Recurrence Measures of Complexity in Energy Market Dynamics

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Abstract: The instability of the price dynamics of the energy market from a theoretical point of view indicates the inadequacy of the dominant paradigm of the quantitative description of pricing processes, and from a practical point of view, it leads to abnormal shocks and crashes. Through the recurrence quantification analysis, we analyze and construct indicators of intermittent events in energy indices, where periods of regular behavior are replaced by periods of chaotic behavior, which could explain the emergence of crisis events. For further analysis, we have chosen daily data of Henry Hub natural gas spot prices, WTI spot prices, and Europe Brent spot prices. Our empirical results present that all of the presented recurrence measures respond in a particular way during crashes and can be effectively implemented for risk management strategies.

1 INTRODUCTION

Global economic and financial systems rely on crude oil to maintain stability, making it a strategic resource for national economic development (Zhang and Wu, 2019; Dong et al., 2018). The importance of examining various factors that may affect crude oil prices is therefore critical to investors, government agencies, and other stakeholders. Many factors contribute to crude oil price fluctuations, including fundamental factors (such as supply and demand of crude oil) (Wu and Zhang, 2014) and non-fundamental factors (such as speculations and investor sentiment) (Ji et al., 2019). Specifically, the global economic environment, political security between oil-producing countries and their neighbors, and economic policy uncertainty prove to have a significant impact on crude oil prices.

Regarding the strategic role of crude oil in economic progress, the market volatility of crude oil prices has had a substantial negative effect on the economy, specifically in those countries that are dependent on imports of crude oil. The impact of many drivers on crude oil price volatility has thus been investigated in a variety of publications, and crude oil market mechanism has become a controversial topic in academia (Coleman, 2012; Sari et al., 2011; Dées et al., 2007). While this was going on, some studies underlined the substantial risks associated with crude oil price changes, as well as their complexity and stochastic nature (Zhang and Wang, 2015; Shahzad et al., 2022; Yin and Wang, 2022; Zhang et al., 2023).

Oil prices are generally referred to as benchmark prices by both WTI and Brent contracts. Hedge funds and traders typically select one or the other contract. Consequently, there is considerable interest in the WTI-Brent pricing structure, including the shapes of the futures curves, the absolute price differences between the two benchmarks, and the degree of integration between the two markets. Hedge funds and financial institutions heavily trade these markets (directly and indirectly). As a result, the prices of jet fuel, heat-

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ing oil, diesel, and gasoline are highly influenced by these markets. The price spread between WTI and Brent is also used as the basis for a number of derivative financial products, such as swaps and options.

The microeconomic theory states that the supply and demand condition determines the fundamental value of crude oil price assets. The financialization of crude oil in the past decade, however, has increased the role of speculation in crude oil prices, making the process of determining crude oil prices more difficult (Kilian, 2009; Flood and Hodrick, 1990).

The natural gas industry has achieved strong growth in recent years because of the large demand market, sufficient low-cost supply, and active global natural gas trade. The forecasting of natural gas prices is one of the most crucial topics in finance since this resource is important for trading, electric power production planning, and regulatory decision-making. Nowadays, Henry Hub in the U.S., NBP in the U.K., and LNG in Japan represent major international trading centers of natural gas. These three centers have become an important reference point for determining the international level of natural gas prices. Among them, Henry Hub has the highest market liquidity, the largest influence, and the best reflection of the market supply and demand. On the other hand, in addition to the basic factors of supply and demand, the price of natural gas is driven by multiple factors such as extreme weather, wars, and geopolitics (Li et al., 2021).

Considering the highly nonlinear and non-stationary characteristics of crude oil and natural gas markets under the influence of complex factors, it is of great research significance to improve the accuracy of early identification of crisis phenomena in those markets. In this paper, we present indicators (indicators-precursors) based on recurrence analysis.

2 METHODOLOGY OF RECURRENCE ANALYSIS

In 1890 Poincaré introduced *Poincaré recurrence theorem* (Poincaré, 2017), which states that certain systems return to their arbitrarily close, or exactly the same initial states after a sufficiently long but finite time. Such property in the case of deterministic behavior of the system allows us to make conclusions regarding its future development.

2.1 Time Delay Method

The state of the system can be described by the set of variables. Its observational state can be expressed through a d-dimensional vector or matrix, where each of its components refers to a single variable that represents a property of the system. After a while, the variables change, resulting in different system states.

Usually, not all relevant variables can be captured from our observations. Often, only a single variable may be observed. *Thakens' theorem* (Takens, 1981) that was mentioned in previous sections ensures that it‘s possible to reconstruct the topological structure of the trajectory formed by the state vectors, as the data collected for this single variable contains information about the dynamics of the whole system.

For an approximate reconstruction of the original dynamics of the observed system, we project the time series onto a Reconstructed Phase Space (Eckmann and Ruelle, 1985; Kantz and Schreiber, 2003; Ott et al., 1994) with the commonly used time delay method (Kantz and Schreiber, 2003) which relied on the *embedding dimension* and *time delay*.

The embedding dimension is being the dimensionality of the reconstructed system (corresponds to the number of relevant variables that may differ from one system to another. The time delay parameter specifies the temporal components of the vector components.

