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Keywords (separated by '-')	Stock market - crypto market - cross-correlations - multifractal analysis - crash - complex systems - indicator-precursor

The Analysis of Multifractal Cross-Correlation Connectedness Between Bitcoin and the Stock Market

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Keywords: Stock market · crypto market · cross-correlations · multifractal analysis · crash · complex systems · indicator-precursor

1 Introduction

After the COVID-19 pandemic and during the Russia-Ukraine war [1, 7, 8, 24, 25, 28, 37, 46, 53], decentralized finance with its one of the most popular representatives Bitcoin (BTC) gained rapid popularity [2, 10, 17, 19, 50]. These innovations gained attention from policymakers, financial regulators, scientists, and ordinary people, who despite the regulatory laws in their country, continue to help people using the benefits of decentralized financial operation and analyze it from the perspective of complex systems [15, 47, 57].

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Since approximately June 2021, the correlation between stock and crypto markets started an upward trend. In particular, this can be connected to the growing number of financial instruments in the crypto market, which makes investors behave in a similar way as to the stock market. Since BTC is still developing digital asset, the fluctuations in it are violent and further dynamics begins to be speculated. Thus, correlations between stock and crypto markets vary across time, demonstrating non-linear dependence.

One of the possible approaches to measure long-term memory (correlations) in time series, which was called rescaled range analysis (R/S), was proposed by Hurst [22]. Lo found that classical R/S analysis was sensitive to short-term memory of a system, which may lead to bias error of nonstationary time series [32]. Considering the limitations of R/S analysis, Peng et al. [40] developed detrended fluctuation analysis (DFA). Kantelhardt extended classical DFA to a multifractal DFA that gives the possibility to study long-term memory of both small and large fluctuations using a range of statistical moments [27]. Interesting for us methods were developed by Podobnik and Stanley [41] and then extended to multifractal version by Zhou [56], that gives the possibility to study long-range cross-correlations between two nonstationary time series such as crypto and stock markets. Classical multifractal DFA (MF-DFA) and multifractal extension of detrended cross-correlation analysis (MF-DCCA) have been widely applied to such complex financial systems as foreign exchange markets, stock markets, crude oil market, carbon and commodity markets, futures, investment strategies, and even for Twitter happiness sentiment, mass and new media [9, 13, 31, 33–36, 54, 55].

The aim of this study is to study the degree of cross-correlation between one of the most capitalized and developed stock markets of the USA and China represented by the Standard and Poor's 500 (S&P 500) and the Hang Seng (HSI) with the cryptocurrency market represented by BTC. All data we take from Yahoo! Finance [48] for the period from September 15, 2014 to May 22, 2022 to make it comparable with the data range of BTC dates provided by the mentioned data source. Also, we present the indicators (indicators-precursors) of crash phenomena in stock and crypto markets based on MF-DCCA.

2 Multifractal Detrended Cross-Correlation Analysis

Multifractal detrended cross-correlation analysis that was derived from standard DCCA gives multifractal characteristics derived from power-law cross-correlations of time series [56]. This approach modifies standard detrended covariance fluctuation function to q th order. For its calculations, we take two time series $\{x_i | i = 1, 2, \ldots, N\}$ and $\{y_i | i = 1, 2, \ldots, N\}$ and find their cumulative profiles $X(i) = \sum_{k=1}^{i} [x_k - \langle x \rangle]$ and $Y(i) = \sum_{k=1}^{i} \left[y_k - \langle y \rangle\right]$, where $\langle \cdot \rangle$ is an average of an analyzed series.

Then, by dividing the series into $N_s \equiv int(N/s)$ non-overlapping segments v of equal length s, we explore how evolves the covariance of the residuals of two systems:

$$f^2(v, s) = \frac{1}{s} \sum_{i=1}^{s} \{X[(v-1)s + i] - \tilde{X}^v(i)\Big\} \times \Big\{Y[(v-1)s + i] - \tilde{Y}^v(i)\Big\}, \quad (1)$$

where $\tilde{X}^v(i)$ and $\tilde{Y}^v(i)$ are m-order polynomials for each sub-series v.

