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FORCED VIBRATIONS OF ELLIPSOIDAL SHELLS REINFORCED WITH TRANSVERSE RIBS UNDER A NONSTATIONARY DISTRIBUTED LOAD

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The problem of the forced nonaxisymmetric vibrations of reinforced ellipsoidal shells under nonstationary loading is considered. A numerical algorithm for solving the problem is developed. Numerical results are obtained and analyzed

Keywords: reinforced ellipsoidal shell, geometrically nonlinear theory, numerical method, forced nonaxisymmetric vibrations

Introduction. The dynamic behavior of ellipsoidal reinforced shells is of interest because they are widely used in modern engineering structures. So far, only the harmonic vibrations of reinforced shells of simple geometry (cylindrical, conical, spherical) have mainly been studied [1–3, 6, 19]. The forced vibrations of reinforced shells under impulsive loads were studied in [7–9, 18]. There are very few studies on the dynamic behavior of reinforced shells of more complex geometry. Noteworthy are the papers [8–14] which report results on the forced vibrations of shells of revolution, including reinforced ellipsoidal shells. Mathematically, both statement and solution of such problems are very complicated (use of the equations of elasticity, formulation of shell–rib interface conditions, development of a numerical problem-solving algorithm, etc.).

Here we will derive the equations describing the nonaxisymmetric vibrations of a rib-reinforced ellipsoidal shell. To describe the casing and ribs, we will use the refined theory of shells and beams based on the Timoshenko hypotheses [9, 15]. To derive the vibration equations, we will use the Hamilton–Ostrogradskii variational principle. The numerical approach to solving the dynamic equations employs the integro-interpolation method to construct finite-difference schemes for an equation with discontinuous coefficients. We will solve, as a numerical example, the problem of the nonaxisymmetric vibrations of a transversely reinforced ellipsoidal shell under a distributed internal load normal to the shell surface.

1. Problem Formulation. Consider an inhomogeneous elastic structure that is an ellipsoidal shell reinforced with transverse ribs. The mid-surface of the casing is described by the following formulas [4, 5]:

$$x = R \sin \alpha_1 \sin \alpha_2,$$

$$y = R \sin \alpha_1 \cos \alpha_2,$$

$$z = kR \cos \alpha_1, \tag{1.1}$$

where α_1 and α_2 are the meridional and circumferential Gaussian curvilinear coordinates on the shell surface; $k = b/a$ is the ellipse aspect ratio; a and b are the ellipse semiaxes.

The expressions for the metrics and the shape of the shell's mid-surface are

$$a_{11} = R^2(\cos^2 \alpha_1 + k^2 \sin^2 \alpha_1), \quad a_{22} = R^2 \sin^2 \alpha_1,$$

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$$b_{11} = kR(\cos^2 \alpha_1 + k^2 \sin^2 \alpha_1)^{-1/2}, \quad b_{22} = kR \sin^2 \alpha_1 (\cos^2 \alpha_1 + k^2 \sin^2 \alpha_1)^{-1/2}. \tag{1.2}$$

The coefficients of the first quadratic form and the curvature of the mid-surface are given by

$$A_1 = a(\cos^2 \alpha_1 + k^2 \sin^2 \alpha_1)^{1/2}, \quad A_2 = a \sin \alpha_1,$$

$$k_1 = \frac{b}{a^2}(\cos^2 \alpha_1 + k^2 \sin^2 \alpha_1)^{-3/2}, \quad k_2 = \frac{b}{a^2}(\cos^2 \alpha_1 + k^2 \sin^2 \alpha_1)^{-1/2}. \tag{1.3}$$

To mathematically model the dynamic deformation of the structure, we will use the geometrically nonlinear Timoshenko-type theory of shells based on the following assumptions.

The variation of the displacements throughout the thickness of the shell in the coordinate system (s_1, s_2, z) is described by

$$u_1^z(s_1, s_2, z) = u_1(s_1, s_2) + z\varphi_1(s_1, s_2),$$

$$u_2^z(s_1, s_2, z) = u_2(s_1, s_2) + z\varphi_2(s_1, s_2),$$

$$u_3^z(s_1, s_2, z) = u_3(s_1, s_2), \quad z \in [-h/2, h/2], \tag{1.4}$$

where $u_1, u_2, u_3, \varphi_1, \varphi_2$ are the components of the generalized displacement vector of the mid-surface; $s_1 = \alpha_1 A_1, s_2 = \alpha_2 A_2$, where A_1 and A_2 are the coefficients of the first quadratic form of the ellipsoidal shell.