2.2 Recurrence Plot

Recurrence plot (RP) have been introduced to study dynamics and recurrence states of complex systems. When we create RP, at first, from recorded time series we reconstruct phase-space trajectory. Then, according to Eckmann et al. (Eckmann et al., 1987), we consider a trajectory $\vec{X}(i)$ on the reconstructed trajectory. The recurrence plot is an array of dots in a $N \times N$ matrix, where dot is placed at (i, j) whenever $\vec{X}(j)$ is sufficiently close to $\vec{X}(i)$, and both axes are time axes which mathematically can be expressed as

$$R_{ij} = \Theta(\varepsilon - \left\| \vec{X}(i) - \vec{X}(j) \right\|), \quad \text{for } i, j = 1, \ldots, N, \tag{1}$$

where $\| \|$ is a norm (representing the spatial distance between the states at times i and j); ε is a predefined recurrence threshold, and $\Theta(\cdot)$ is the Heaviside function. As a result, the matrix captures a total of N^2 binary similarity values.

Typically, L_p-norm is applied to determine the pairwise similarity between two vectors. According to Webber and Zbilut (Webber and Zbilut, 2005), the L_1-norm (Taxicab metric), the L_2-norm (Euclidean metric), and the L_∞-norm (Chebyshev metric) can serve as candidates for measuring distance between trajectories in phase space.

Also, as it can be seen from equation (1), the similarity between vectors is determined by a threshold

ε. The choice of $\varepsilon > 0$ ensures that all vectors that lie within this radius are similar to each other, and that dissimilarity up to a certain error is permitted (Poincaré, 2017).

The fixed radius for recurrent states is the commonly used condition, which leads to equally sized ε-neighborhoods. The shape in which neighborhoods lie is determined by the distance metric. Applying the fixed threshold with the distance metric, we define recurrence matrices that are symmetric along the middle diagonal. The self-similarity of the multi-dimensional vectors reflects in the middle diagonal, which is commonly referred to as the line of identity (LOI). In contrast, it is not guaranteed that a recurrence matrix is symmetric if the condition of the fixed number of nearest neighbors is applied. For specific purposes (e.g., quantification of recurrences), it can be useful to exclude the LOI from the RP, as the trivial recurrence of a state with itself might not be of interest (Charles et al., 2015).

2.2.1 Recurrence Plots and their Structures

The main purpose of RP is the visualization of trajectories and hidden patterns of the systems (Marwan et al., 2007; Charles et al., 2015).

The dots within RP, representing the time evolution of the trajectories, exhibit characteristic large-scale and small-scale patterns. Large-scale patterns of RP can be classified as

- *homogeneous* – autonomous and stationary systems, which consist of many recurrence points that are homogeneously distributed (relaxation times are short);
- *periodic* – long, uninterrupted, and diagonally oriented structures that represent which indicate periodic behavior. These lines are usually distributed regularly;
- *drift* – systems with patterns paling or darkening from the LOI to the outer corners of RP;
- *disrupted* – systems with drastic changes as well as extreme events in the system dynamics.

The small-scale clusters can represent a combination of *isolated dots* (abrupt events). Similar evolution at different periods in time or in reverse temporal order will present *diagonal lines* (deterministic structures) as well as *vertical/horizontal lines* to inscribe laminar states (intermittency) or systems that paused at singularities. For the quantitative description of the system, such small-scale clusters serve the base of the *recurrence quantification analysis* (RQA).

2.3 Recurrence Quantification Analysis

The graphic representation of the system suits perfectly for a qualitative description. However, the main disadvantage of graphical representation is that it forces users to subjectively intuit and interpret patterns and structures presented within the recurrence plot. Also, with the increasing size of RP, they can be hardly depicted on graphical display as a whole. As a result, we need to work with separated parts of the original plot. Analysis in such a way may create new defects, which should distort objectivity of the observed patterns and lead to incorrect interpretations. To overcome such limitation and spread an objective assessment among observers, in the early 1990s by Webber and Zbilut (Webber and Zbilut, 1994; Zbilut and Webber, 1992) were introduced definitions and procedures to quantify RP's complexity, and later, it has been extended by Marwan et al. (Marwan et al., 2002).

The first known measure of the RQA is *recurrence rate*, which measures the probability that the studied process will recur (*RR*):

$$RR = \frac{1}{N^2}\sum_{i,j=1}^{N} R_{i,j}. \tag{2}$$

Another measure is based on frequency distribution of line structures in the RP. First, we consider the histogram of the length of the diagonal structures in the RP

$$\begin{aligned} P(l) = \sum_{i,j=1}^{N} &(1 - R_{i-1,j-1}) \\ &\times (1 - R_{i+l,j+l}) \prod_{k=0}^{l-1} R_{i+k,j+k}. \end{aligned} \tag{3}$$

The percentage of recurrence points that form diagonal segments of minimal length l_{min} parallel to the main diagonal is the measure of *determinism* (DET):

$$DET = \sum_{l=l_{min}}^{N} lP(l) \Bigg/ \sum_{l=1}^{N} lP(l). \tag{4}$$

Systems that are characterized by long diagonal lines are presented to be periodic. From chaotic signals, we would expect short diagonal lines, and stochastic processes would not present any diagonal lines. Performing the RQA, typically, we rely on the lines with minimal length, which excludes the shorter lines, which may be spurious for characterizing deterministic processes. In our case, $l_{min} = 2$ is considered. In case when $l_{min} = 1$, DET and RR are identical.

