-Step Approach to Engaged Thinking and Learning, K-8*, Teacher College Press.

Harrington, D. (2009). *Confirmatory factor analysis*. Oxford University Press.

Hemmi, K., Krzywacki, H., & Koljonen, T. (2017). Investigating Finnish teacher guides as a resource for mathematics teaching. *Scandinavian Journal of Educational Research*, *62*(6), 911–928. https://doi.org/10.1080/00313831.2017.1307278

Ingram, J., Andrews, N., & Pitt, A. (2019). When students offer explanations without the teacher explicitly asking them to. *Educational Studies in Mathematics*, *101*, 51–66. https://doi.org/10.1007/s10649-018-9873-9

Kress, N. E. (2017). 6 Essential Questions for Problem Solving. *The Mathematics Teacher*, *111*(3), 190-196. http://www.math.lsa.umich.edu/~glarose/dept/teaching/resources/KressMathematicsTeacher.pdf

Kulbyakina, L. Y., & Zotova, T. N. (2004). Questions in the methodology of teaching mathematics. *Primary School*, *7*, 117-121.

Lebedeva, I. P. (2003). On systems of educational questions as a means of stimulating the thinking of junior students. In *Modern problems of primary education* (pp. 54–68). Perm.

Moberg, E. M. (2008). *Socrates Lives: Dialogue as a Means of Teaching and Learning*. ERIC. https://eric.ed.gov/?id=ED507870

Nasledov, A. (2013). *IBM SPSS Statistics 20 and AMOS: professional statistical data analysis*. Peter.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. NCTM.

National Council of Teachers of Mathematics (NCTM). (2014). *Principles to action. Ensuring mathematical success for all*. NCTM.

Pape, S., Bell, C., & Yetkin, İ. (2003). Developing Mathematical Thinking and Self-Regulated Learning: A Teaching Experiment in a Seventh-Grade Mathematics Classroom. *Educational Studies in Mathematics*, *53*, 179–202. https://doi.org/10.1023/A:1026062121857

Piccolo, D. L., Harbaugh, A. P., Carter, T. A., Capraro, M. M., & Capraro, R. M. (2008). Quality of Instruction: Examining Discourse in Middle School Mathematics Instruction. *Journal of Advanced Academics*, *19*(3), 376-410. https://doi.org/10.4219/jaa-2008-809

Pólya, G. (1945). *How to Solve it: A New Aspect of Mathematical Method*. Princeton University Press.

Ryve, A., & Hemmi, K. (2019). Educational policy to improve mathematics instruction at scale: conceptualizing contextual factors. *Educational Studies in Mathematics*, *102*, 379–394 https://doi.org/10.1007/s10649-019-09887-6

Ryve, A., Hemmi, K., & Kornhall, P. (2016). *Skola på vetenskaplig grund [School on a scientific basis]*. Natur & Kultur.

Schuster, L., & Anderson, N. C. (2005). *Good questions for math teaching: why ask them and what to ask, grades 5–8 Math*. Math Solutions Publications.

Sigel, I. E., & Saunders, R. (1977). *An Inquiry Into Inquiry: Question-Asking as an Instructional Model*. ERIC. https://eric.ed.gov/?id=ED158871

Sullivan, P., & Lilburn, P. (2002). *Good Questions for Math Teaching: Why Ask Them and What to Ask, K–6*. Math Solutions Publications.

Tarasenkova, N. (2002). *The use of sign-symbolic means in teaching mathematics*. Vidlunnya-Plus.

Webb, D. C. (2016). *Applying Principles for Active Learning to Promote Student Engagement in Undergraduate Calculus* [Conference presentation]. 13th International Congress on Mathematical Education*,* Hamburg, 24-31 July 2016.

Wood, T. (1998). Alternative patterns of communication in mathematics classes: funneling or focusing? In H. Steinbring, M. Bussi, & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 167-178). NCTM.