The expressions for the quadratic approximation of the strains in the shell can be found in [16]:

$$\varepsilon_{11} = \frac{\partial u_1}{\partial s_1} + k_1 u_3 + \frac{1}{2}\theta_1^2, \quad \varepsilon_{22} = \frac{\partial u_2}{\partial s_2} + \frac{1}{A_2}\frac{\partial A_2}{\partial s_1} u_1 + k_2 u_3 + \frac{1}{2}\theta_1^2,$$

$$\varepsilon_{12} = \omega + \theta_1 \theta_2, \quad \varepsilon_{13} = \varphi_1 + \theta_1, \quad \varepsilon_{23} = \varphi_2 + \theta_2,$$

$$\omega = \omega_1 + \omega_2, \quad \omega_1 = \frac{\partial u_2}{\partial s_1}, \quad \omega_2 = \frac{\partial u_1}{\partial s_2} - \frac{1}{A_2}\frac{\partial A_2}{\partial s_1} u_2,$$

$$\theta_1 = \frac{\partial u_3}{\partial s_1} - k_1 u_1, \quad \theta_2 = \frac{\partial u_3}{\partial s_2} - k_2 u_2,$$

$$\chi_{11} = \frac{\partial \varphi_1}{\partial s_1}, \quad \chi_{22} = \frac{\partial \varphi_2}{\partial s_2} + \frac{1}{A_2}\frac{\partial A_2}{\partial s_1}\varphi_1, \quad \chi_{12} = \tau_1 + \tau_2 + \kappa_1 \omega_1 + \kappa_2 \omega_2,$$

$$\tau_1 = \frac{\partial \varphi_2}{\partial s_1}, \quad \tau_2 = \frac{\partial \varphi_1}{\partial s_2} - \frac{1}{A_2}\frac{\partial A_2}{\partial s_1}\varphi_2. \tag{1.5}$$

2. Basic Equations. To mathematically model the deformation of the jth rib aligned with the α_2-axis, we will hypothesize that the rib cross-section is undeformable, as per the geometrically nonlinear Timoshenko beam theory. We will use the following approximation of the displacements in the cross-section of the jth rib:

$$U_{1j}^{xz}(x, s_2, z) = U_{1j}(s_2) + z\varphi_{1j}(s_2),$$

$$U_{2j}^{xz}(x, s_2, z) = U_{2j}(s_2) + z\varphi_{2j}(s_2),$$

$$U_{3j}^{xz}(x, s_2, z) = U_{3j}(s_2), \tag{2.1}$$

where $U_{1j}, U_{2j}, U_{3j}, \varphi_{1j}, \varphi_{2j}$ are the components of the generalized displacement vector of the cross-sectional center of gravity of the jth rib.

The expressions for the quadratic approximation of the strains in the ribs become

$$\varepsilon_{22j} = \frac{\partial u_2}{\partial s_2} \pm h_{cj} \frac{\partial \varphi_2}{\partial s_2} + k_{2j} u_3 + \frac{1}{2}\theta_{1j}^2 + \frac{1}{2}\theta_{2j}^2,$$

$$\varepsilon_{21j} = \theta_{2j}, \quad \varepsilon_{23j} = \varphi_2 + \theta_{1j},$$

$$\theta_{1j} = \frac{\partial u_3}{\partial s_2} - k_{2j}(u_2 \pm h_{cj}\varphi_2), \quad \theta_2 = \frac{\partial u_1}{\partial s_2} \pm h_{cj} \frac{\partial \varphi_1}{\partial s_2},$$

$$\chi_{21j} = \frac{\partial \varphi_1}{\partial s_2}, \quad \chi_{22j} = \frac{\partial \varphi_2}{\partial s_2}. \tag{2.2}$$

The interface conditions relate the components of the displacement vector of the cross-sectional center of gravity of the jth rib aligned with the α_2-axis and the components of the generalized displacement vector of the mid-surface [1, 3, 8, 9]:

$$U_{1j}(s_2) = U_1(s_{1j}, s_2) \pm h_{cj}\varphi_2(s_{1j}, s_2),$$

$$U_{2j}(s_2) = U_2(s_{1j}, s_2) \pm h_{cj}\varphi_1(s_{1j}, s_2),$$

$$U_{3j}(s_2) = U_3(s_{1j}, s_2),$$

$$\varphi_{1j}(s_2) = \varphi_2(s_{1j}, s_2),$$

$$\varphi_{2j}(s_2) = \varphi_1(s_{1j}, s_2), \tag{2.3}$$

where $h_{cj} = 0.5(h + h_j)$ is the distance from the mid-surface to the centroidal line of the jth rib; h_j is the height of the jth rib aligned with the α_2-axis; α_{1j} is the coordinate of the projection line of the cross-sectional center of gravity of the jth rib onto the coordinate mid-surface of the casing.