Considering diagonal line segments, we can emphasize the longest one – L_{max}. This indicator measures the maximum time that two trajectories remain close to each other and can be interpreted as the maximum prediction time:

$$L_{max} = \max\left(\{l_i \,|\, i = 1,\ldots,N_l\}\right), \quad (5)$$

where $N_l = \sum_{l \geqslant l_{min}} P(l)$ is the total number of diagonal lines.

Divergence (DIV) is the inverse of L_{max} characterizes the exponential divergence of the phase space trajectory (Goldberger et al., 2000; Kirchner et al., 2014):

$$DIV = 1 \Big/ L_{max}. \quad (6)$$

For longer diagonal lines system is more deterministic and, therefore, the measure of divergence is also lower. The smaller L_{max}, the more divergent are trajectories and more chaotic the studied system. According to Eckmann et al. (Eckmann et al., 1987), DIV can be used to estimate the largest positive Lyapunov exponent.

Another measure which is related to the diagonal line segments is the *average diagonal line length* (L_{mean}):

$$L_{mean} = \sum_{l=l_{min}}^{N} lP(l) \Big/ \sum_{l=l_{min}}^{N} P(l) \quad (7)$$

It can be interpreted as the mean prediction horizon of the system, and it measures average time that two trajectories remain close to each other.

Using the classic Shannon entropy, we can measure the hidden complexity of recurrence structures in the RP. In accordance with this study, the entropy of diagonal line histogram (*DLEn*) is of the greatest interest. It can be defined as:

$$DLEn = -\sum_{l=l_{min}}^{N} p(l) \ln p(l) \quad (8)$$

and

$$p(l) = P(l) \Big/ \sum_{l=l_{min}}^{N} P(l), \quad (9)$$

where $p(l)$ captures the probability that a diagonal line has exactly length l, and *DLEn* reflects the complexity of deterministic structure in the system. The more uniform is the frequency distribution of diagonal lines, the higher the value of *DLEn*. If there is predominant deterministic behavior with a particular period l, then *DLEn* becomes lower.

As it was mentioned, the RP structure consists of vertical (horizontal lines). For them Marwan and Webber (Marwan and Webber, 2015) proposed additional recurrence measures. The first of them is the *laminarity* (*LAM*) Analogously to the equation (4), which measures the percentage of diagonal lines with minimal length l_{min} in the RP, we can calculate the fraction of recurrence points forming vertical structures of minimal length v_{min}:

$$LAM = \sum_{v=v_{min}}^{N} vP(v) \Big/ \sum_{v=1}^{N} vP(v) \quad (10)$$

with

$$P(v) = \sum_{i,j=1}^{N} (1 - R_{i,j-1}) \times (1 - R_{i,j+v}) \prod_{k=0}^{v-1} R_{i,j+k} \quad (11)$$

as the histogram of lengths of vertical lines.

Since it measures the overall amount of vertical lines, it characterizes the percentage of laminar states within the system. If *LAM* increases, then there are more vertical or diagonal structures than isolated recurrent points.

Similarly to L_{max}, we can define the measure which will indicate the maximum time that a system holds an unchangeable pattern – the *maximal vertical lines length* (V_{max}):

$$V_{max} = \max\left(\{v_i \,|\, i = 1,\ldots,N_v\}\right), \quad (12)$$

where $N_v = \sum_{v \geqslant v_{min}} P(v)$ is the total number of vertical lines.

Vertical line divergence ($VDIV$) is the analogous to (6), which can be related to the rate of divergence from laminar state:

$$VDIV = 1 \Big/ V_{max}. \quad (13)$$

Consequently, we can define the average time that two trajectories remain at a specific state – *trapping time* (TT):

$$TT = \sum_{v=v_{min}}^{N} vP(v) \Big/ \sum_{v=v_{min}}^{N} P(v). \quad (14)$$

For high TT values we would expect the system to consist of more laminar states, whereas low TT values would indicate abrupt changes in the system's dynamics.

The variability of laminar states with different duration time can be measured in the same way as for diagonal lines – using Shannon entropy. The complexity of vertical lines can be measures according to the following equation:

$$VLEn = -\sum_{v=v_{min}}^{N} p(v) \ln p(v) \quad (15)$$

with

$$p(v) = P(v) \Big/ \sum_{v=v_{min}}^{N} P(v) \quad (16)$$

indicating the probability of a vertical line to have length $v \geqslant v_{min}$.

In the same manner, we can quantify the variation (complexity) of abrupt changes during the studied periods in the energy markets. Regarding equation (7), we can quantify the average time of divergence when two trajectories in the phase-space remain out of recurrence threshold ε. This measure can be called as *average white vertical line length* (WVL_{mean}):

$$WVL_{mean} = \sum_{w=w_{min}}^{N} wP(w) \Big/ \sum_{w=w_{min}}^{N} P(w), \qquad (17)$$

where $P(w)$ is the frequency of white vertical lines in the RP. This measure can be interpreted as the mean horizon of unpredictability of the system.