Since N is usually not an integer multiple of s, we may neglect the last part of a time series. Thus, we have to repeat the procedure of division from the end of the series and obtain $2N_s$ sub-series v ($v = 1, \ldots, 2N_s$). Then, we apply the equation above to the reversed segments:

$$f^2(v,s) = \frac{1}{s}\sum_{i=1}^{s} \begin{array}{l} \{X[N-(v-1)s+i] - \tilde{X}^v(i)\} \\ \times\left\{Y[N-(v-1)s+i] - \tilde{Y}^v(i)\right\} \end{array}. \tag{2}$$

As the result, we calculate the fluctuation function $F_q(s)$ for the combination of various scales s and statistical moments q:

$$F_q(s) = \begin{cases} \left(\frac{1}{2N_s}\sum_{v=1}^{2N_s}\left[f^2(v,s)\right]^{q/2}\right)^{1/q}, & q \neq 0, \\ exp\left(\frac{1}{4N_s}\sum_{v=1}^{2N_s}\ln\left[f^2(v,s)\right]\right), & q = 0. \end{cases} \tag{3}$$

By analyzing the log-log plots of $F_q(s)$ versus s, we can get the scaling behavior of the fluctuation function. Particularly, if time series are power-law cross-correlated, then $F_q(s) \propto s^{h_{xy}(q)}$, where $h_{xy}(q)$ represents a multifractal generalization of power-law cross-correlation Hurst exponent. For $q = 2$, it is the cross-correlation scaling exponent, which is similar to the known Hurst exponent H [22].

This extension of the Hurst exponent works in the same way:

1. If $h_{xy}(2) > 0.5$, the cross-correlations between time series are presented to be persistent: an increase (a decrease) in one time series is followed by an increase (a decrease) in another time series.
2. If $h_{xy}(2) < 0.5$, the cross-correlations between time series are presented to be anti-persistent: an increase in one time series is likely to be followed by a decrease in the other time series.
3. If $h_{xy}(2) \approx 0.5$, both time series follow a random walk, i.e., there are no correlations between them.
4. If $h_{xy}(2) > 1$, both time series are presented to be highly correlated and non-stationary.

Values of q emphasize the density of small (large) fluctuations. If those values are negative, we make an accent on scaling properties of small fluctuations. For positive values, scaling properties of the large magnitudes dominate. Generally, if our multifractal characteristics do not depend on q values, the studied time series is presented to be monofractal.

Except for the cross-correlation Hurst exponent, using the standard DCCA algorithm, we can compute the standard DCCA cross-correlation coefficient $\rho_{DCCA}(s)$ between time series [52]:

$$\rho_{DCCA}(s) = \frac{F^2_{DCCA}(s)}{F_{DFA_{\{x\}}}(s) \times F_{DFA_{\{y\}}}(s)}. \tag{4}$$

In (4), $F^2_{DCCA}(s)$ is the detrended covariance function between x and y from DCCA; $F_{DFA}(s)$ is the standard DFA [40] and $-1 \leq \rho_{DCCA}(s) \leq 1$. In a similar way to the classical correlation coefficient, $\rho_{DCCA} = 1$ means that time series are positively correlated and co-move synchronically; $\rho_{DCCA} = -1$ denotes that time series move anti-persistently; $\rho_{DCCA} = 0$ presents that there is no correlation between two time series.

For further calculations, through the multifractal (Rényi) mass exponent $\tau(q) = qh_{xy}(q) - 1$ [38], we define the singularity strength (Hölder exponent) through a Legendre transform [18, 20, 21]:

$$\alpha(q) = h_{xy}(q) + q\left[\frac{dh_{xy}(q)}{dq}\right] \tag{5}$$

and the singularity (multifractal) spectrum [18, 21]:

$$f(\alpha) = q\big[\alpha(q) - h_{xy}(q)\big] + 1. \tag{6}$$

If critical events dominate in our system, the singularity spectrum has a long-left tail that indicates the dominance of large events. The right-tailed multifractal spectrum indicates sensitivity to events of small magnitude. The symmetrical spectrum represents an equal distribution of small and large fluctuations.