To derive the equations of motion, we will use interface conditions in integral form [12].

To derive the equations of the vibrations of the discretely reinforced structure, we will use the Hamilton–Ostrogradskii variational principle:

$$\int_{t_1}^{t_2} \left[\delta(\Pi - K) + \delta A\right] dt = 0,$$

$$\Pi = \Pi_0 + \sum_{j=1}^{n_2} \Pi_j, \quad K = K_0 + \sum_{j=1}^{n_2} K_j, \tag{2.4}$$

where Π_0 and K_0 are the potential and kinetic energy of the casing; Π_j and K_j are the potential and kinetic energy of the jth rib; A is the work done by external forces.

The expressions for δK and $\delta\Pi$ are

$$\delta\Pi = \delta\Pi_0 + \sum_{j=1}^{n_2} \delta\Pi_j, \quad \delta K = \delta K_0 + \sum_{j=1}^{n_2} \delta K_j,$$

$$\delta\Pi_0 = \iint_S \left[T_{11}\delta\varepsilon_{11} + T_{22}\delta\varepsilon_{22} + S\delta\varepsilon_{12} + T_{13}\delta\varepsilon_{13} + T_{23}\delta\varepsilon_{23}\right.$$

$$\left. + M_{11}\delta\kappa_{11} + M_{22}\delta\kappa_{22} + H\delta(\tau_1 + \tau_2)\right] ds,$$

$$\delta\Pi_j = \int_{l_2} \left[T_{21j}\delta\varepsilon_{21j} + T_{22j}\delta\varepsilon_{22j} + T_{23j}\delta\varepsilon_{23j} + M_{21j}\delta\kappa_{21j} + M_{22j}\delta\kappa_{22j}\right] dl_2,$$

$$\delta K_0 = \rho h \iint_S \left[\frac{\partial U_1}{\partial t} \delta \frac{\partial U_1}{\partial t} + \frac{\partial U_2}{\partial t} \delta \frac{\partial U_2}{\partial t} + \frac{\partial U_3}{\partial t} \delta \frac{\partial U_3}{\partial t} + \frac{h^2}{12} \left(\frac{\partial \varphi_1}{\partial t} \delta \frac{\partial \varphi_1}{\partial t} + \frac{\partial \varphi_2}{\partial t} \delta \frac{\partial \varphi_2}{\partial t} \right) dS \right],$$

$$\delta K_j = \rho_j h_j \int_{l_2} \left[\frac{\partial U_{1j}}{\partial t} \delta \frac{\partial U_{1j}}{\partial t} + \frac{\partial U_{2j}}{\partial t} \delta \frac{\partial U_{2j}}{\partial t} + \frac{\partial U_{3j}}{\partial t} \delta \frac{\partial U_{3j}}{\partial t} \right.$$

$$\left. + \frac{I_{crj}}{F_j} \frac{\partial \varphi_{1j}}{\partial t} \delta \frac{\partial \varphi_{1j}}{\partial t} + \frac{I_{2j}}{F_j} \frac{\partial \varphi_{2j}}{\partial t} \delta \frac{\partial \varphi_{2j}}{\partial t} \right] dl_2. \tag{2.5}$$

Performing the standard operations of variation and integration and taking the interface conditions (2.3) into account, we obtain two groups of equations:

(i) the equations of vibrations of the casing between ribs:

$$\frac{1}{A_2} \left[\frac{\partial}{\partial s_1} (A_2 T_{11}) - \frac{\partial A_2}{\partial s_1} T_{22} \right] + k_1 \overline{T}_{13} + \frac{1}{A_1} \frac{\partial}{\partial s_2} (A_1 T_{21}) = \rho h \frac{\partial^2 u_1}{\partial t^2},$$

$$\frac{1}{A_2} \left[\frac{\partial}{\partial s_1} (A_2 T_{12}) - \frac{\partial A_2}{\partial s_1} T_{21} \right] + k_2 \overline{T}_{23} + \frac{1}{A_1} \frac{\partial}{\partial s_2} (A_1 T_{22}) = \rho h \frac{\partial^2 u_2}{\partial t^2},$$

$$\frac{1}{A_2} \frac{\partial}{\partial s_1} (A_2 \overline{T}_{13}) - k_1 T_{11} - k_2 T_{22} + P_3 + \frac{1}{A_1} \frac{\partial}{\partial s_2} (A_1 \overline{T}_{23}) = \rho h \frac{\partial^2 u_3}{\partial t^2},$$

$$\frac{1}{A_2} \left[\frac{\partial}{\partial s_1} (A_2 M_{11}) - \frac{\partial A_2}{\partial s_1} M_{22} \right] - T_{13} + \frac{1}{A_1} \frac{\partial}{\partial s_2} (A_1 M_{21}) = \rho \frac{h^3}{12} \frac{\partial^2 \varphi_1}{\partial t^2},$$