This kind of complexity is associated with the white vertical lines in the RP and can be quantified in the following way:

$$WVLEn = -\sum_{w=w_{min}}^{N} p(w) \ln p(w) \qquad (18)$$

with

$$p(w) = P(w) \Big/ \sum_{w=w_{min}}^{N} P(w) \qquad (19)$$

indicating the probability of a white vertical line to have length $w \geqslant w_{min}$.

The further measure is based on the ration between DET and RR, and known as *ratio* (DET/RR):

$$DET/RR = N^2 \sum_{l=l_{min}} P(l) \Big/ \left(\sum_{l=1}^{N} lP(l) \right)^2 \qquad (20)$$

In the same manner, we can define another measure which is based on the ratio between LAM and DET:

$$LAM/DET = \sum_{v=v_{min}}^{N} vP(v) \cdot \sum_{l=1}^{N} lP(l) \Big/ \sum_{v=1}^{N} vP(v) \cdot \sum_{l=l_{min}}^{N} lP(l). \qquad (21)$$

This measures can be used to uncover hidden transitions in the dynamics of the system (Webber and Zbilut, 1994).

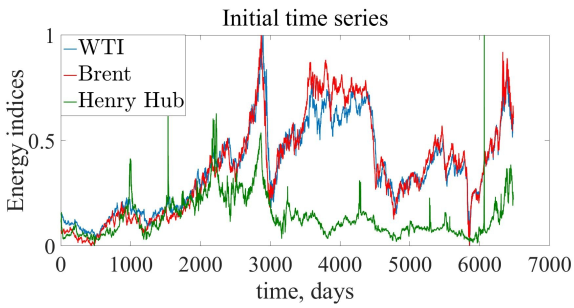
3 RESULTS AND ANALYSIS

Regarding previous studies, we present additional analysis on co-movement between 3 energy-related indices and construct indicators or indicators-precursors based on the using recurrence analysis.

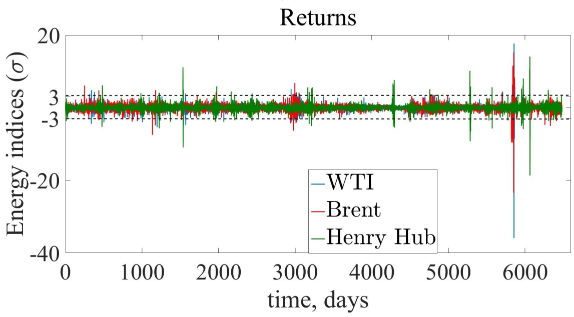
The presented work uses daily data of Henry Hub natural gas spot prices (US$/MMBTU) ranged from 7 February 1997 to 18 October 2022; Cushing, OK WTI spot prices FOB (US$/BBL) ranged from 20 May 1987 to 17 October 2022; Europe Brent spot prices FOB (US$/BBL) ranged from 20 May 1987 to 17 October 2022 (U.S. Energy Information Administration, 1997, 1986).

In figure 1 are presented:

- the dynamics of the initial time series;
- standardized returns, where returns can be calculated as $G(t) = [x(t+\Delta t) - x(t)]/x(t)$ and their standardized version as $g(t) = [G(t) - \langle G \rangle]/\sigma$;
- probability density function of the standardized returns.



(a)



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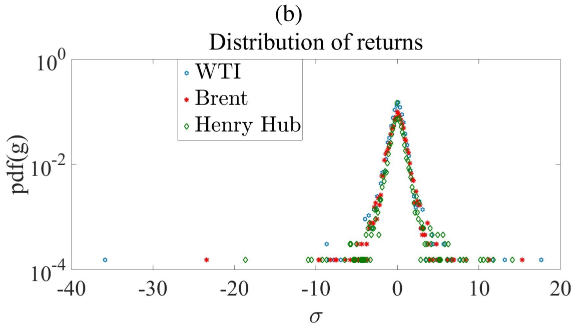


(c)

Figure 1: Initial time series (a), standardized returns (b), and pdf of standardized returns of WTI spot prices (WTI), Europe Brent spot prices (Brent), and Henry Hub natural gas spot prices (Henry Hub).

We can see that most periods in energy markets are defined by events that exceed $\pm 3\sigma$. Both WTI and Brent returns are characterized by much more extensive crashes. Previous studies pointed out that such events are located in fat-tails of the probability distribution. Such crashes are the main source of high complexity and non-linearity in the studied systems.

Most of our results are based on the sliding window approach. The idea here is to take a sub-window of a predefined length w. For that sub-window, we perform recurrence quantification analysis, get necessary indicators that are appended to the array. Then, the window is shifted by a predefined time step h, and the procedure is repeated until the time series is completely exhausted.

We have performed RQA under sliding window procedure for standardized returns and standardized initial time series (Soloviev et al., 2020; Bielinskyi and Soloviev, 2018; Bielinskyi et al., 2022, 2021c,b, 2020). We have found that standardized initial time series better expresses internal complexity and recurrent properties of the energy market indices.

RQA was performed for the following parameters:

- embedding dimension $d_E = 1$;
- time delay $\tau = 1$;
- recurrence threshold $\varepsilon = 0.3$;
- L_2-norm as a candidate for measuring distance between trajectories in phase space;
- minimum diagonal line length $l_{min=2}$;
- minimum vertical line length $v_{min} = 2$;
- minimum white vertical line length $w_{min} = 2$;
- sliding window length $w = 500$ days;
- sliding window time step $h = 1$ day.