Except for those characteristics that were presented before, we would like to calculate the width of the multifractal spectrum which can be defined as

$$\Delta\alpha = \alpha_{max} - \alpha_{min}. \tag{7}$$

In (7), α_{min} and α_{max} are the ends of $f(\alpha)$. The wider $\Delta\alpha$ is, the more complex structure, the more uneven distribution we have, and the more violent fluctuations on the surface of our time series. On the contrary, smaller multifractal width indicates that the time series are uniformly distributed. Thus, their structure is much simpler.

Except $\Delta\alpha$ of the whole spectrum, we can calculate widths of its left (L) and right (R) tails:

$$\begin{cases} L = \alpha_0 - \alpha_{min} \\ R = \alpha_{max} - \alpha_0 \end{cases}, \tag{8}$$

for which $\alpha_0 = \text{argmax}_\alpha f(\alpha)$. The wider one of this width is, the more uneven distribution we have. Greater value of L points on the wider right tail of the multifractal, which corresponds to higher complexity due to large fluctuations. On the contrary, if R becomes wider, we have higher complexity due to fluctuations with small magnitude. If both values are equal, both small and large fluctuations are uniformly distributed. Such asymmetry (skewness) is better reflected by the long tail type ΔS [23]:

$$\Delta S = R - L \tag{9}$$

and asymmetry coefficient A [14, 16, 39]:

$$A = \frac{L - R}{L + R}. \tag{10}$$

Negative value for A would highlight the dominance of small fluctuations (right-sided asymmetry). Consequently, positive values for A would denote an increase of heterogeneity for large fluctuations (left-sided asymmetry). When $A = 0$, the spectrum is presented to be symmetric.

For $\Delta S < 0$, we have wider left tail, which tells about insensitiveness of a time series to small fluctuations, while for $\Delta S > 0$, we expect to have less fluctuated time series.

Another option is to find the difference between the maximum and the minimum probability subsets Δf [11, 12, 51]:

$$\Delta f = f(\alpha_{min}) - f(\alpha_{max}). \tag{11}$$

For $\Delta f < 0$, we have the higher chance of occurring decreasing direction, while for $\Delta f > 0$ we have the opposite relation.

3 Experiments and Empirical Results

Further, for measuring the degree of multifractal cross-correlations between S&P 500, HSI, and BTC, we present the comparative dynamics of the described indicators calculated with the usage of the sliding window approach [5, 6] along with the studied series. The presented measures are calculated for the standardized returns of S&P 500, HSI, and BTC, where returns are calculated as

$$G(t) = \ln x(t + \Delta t) - \ln x(t) \cong [x(t + \Delta t) - x(t)]/x(t) \tag{12}$$

for $t = 1, \ldots, N-1$. Here, N is the length of the initial time series, and the standardized version of G can be calculated as $g(t) \cong [G(t) - \langle G \rangle]/\sigma$ with σ representing standard deviation of G; Δt – time lag (in our case $\Delta t = 1$); $\langle \cdots \rangle$ – average over the time period under study.

Figures below include such measures as:

- the cross-correlation multifractal function $F_q(s)$, the generalized cross-correlation Hurst exponent $h_{xy}(q)$, the multifractal cross-correlation Rényi exponent $\tau(q)$, and the multifractal cross-correlation spectrum $f(\alpha)$;
- the DCCA correlation coefficient ρ_{DCCA} for a long-term ($s = 250$ days $- \rho_{last}$) and midterm ($s = 125$ days $- \rho_{middle}$);
- the generalized cross-correlation Hurst exponent ($h_{xy}(2)$);
- the width of multifractal spectrum $\Delta\alpha$;
- the singularity exponents $\alpha_0, \alpha_{min}, \alpha_{mean}, \alpha_{max}$;
- the widths of left (L) and right (R) tails of $f(\alpha)$;
- the long tail type ΔS;
- the asymmetry coefficient A;
- the height of the multifractal spectrum Δf.