$$\frac{1}{A_2} \left[\frac{\partial}{\partial s_1} (A_2 M_{12}) + \frac{\partial A_2}{\partial s_1} M_{21} \right] + \frac{1}{A_1} \frac{\partial}{\partial s_2} (A_1 M_{22}) - T_{23} = \rho \frac{h^3}{12} \frac{\partial^2 \varphi_2}{\partial t^2}, \tag{2.6}$$

$$T_{11} = B_{11} \varepsilon_{11} + B_{12} \varepsilon_{22}, \quad T_{22} = B_{21} \varepsilon_{11} + B_{22} \varepsilon_{22},$$

$$T_{12} = S + k_2 H, \quad T_{21} = S + k_1 H,$$

$$T_{13} = B_{13} \varepsilon_{13}, \quad T_{23} = B_{23} \varepsilon_{23},$$

$$\overline{T}_{13} = T_{13} + T_{11} \theta_1 + S \theta_2, \quad \overline{T}_{23} = T_{23} + T_{22} \theta_2 + S \theta_1,$$

$$S = B_s \varepsilon_{12},$$

$$M_{11} = D_{11} \chi_{11} + D_{12} \chi_{22}, \quad M_{22} = D_{21} \chi_{11} + D_{22} \chi_{22},$$

$$M_{12} = M_{21} = H, \quad H = D_s \chi_{12}, \tag{2.7}$$

(ii) the equations of vibrations of the jth rib aligned with the α_2-axis;

$$\frac{\partial \overline{T}_{21j}}{\partial s_2} + [T_{11}]_j = \rho_j F_j \left(\frac{\partial^2 U_1}{\partial t^2} \pm h_{cj} \frac{\partial^2 \varphi_1}{\partial t^2} \right),$$

$$\frac{\partial T_{22j}}{\partial s_2} + k_{2j} \overline{T}_{23j} + [S]_j = \rho_j F_j \left(\frac{\partial^2 U_2}{\partial t^2} \pm h_{cj} \frac{\partial^2 \varphi_2}{\partial t^2} \right),$$

$$\frac{\overline{\partial T}_{23j}}{\partial s_2} - k_{2j} T_{22j} + [\overline{T}_{13}]_j = \rho_j F_j \frac{\partial^2 U_3}{\partial t^2},$$

$$\frac{\partial M_{21j}}{\partial s_2} \pm h_{cj} \frac{\partial \overline{T}_{21j}}{\partial s_2} + [M_{11}]_j = \rho_j F_j \left(\pm h_{cj} \frac{\partial^2 U_1}{\partial t^2} + \left(h_{cj}^2 + \frac{I_{crj}}{F_j} \right) \frac{\partial^2 \varphi_1}{\partial t^2} \right),$$

$$\frac{\partial M_{22j}}{\partial s_2} - T_{23j} \pm h_{cj} \left(\frac{\partial T_{22j}}{\partial s_2} + k_{2j} \overline{T}_{23j} \right) + [H]_j = \rho_j F_j \left(\pm h_{cj} \frac{\partial^2 U_2}{\partial t^2} + \left(h_{cj}^2 + \frac{I_{2j}}{F_j} \right) \frac{\partial^2 \varphi_2}{\partial t^2} \right), \tag{2.8}$$

$$\overline{T}_{21j} = T_{21j} + T_{22j}\theta_{1j}, \quad T_{21j} = G_j F_j \varepsilon_{21j},$$

$$T_{22j} = E_j F_j \varepsilon_{22j}, \quad \overline{T}_{23j} = T_{23j} + T_{22j}\theta_{2j}, \quad T_{23j} = G_j F_j k_j^2 \varepsilon_{23j},$$

$$M_{21j} = G_j I_{crj} \chi_{21j}, \quad M_{22j} = E_j I_{2j} \chi_{22j}. \tag{2.9}$$

Equations (1.11) and (1.13) are supplemented with the natural boundary and initial conditions [12].

3. Procedure for Numerical Solution of Nonlinear Problems. Equations (2.6) and (2.8) constitute a system of nonlinear partial differential equations for the variables s_1, s_2, and t with discontinuities in s_1. The discontinuities are the projection lines of the cross-sectional centers of gravity of the transverse ribs onto the mid-surface of the ellipsoidal shell. In this connection, the numerical algorithm for solving the original problem is as follows: find the solution in the smooth region between ribs (2.6) and on the discontinuity lines (2.8) [8, 9]. The difference algorithm is based on the integro-interpolation method for the construction of difference schemes with respect to the space coordinates s_1 and s_2 and an explicit finite-difference scheme with respect to the time coordinate t [17]. The components of the generalized displacement vector are approximated at the integer points of the difference mesh, while the strains and forces at half-integer points. Such an approach maintains the divergent difference representation of the differential equations and ensures the conservation of total mechanical energy [3]. The continuous system is reduced to the finite-difference one in two steps.