Worth to mention that the experiments were performed for sliding window lengths of 250 days and 500 days. We have chosen the second option since it represents a more reliable and smoother dynamics of all the presented indicators. All described measures result into highly volatile variation with the sliding window of 250 days that difficult to interpret.

In figure 2 are presented RPs for the studied series.

Recurrence plots in figure 2 represent that the studied energy markets are highly inhomogeneous. As it was expected, nonlinear structure of WTI and Brent is presented to be very similar, comparing to Henry Hub. Recurrence structure of all indices varies across time. They do not follow a certain pattern, presented to be non-periodic, and there are differences in the patterns that concern the frequency of their appearance, shape, and size. It should be noticed that for the oil markets first 4000 days are presented to be

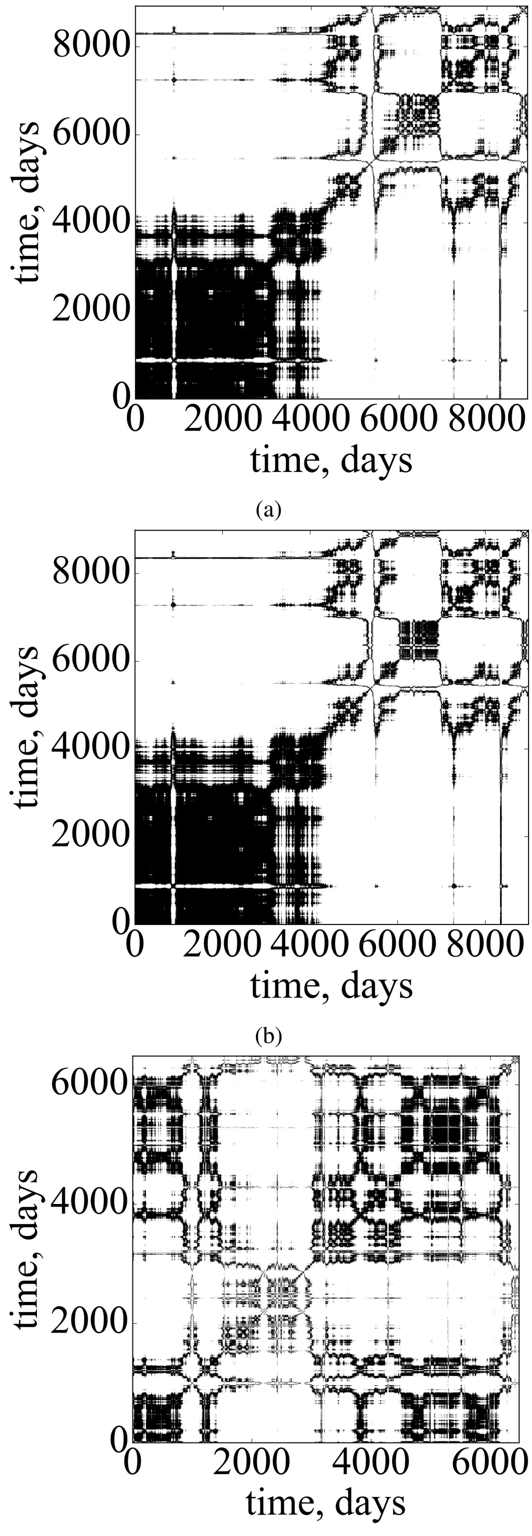


Figure 2: Recurrence plots calculated for WTI (a), Brent (b), and Henry Hub (c) standardized time series.

highly recurrent, while the remaining days seem to be more volatile, which is indicated by high proportion of white regions. The recurrence structure of Henry Hub index is presented to be more uniformly distributed. The variations of recurrence patterns should be more noticeable during crashes. Recurrence quantitative indicators should give a more accurate representation of the complex, chaotic structure of the studied markets.

Figure 3 represents recurrence measures of determinism (DET) and laminarity (LAM).

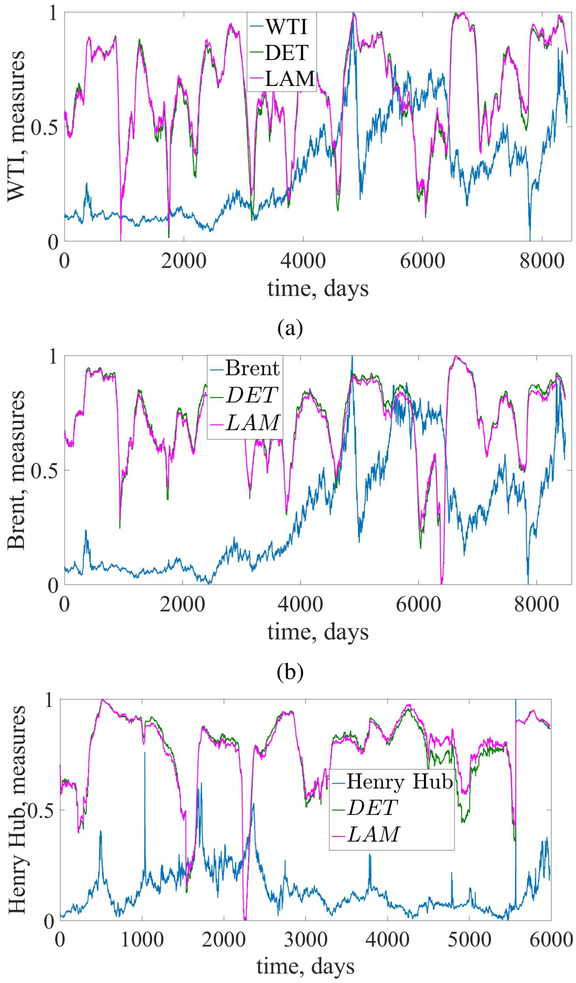


Figure 3: Recurrence measures of determinism (DET) and laminarity (LAM) calculated for WTI (a), Europe Brent (b), and Henry Hub (c) indices.