We expect our indicators to behave in a particular way during the critical event: increase or decrease. Cross-correlation measures are estimated with:

- the sliding window of 250 days and step size of 1 day;
- $m = 2$ for fitting local trends in Eqs. (1) and (2);

- the values of $q \in [-5; 5]$ with a delay 0.1 to have a better view of scales with small and large fluctuation density;
- the time scale s varies from 10 to 1000 days for the whole time series and from 10 to 250 days for the sliding window method.

In Fig. 1 multifractal characteristics of the pair S&P 500-BTC are presented.

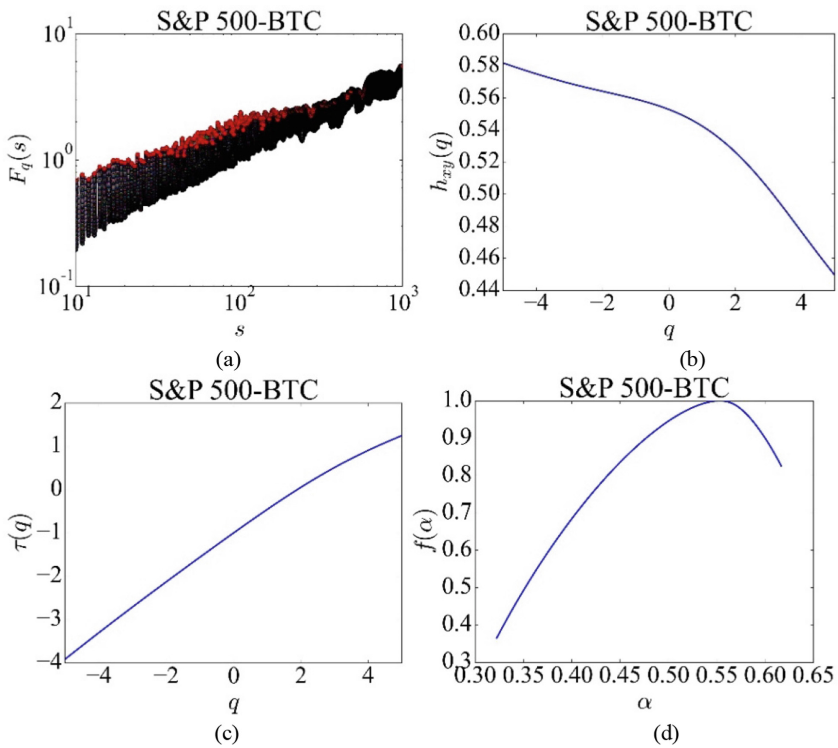


Fig. 1. The log-log plot of the cross-correlation fluctuation function $F_q(s)$ versus time scale s (a), the generalized cross-correlation Hurst exponent $h_{xy}(q)$ versus the order q (b), the multifractal cross-correlation mass exponent $\tau(q)$ versus the order q (c), and the multifractal cross-correlation spectrum $f(\alpha)$ versus the cross-correlation singularity exponent α for the pair S&P 500-BTC

In Fig. 1, a the fluctuation function $F_q(s)$ follows the power-law, and it appears to be wide for $s < 100$.

In Fig. 1, b we see that $h_{xy}(q)$ appears to be non-linear. For $q < 0$, the generalized cross-correlation Hurst exponent responds precisely persistent dynamics, whereas for large fluctuations ($q > 0$) between two indices we should expect anti-persistent dynamics.

In Fig. 1, c the multifractal Rényi exponent remains mostly linear for $q < 0$, which is an indicator of mostly monofractal cross-correlation dynamics of small fluctuations, while for $q > 0$, cross-correlations dynamics demonstrates higher multifractality.

Figure 1, d demonstrates that $f(\alpha)$ is broad, which is an indicator of highly complex non-linear dynamics of two systems. Moreover, this spectrum is skewed toward left. We can conclude that multifractal cross-correlation structure formed by S&P 500 and BTC has higher sensitivity to larger local fluctuations.

In Fig. 2 multifractal characteristics of the pair HSI-BTC are presented.