The first step is the finite-difference approximation of the divergent vibration equations written for forces and moments.

Integrating Eqs. (2.6) and using the explicit approximation with respect to the time coordinate, we obtain the following difference equations in the smooth region of the ellipsoidal shell:

$$\frac{1}{A_{2\,l}} \left(\frac{A_{2\ l+1/2} T_{11\,l+1/2,m}^n - A_{2\ l-1/2} T_{11\ l-1/2,m}^n}{\Delta s_1} \right) - \frac{1}{A_{2\ l}} \frac{A_{2\ l+1/2} - A_{2\ l-1/2}}{\Delta s_1} T_{22\ l,m}^n$$

$$+ \frac{1}{A_{1\,l}} \frac{A_{1\ l}\, T_{21\,l,m+1/2}^n - A_{1\,l}\, T_{21\,l,m-1/2}^n}{\Delta s_2} + k_{1\ l}\, T_{13\ l,m}^n = \rho h (u_{1\ l,m}^n)_{\bar{t}t},$$

$$\frac{1}{A_{2\ l}} \left(\frac{A_{2\ l+1/2} T_{12\,l+1/2,m}^n - A_{2\,l-1/2} T_{12\ l-1/2,m}^n}{\Delta s_1} \right) - \frac{1}{A_{2\ l}} \frac{A_{2\ l+1/2} - A_{2\ l-1/2}}{\Delta s_1} T_{21\ l,m}^n$$

$$+ \frac{1}{A_{1\,l}} \left(\frac{A_{1\ l}\, T_{22\,l,m+1/2}^n - A_{1\ l}\, T_{22\ l,m-1/2}^n}{\Delta s_2} \right) + k_{2\ l}\, T_{23\ l,m}^n = \rho h (u_{2\ l,m}^n)_{\bar{t}t},$$

$$\frac{1}{A_{2\ l}} \left(\frac{A_{2\ l+1/2} T_{13\ l+1/2,m}^n - A_{2\ l-1/2} T_{13\ l-1/2,m}^n}{\Delta s_1} \right) - k_{1\ l}\, T_{11\ l,m}^n$$

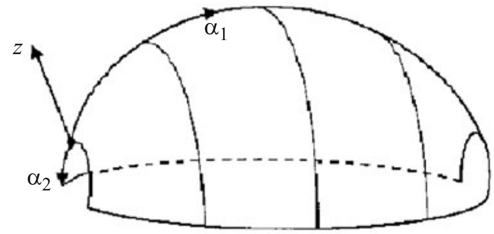

Fig. 1

$$+\frac{1}{A_{1\ l}}\left(\frac{A_{1\ l}\,T^{n}_{23\ l,m+1/2}-A_{1\ l}\,T^{n}_{23\ l,m-1/2}}{\Delta s_2}\right)-k_{2\ l}\,T^{n}_{22\ l,m}+P^{n}_{3\,l,m}=\rho h(u^{n}_{3\ l,m})_{\bar{t}t},$$

$$\frac{1}{A_{2\ l}}\left(\frac{A_{2\ l+1/2}M^{n}_{11\ l+1/2,m}-A_{2\ l-1/2}M^{n}_{11\ l-1/2,m}}{\Delta s_1}\right)-\frac{1}{A_{2\ l}}\frac{A_{2\ l+1/2}-A_{2\ l-1/2}}{\Delta s_1}M^{n}_{22\ l,m}$$

$$+\frac{1}{A_{1\ l}}\left(\frac{A_{1\ l}\,M^{n}_{21\ l,m+1/2}-A_{1\ l}\,M^{n}_{21\ l,m-1/2}}{\Delta s_2}\right)-T^{n}_{13\ l,m}=\frac{\rho h^3}{12}(\varphi^{n}_{1\ l,m})_{\bar{t}t},$$

$$\frac{1}{A_{2\ l}}\left(\frac{A_{2\ l+1/2}M^{n}_{12\ l+1/2,m}-A_{2\ l-1/2}M^{n}_{12\ l-1/2,m}}{\Delta s_1}\right)-\frac{1}{A_{2\ l}}\frac{A_{2\ l+1/2}-A_{2\ l-1/2}}{\Delta s_1}M^{n}_{21\ l,m}$$