In figure 3 we see that DET and LAM increase during crisis events of all markets. We may conclude that those critical states are characterized by high degree of laminarity and determinism. Crashes are presented to be highly complex and deterministic. Their degree of predictability becomes higher, and corresponding recurrence measures seem to be indicators or even indicators-precursors of such changes.

Figure 4 represents recurrence measures of ratios DET/RR and LAM/DET.

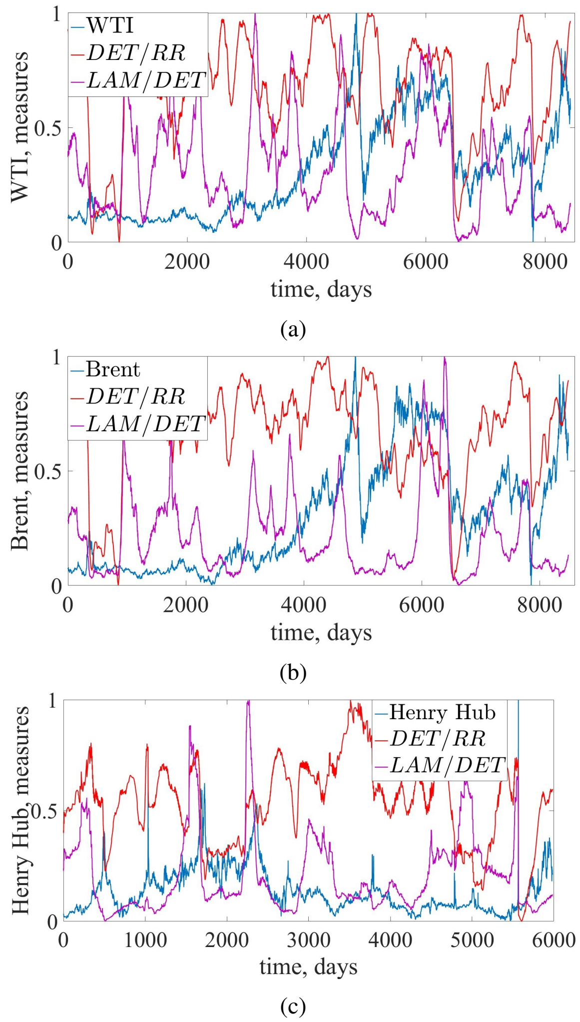


Figure 4: Recurrence measures (DET/RR) and (LAM/DET) calculated for WTI (a), Europe Brent (b), and Henry Hub (c) indices.

From figure 4 we can see that both measures decrease during crisis events of energy indices. For ratio DET/RR we may say that the overall percentage of recurrence points in RP becomes higher than the percentage of only diagonal structures in RP. For ratio LAM/DET we see precisely the same behavior during crashes, i.e., it starts to decline during crisis or even in advance. Thus, it can be seen that the overall determinism of the system during crashes is much higher than the degree of laminarity.

Figure 5 shows recurrence measures of diagonal

(DIV) and vertical line ($VDIV$) divergences.

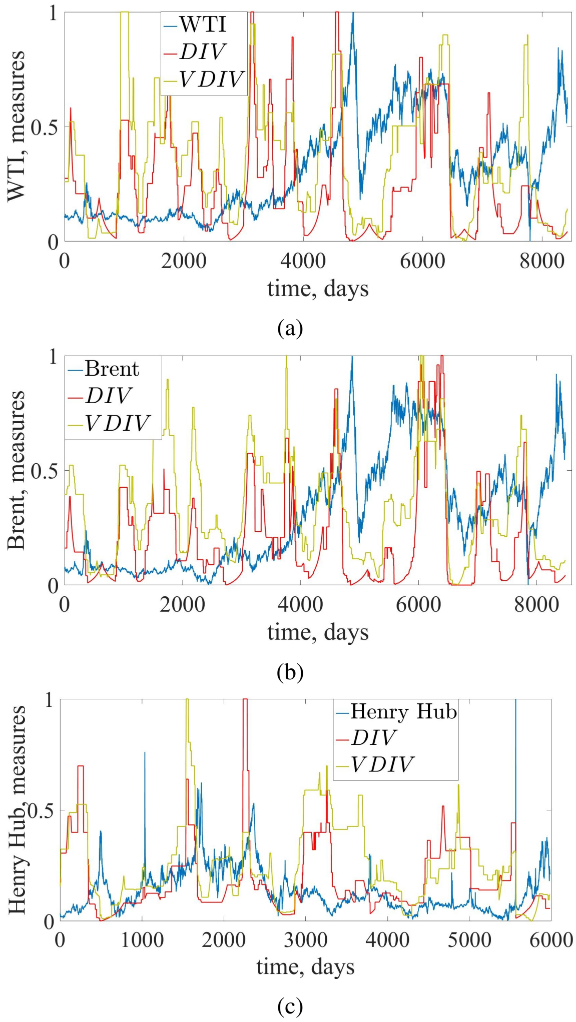


Figure 5: Recurrence measures of diagonal line divergence (DIV) and vertical line divergence ($VDIV$) calculated for WTI (a), Europe Brent (b), and Henry Hub (c) indices.