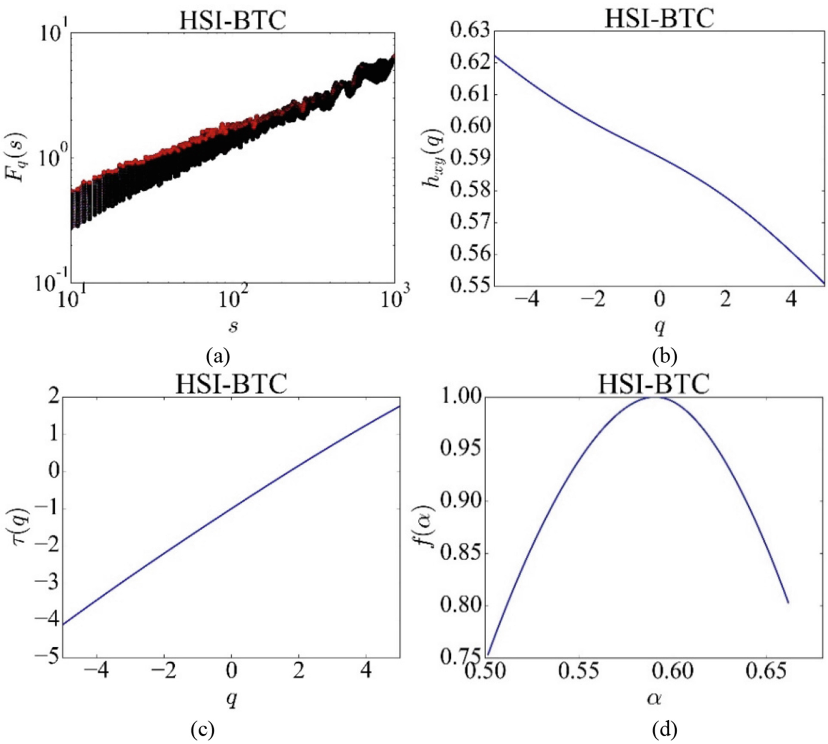


Fig. 2. The log-log plot of the cross-correlation fluctuation function $F_q(s)$ versus time scale s (a), the generalized cross-correlation Hurst exponent $h_{xy}(q)$ versus the order q (b), the multifractal cross-correlation mass exponent $\tau(q)$ versus the order q (c), and the multifractal cross-correlation spectrum $f(\alpha)$ versus the cross-correlation singularity exponent α for the pair HSI-BTC

In Fig. 2 we can see that multifractal cross-correlations for HSI-BTC are presented to be weak.

The fluctuation function $F_q(s)$ in Fig. 2, a follows power-law, but in general, it appears to be narrow for all scales.

In Fig. 2, b we see that $h_{xy}(q)$ is presented to be non-linear. For $q < 0$, the generalized exponent $h_{xy}(q) > 0.59$, which implies that small fluctuations between two indices behave persistently. For large fluctuations ($q > 0$), the generalized exponent $h_{xy}(q) > 0.55$, which shows that even large fluctuations between two markets remain correlated.

In Fig. 2, c the multifractal Rényi exponent remains mostly linear across different statistical moments q, which demonstrates that most of the cross-correlation multifractal behavior between two markets is weak.

Figure 2, d demonstrates that $f(\alpha)$ is not concentrated in one point, which proves that cross-correlation dynamics of the studied systems demonstrates multifractal characteristics. The spectrum demonstrates a precisely uniform shape, which indicates a relatively uniform contribution of large and small fluctuations. However, compared to other studied pairs, the spectrum of HSI-BTC looks narrow.

We conclude that for these indices, over a range of q and s values, multifractal cross-correlations are presented to be insignificant.

Next, in Fig. 3 we present multifractal characteristics of two stock indices – S&P 500 and HSI.