$$+\frac{1}{A_{1\ l}}\left(\frac{A_{1\ l}\,M^{n}_{22\ l,m+1/2}-A_{1\ l}\,M^{n}_{22\ l,m-1/2}}{\Delta s_2}\right)-T^{n}_{23\ l,m}=\frac{\rho h^3}{12}(\varphi^{n}_{2\ l,m})_{\bar{t}t}, \qquad (3.1)$$

where the components of the generalized displacement vector $\overline{U}=(u_1,u_2,u_3,\varphi_1,\varphi_2)^T$ of the mid-surface of the ellipsoidal shell are calculated at the integer points of the difference mesh $\overline{U}_{l,m}=(u_{1\,l,m},u_{2\,l,m},u_{3\,l,m},\varphi_{1\,l,m},\varphi_{2\,l,m})^T$ with respect to the space coordinates.

Integrating Eqs. (2.8) and using the explicit approximation with respect to the time coordinate, we obtain the following difference equations for the jth rib:

$$\frac{T^{n}_{21\ j\ m+1/2}-T^{n}_{21\ j\ m-1/2}}{\Delta s_2}+[T_{11}]^{n}_{j}=\rho_{\ j}F_{j}\Big[(u^{n}_{1\ \ m})_{\bar{t}t}\pm h_{ci}(\varphi^{n}_{1\ \ m})_{\bar{t}t}\Big],$$

$$\frac{T^{n}_{22\ j\ \ m+1/2}-T^{n}_{22\ j\ \ m-1/2}}{\Delta s_2}+k_{2\ j\ \ m}T^{n}_{23\ j\ \ m}+[S]^{n}_{j}=\rho_{\ j}F_{j}\Big[(u^{n}_{2\ \ m})_{\bar{t}t}\pm h_{ci}(\varphi^{n}_{2\ \ m})_{\bar{t}t}\Big],$$

$$\frac{T^{n}_{23\ j\ \ m+1/2}-T^{n}_{23\,j\ \ m-1/2}}{\Delta s_2}-k_{2\ j\ \ m}T^{n}_{22\ j\ \ m}+[T_{13}]^{n}_{j}=\rho_{\ j}F_{j}(u^{n}_{3\ \ \ m})_{\bar{t}t},$$

$$\frac{M^{n}_{21\,j\ \ m+1/2}-M^{n}_{21j\ \ m-1/2}}{\Delta s_2}\pm h_{cj}\,\frac{T^{n}_{21\ j\ \ m+1/2}-T^{n}_{21j\ \ m-1/2}}{\Delta s_2}+[M_{11}]^{n}_{j}$$

$$=\rho_{\ j}F_{j}\left[\pm h_{cj}(u^{n}_{1\ \ m})_{\bar{t}t}+\left(h^{2}_{cj}+\frac{I_{crj}}{F_{j}}\right)(\varphi^{n}_{1\ \ m})_{\bar{t}t}\right],$$