Figure 5 demonstrates that the divergence of deterministic and laminar structure of energy-related markets becomes lower during critical states. Since both measures are inverse quantities to maximum diagonal and vertical line length (L_{max} and V_{max}), such behavior has to be obvious. Previous measures have made it clear to us that the crisis phenomena of energy indices are characterized by a high degree of determinism and laminarity. In this case, the lengths of diagonal and vertical lines should also increase, which indicate an increase in the horizon of predictability and immutability.

Figure 6 represents recurrence measures of recurrence rate (RR), average diagonal line length (L_{mean}), and trapping time (TT).

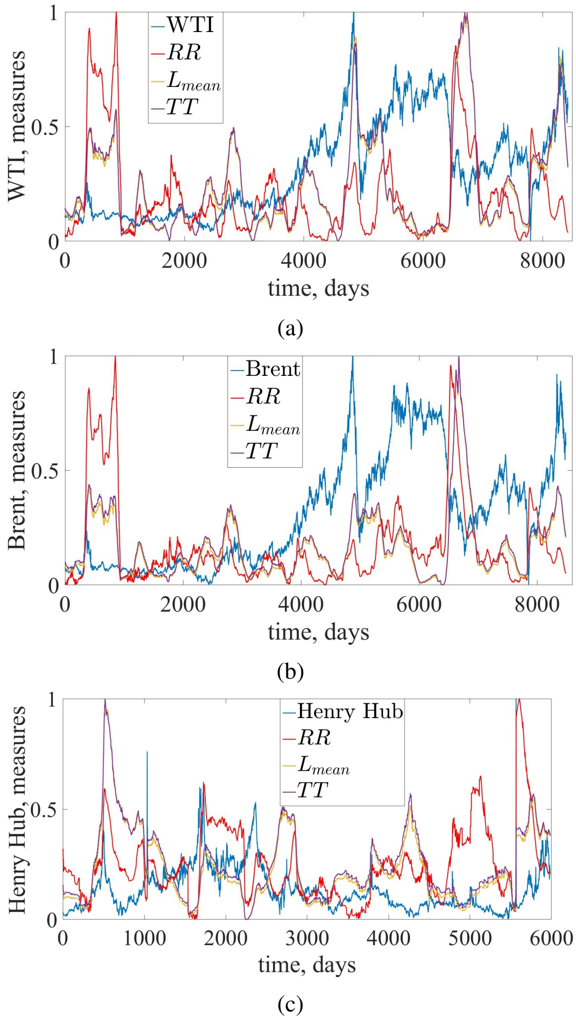
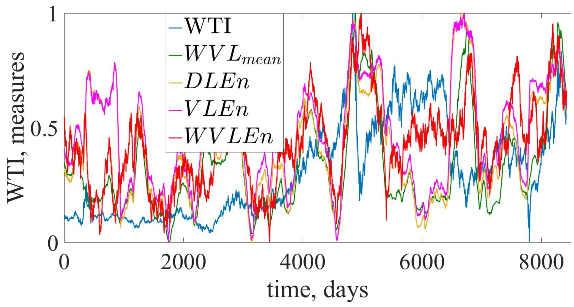


Figure 6: Recurrence measures of recurrence rate (RR), average diagonal line length (L_{mean}), and trapping time (TT) calculated for WTI (a), Europe Brent (b), and Henry Hub (c) indices.

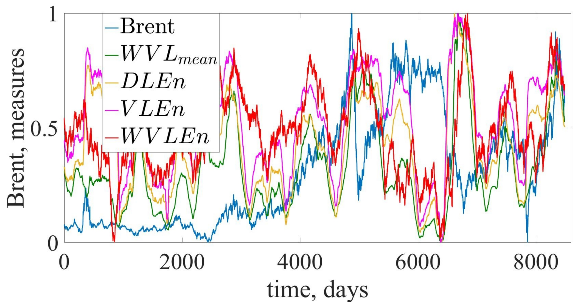
In figure 6 we see that recurrence rate increases during crisis phenomena. This means that the total number of trajectories in the phase space that are close enough to each other becomes larger on the eve of a crisis or at the moment of its onset. Thus, the probability of recurrence state increases during crash. Regarding previous measures, RR and L_{mean}, we see that the average degree of predictability during crisis increases. The same can be seen for trapping time: average degree of changeability increases during crashes. Based on this indicator, we may conclude that the system is 'trapped' in a state of crisis.

Figure 7 presents recurrence measures of average white vertical line length (WVL_{mean}), and diagonal, vertical and white vertical line entropies ($DLEn$, $VLEn$, and $WVLEn$).