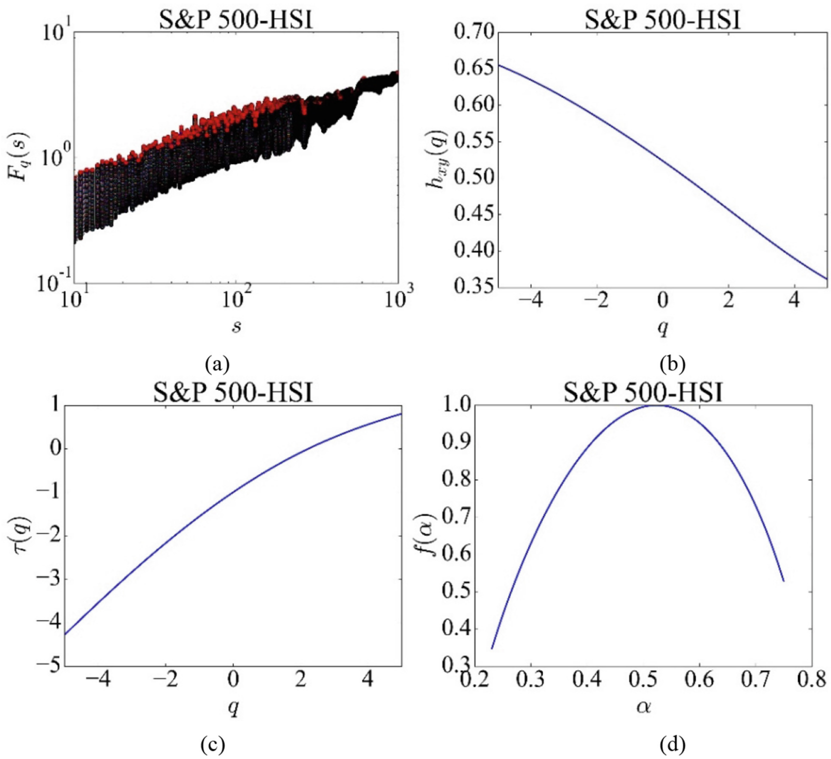


Fig. 3. The log-log plot of the cross-correlation fluctuation function $F_q(s)$ versus time scale s (a), the generalized cross-correlation Hurst exponent $h_{xy}(q)$ versus the order q (b), the multifractal cross-correlation mass exponent $\tau(q)$ versus the order q (c), and the multifractal cross-correlation spectrum $f(\alpha)$ versus the cross-correlation singularity exponent α for the pair S&P 500-HSI

In Fig. 3 we can see that multifractal cross-correlations for S&P 500-HSI are presented to be strong.

The fluctuation function $F_q(s)$ in Fig. 3, a follows power-law, and it appears to be wide for a range of many scales.

In Fig. 3, b we see that $h_{xy}(q)$ is presented to be non-linear. For $q < 0$, the generalized exponent $h_{xy}(q)$ demonstrates that small fluctuations between two stocks represent persistent dynamics, whereas for large fluctuations ($q > 0$), the generalized cross-correlation Hurst exponent $h_{xy}(q)$ has the tendency to be less than 0.50.

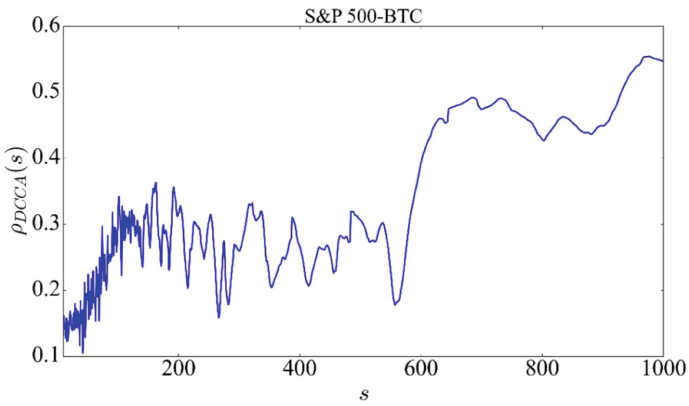
In Fig. 3, c the multifractal Rényi exponent remains mostly non-linear across different statistical moments q, which demonstrates that most of the cross-correlation behavior of two markets demonstrates high degree of multifractality.

Figure 3, d demonstrates that $f(\alpha)$ is the broadest among other spectrums, which implies that multifractal cross-correlation dependence in the stock market has to be the biggest. According to the presented spectrum, fluctuations with small and large magnitudes have approximately equal influence on each other.