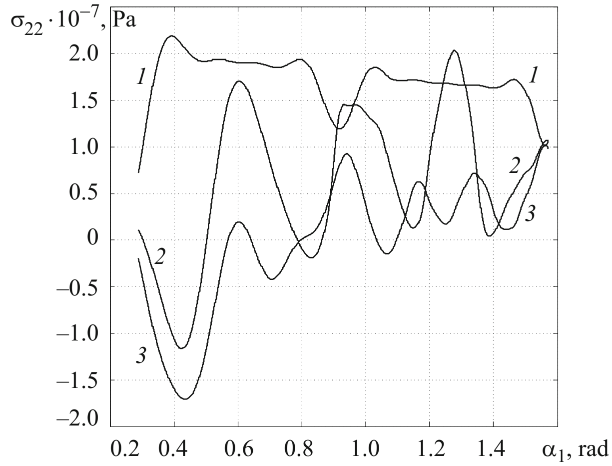

Fig. 2

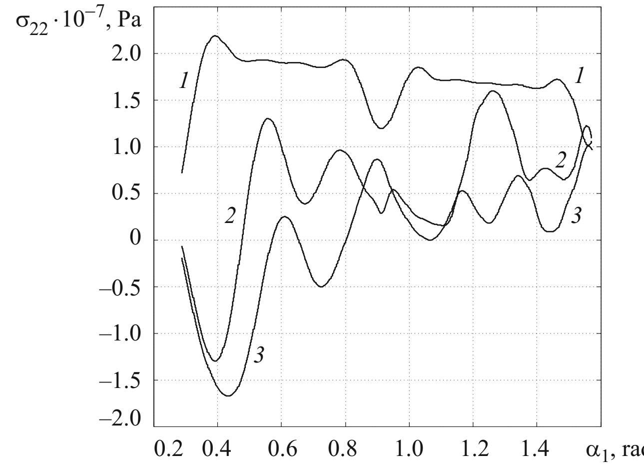

Fig. 3

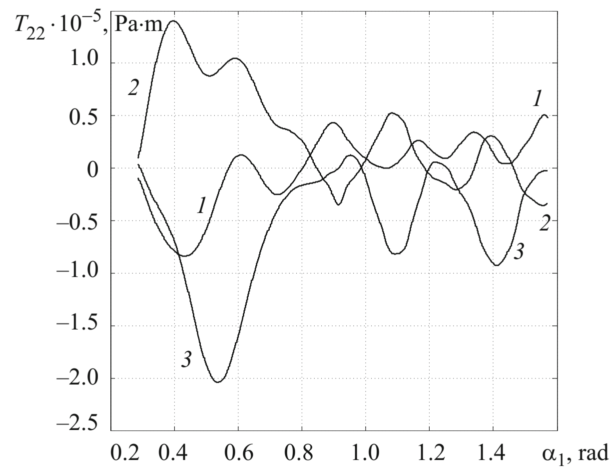

Fig. 4

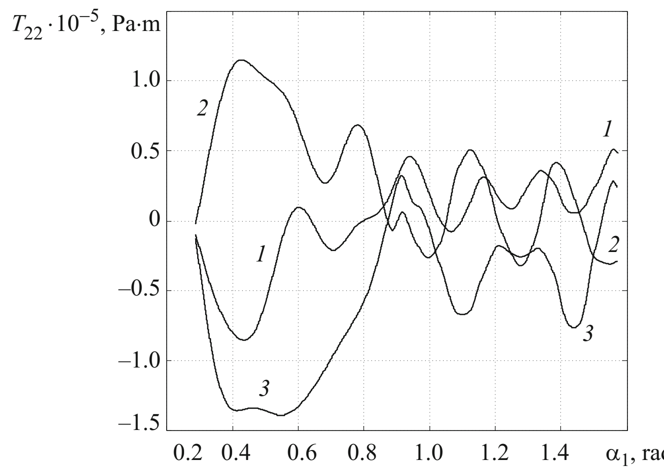

Fig. 5

$$\frac{M^n_{22\,j\;m+1/2} - M^n_{22\,j\;m-1/2}}{\Delta s_2} - T^n_{23\,j\;m} \pm h_{cj}\left(\frac{T^n_{22\,j\;m+1/2} - T^n_{22\,j\;m-1/2}}{\Delta s_2} + k_{2\,j\;m} T^n_{23\,j\;m}\right) + [H]^n_j$$

$$= \rho_j F_j \left[\pm h_{cj} (u^n_{2\;m})_{\bar{t}t} + \left(h^2_{cj} + \frac{I_{2j}}{F_j} \right) (\varphi^n_{2\;m})_{\bar{t}t} \right], \quad (3.2)$$

where the components of the generalized displacement vector $\overline{U}_j = (u_{1j}, u_{2j}, u_{3j}, \varphi_{1j}, \varphi_{2j})^T$ of the cross-sectional centers of mass of the *j*th rib are calculated at the integer points of the difference mesh with respect to the space coordinates.

The second step is the finite-difference approximation of the forces and moments and the strains for the finite-difference energy equation [15]. Equations (2.7) and (2.9) are approximated as in [12].

To analyze the stability of linearized difference equations, we will use the necessary stability conditions,

$$\Delta t \le 2/\omega, \quad (3.3)$$

where $\omega = \max(\omega_0, \omega_j)$, $j = \overline{1, J}$, are the maximum natural frequencies of the discrete-difference system of, respectively, the casing and the *j*th rib.

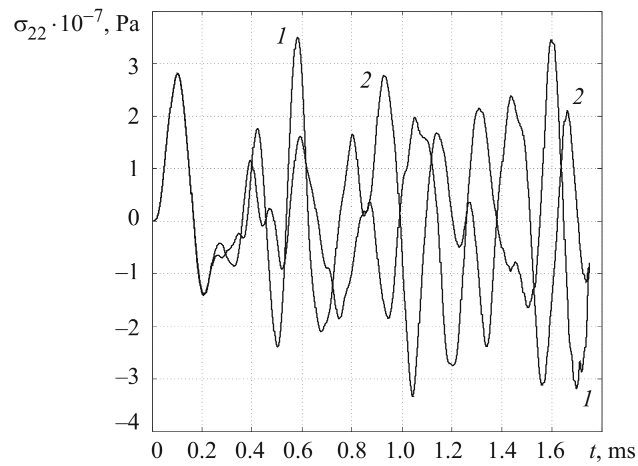

Fig. 6

4. Numerical Example. Let us consider, as a numerical example, the forced vibrations of a rib-reinforced ellipsoidal shell (Fig. 1) with clamped edges in the domain $D = \{\alpha_{10} \le \alpha_1 \le \alpha_{1N}, \alpha_{20} \le \alpha_2 \le \alpha_{2N}\}$ under a distributed normal load $P_3(\alpha_1, \alpha_2, t)$. The boundary conditions $\overline{U}(\alpha_{10}, \alpha_2) = \overline{U}(\alpha_{1N}, \alpha_2) = 0, \overline{U}(\alpha_1, \alpha_{20}) = \overline{U}(\alpha_1, \alpha_{2N}) = 0$. The initial conditions for all the components of the generalized displacement vector at $t = 0$:

$$u_1(\alpha_1, \alpha_2) = u_2(\alpha_1, \alpha_2) = u_3(\alpha_1, \alpha_2) = \varphi_1(\alpha_1, \alpha_2) = \varphi_2(\alpha_1, \alpha_2) = 0,$$

$$\frac{\partial u_1(\alpha_1, \alpha_2)}{\partial t} = \frac{\partial u_2(\alpha_1, \alpha_2)}{\partial t} = \frac{\partial u_3(\alpha_1, \alpha_2)}{\partial t} = \frac{\partial \varphi_1(\alpha_1, \alpha_2)}{\partial t} = \frac{\partial \varphi_2(\alpha_1, \alpha_2)}{\partial t} = 0.$$

The distributed normal load $P_3(\alpha_1, \alpha_2, t)$ is given by

$$P_3(\alpha_1, \alpha_2, t) = A \cdot \sin\frac{\pi t}{T}[\eta(t) - \eta(t - T)],$$

where A and T are the amplitude and duration of the load ($A = 10^6$ Pa, $T = 50 \cdot 10^{-6}$ sec).

The geometrical and mechanical parameters of the shell:

$$\alpha_{10} = \frac{\pi}{12}, \quad \alpha_{1N} = \pi - \frac{\pi}{12}, \quad \alpha_{20} = -\frac{\pi}{2}, \quad \alpha_{2N} = \frac{\pi}{2}, \quad \frac{a}{h} = 60, \quad \frac{b}{a} = 1.5,$$

$$E_1 = E_2 = 7 \cdot 10^{10} \text{ Pa}, \quad \nu_{12} = \nu_{21} = 0.33, \quad \rho = 2.7 \cdot 10^3 \text{ kg/m}^3.$$

The mechanical parameters of the ribs: $E_j = E_1$, $\rho_j = \rho$. The transverse ribs are located in the sections $\alpha_{1j} = \frac{7}{24}\pi + \frac{5}{24}\pi j$, $j = 0, 1, 2$, along the α_2-axis.

Figures 2–5 show the most typical curves for the stress σ_{22} and the force T_{22} for $t_N = 35T$, $\alpha_{10} \le \alpha_1 \le \pi/2$ (due to symmetry with respect to α_1), and $\alpha_2 = 0$. They can be used to analyze the stress state of the structure.

Figures 2 and 3 show the stress σ_{22} as a function of α_1 for outside and inside arrangement of ribs, respectively. Curves *1*, *2*, and *3* correspond to $t_1 = T$, $t_2 = 3T$, and $t_3 = 8T$, respectively.

Figures 4 and 5 show similar curves *1*, *2*, and *3* for the force T_{22} at $t_1 = 3T$, $t_2 = 6T$, and $t_3 = 9T$.

It can be seen where exactly the ribs in the sections α_{1j} $(j = 0, 1, 2)$ are located. The maximum amplitudes of the stress σ_{22} in the shells with inside and outside ribs differ by 25% (curve *2* in Figs. 2 and 3), while the values of the force T_{22} differ by 47% (curve *3* in Figs. 4 and 5).

Figure 6 shows the time dependence of the stress σ_{22} at the characteristic point $(\alpha_1 = \pi/2, \alpha_2 = 0)$ at which the quantities of interest are maximum in magnitude for $t_N = 35T$. Curves *1* and *2* represent the outside and inside arrangement of ribs, respectively. The maximum amplitudes of the stress σ_{22} in the shells reinforced with outside and inside ribs differ by 10%.

Conclusions. Forced nonaxisymmetric vibrations of a rib-reinforced ellipsoidal shell have been studied. The casing and ribs have been described using the refined theory of shells and beams based on the Timoshenko hypotheses. To derive the vibration equations, the Hamilton–Ostrogradskii variational principle have been used. The numerical approach to solving the dynamic equations employs the integro-interpolation method to construct finite-difference schemes for an equation with discontinuous coefficients. Results on the nonaxisymmetric vibrations of a transversely reinforced ellipsoidal shell under a distributed internal load have been presented as a numerical example.

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