From figure 7 we can see that all the presented



(a)



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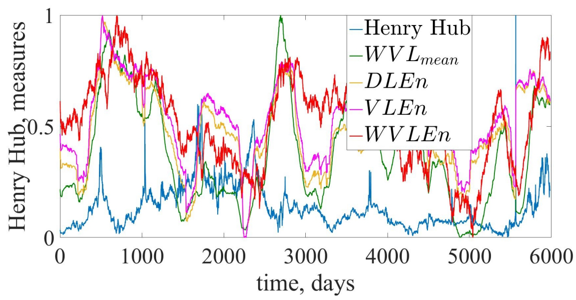

(c)

Figure 7: Recurrence measures of average white vertical line length (WVL_{mean}), diagonal line entropy ($DLEn$), vertical line entropy ($VLEn$), and white vertical line entropy ($WVLEn$) calculated for WTI (a), Europe Brent (b), and Henry Hub (c) indices.

quantitative measures of recurrence begin to increase during crises, indicating a special state of the market at these points in time. The average white vertical line length shows that crisis events are characterized not only by the determinism of the dynamics of market movement, but also by the dissimilarity of these events to many previous ones, since the length of the white vertical lines is becoming an increasing trend. It can also be said that the market represents a much more deterministic structure than a laminar one. Also, the degree of volatility of these events can knock the market dynamics out of the limits of the epsilon value.

The diagonal line entropy also shows an increasing trend. Since the Shannon entropy is maximal with a uniform distribution, it can be concluded that the collapse events of energy indices are characterized by different horizons of predictability. That is, in the pre-crisis dynamics there is no black diagonal line of the same length, which is the dominant one. During a crisis, horizons of determinism appear, which gain even more weight if compared with the rest.

The vertical line entropy increases similarly to $DLEn$. We may assume that similarly to diagonal lines laminar states have different horizons of invariability during crash events, and these horizons of invariability have greater tendency to uniform distribution.

The white vertical line entropy increases similarly to other entropies. This dynamics is consistent with the WVL_{mean} measure.

4 CONCLUSIONS

In this paper, we have studied highly nonlinear and nonstationary dynamics of oil and gas markets from the perspective of the recurrence analysis. Taking into account daily data of Henry Hub natural gas spot prices from 7 February 1997 to 18 October 2022, WTI spot prices from 20 May 1987 to 17 October 2022, and Europe Brent spot prices for the same period as WTI, we have drown some conclusions from the empirical results.

Firstly, recurrence plots presented that the studied markets demonstrate highly inhomogeneous. As it was expected, nonlinear structure of WTI and Brent is presented to be very similar, comparing to Henry Hub. Recurrence structure of all indices varies across time. They do not follow a certain pattern, and there are differences in frequency, shape, and size of black- and white-dot patterns that appear across time.

From quantitative measures of complexity, we have drawn the following conclusions:

1. Crash events of energy-related indices are characterized by high degree of laminarity and determinism. Crashes are presented to be highly complex and deterministic.
2. The overall percentage of recurrence points in RP becomes higher than the percentage of only diagonal structures in RP. At the same time, the percentage of diagonal lines in RP during crises is much higher than the percentage of vertical lines. Thus, the overall degree of determinism is larger than laminarity.
3. The divergence of deterministic and laminar structure of WTI, Brent, and Henry Hub becomes lower during critical states that indicate higher degree of repeatability in the dynamics of the studied systems. Also, it gives understanding that

the phase-space trajectories become close to each other during critical phenomena of financial systems.

4. Such measures as recurrence rate, mean diagonal line length, and trapping time also increase during crisis phenomena. This means that the total number of trajectories in the phase space that are close enough to each other becomes larger before or during crash. Therefore, the probability of recurrence state increases, and the average degree of predictability becomes higher. A larger portion of vertical lines indicates that the system is 'trapped' in a state of crisis for a particular period of time.
5. Entropy-based measures and, particularly, white vertical line measures show that energy-related indices represent complex nonlinear patterns that combine not only horizons of determinism and laminarity, but also some dissimilarity patterns reflected into white lines.

The applied approach to WTI, Brent, and Henry Hub indices approve that the energy market is an open, highly complex, chaotic, and nonlinear system that depends on different technical and fundamental factors. Although RPs and RQA give promising results for crisis prediction and the construction of early-warning indicators, it needs further development to give applicable trading strategies relying on recurrence indicators and further development of autonomous trading bots.

Also, since the proposed recurrence measures are only indicators (indicators-precursors) that give the possibility to monitor crisis phenomena at a particular moment of the market's existence, forecasting of such events requires integration of the proposed indicators with the particular forecasting models (Yin and Wang, 2022; Fang et al., 2023; Li et al., 2021; Zhang et al., 2023; Zou et al., 2023; Guliyev and Mustafayev, 2022; Kiv et al., 2021). It seems a promising direction at the junction of artificial intelligence and fuzzy logic methods (Bielinskyi et al., 2021a; Bondarenko, 2021; Kmytiuk and Majore, 2021; Kobets and Novak, 2021; Kucherova et al., 2021; Lukianenko and Strelchenko, 2021; Miroshnychenko et al., 2021).

At the same time, we intend to investigate cross-recurrences between energy indices and different technical and fundamental indicators using such approaches as cross- and joint-recurrence quantification analysis (Ashe and Egan, 2023; He and Huang, 2020; Romano et al., 2004).

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