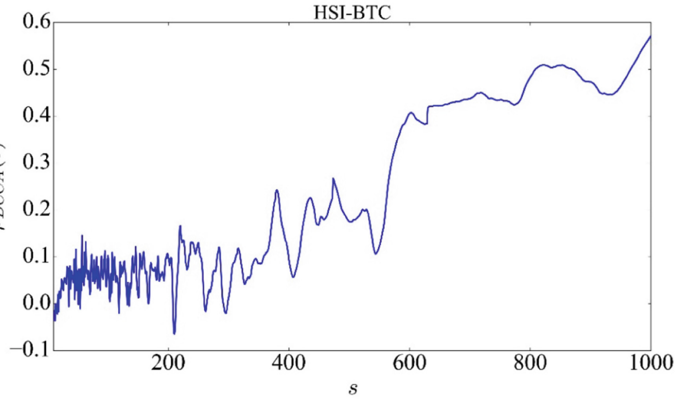
We conclude that for these indices, over a range of q and s values, multifractal cross-correlations are presented to be significant.

In Fig. 4 we would like to present the plots of ρ_{DCCA} versus time scale s for S&P 500-BTC, HSI-BTC, and S&P 500-HSI.

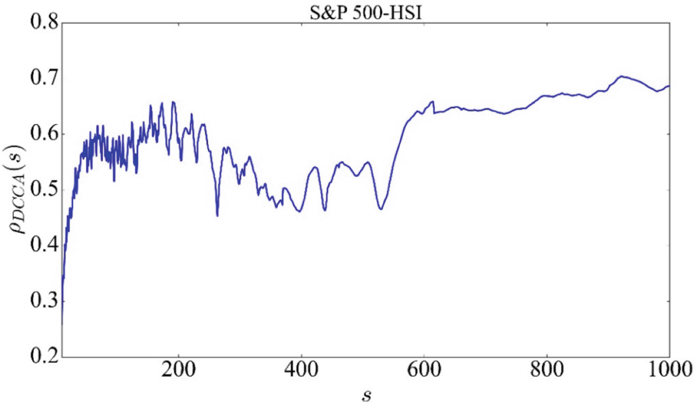
Our analysis of the DCCA correlation coefficient across different time scales demonstrates that the cross-correlations between stock indices and BTC are presented to be weak in the short term (less than 100 days), but tend to increase for a long-term period. For the pair HSI-BTC, cross-correlations are presented to be weak even for $s < 600$ days. Since then, the cross-correlation coefficient $\rho_{DCCA} > 0.3$. As is expected for the stock market, both S&P 500 and HSI demonstrate a high degree of correlation across many time scales. Using the sliding approach, we can track across time how the non-linear dynamics of two systems change dependently on each other.



(a)



(b)



(c)

Fig. 4. The DCCA cross-correlation coefficient $\rho_{DCCA}(s)$ versus time scale s for the pairs: S&P 500-BTC (a), HSI-BTC (b), and S&P 500-HSI (c)

Figure 5 presents the comparative dynamics of coefficient ρ_{DCCA} along with S&P 500 and HSI calculated for pairs S&P 500-BTC, HSI-BTC, and S&P 500-HSI.

Noticeable long-term correlation can be observed for the periods since 2018. For the crisis, because of the coronavirus pandemic, this correlation is also observed. By the end of 2021, both mid and long-term cross-correlation coefficients were decreasing and their dynamics started to demonstrate an upward trend.

Figure 6 demonstrates the performance of the cross-correlation Hurst exponent calculated for S&P 500-BTC, HSI-BTC, and S&P 500-HSI.

The cross-correlation Hurst exponent decreased during the most noticeable crashes in stock markets. The same we can see for the upward trend in stocks. Therefore, most of the time BTC behaved asymmetrically. Also, for HSI and S&P 500 anti-persistent behavior during critical phenomena of both indices is noticeable.

The dynamics of $\Delta\alpha$ calculated for each pair is presented in Fig. 7.

The width of multifractality remains a reliable indicator for critical phenomena of all indices. All of the figures demonstrate an increase in the width of multifractality during crash phenomena. Especially that is noticeable during the COVID-19 crisis. However, the degree of multifractality decreased for 2021, where visually BTC had noticeable drops in price. This period in the BTC market will require further research.

In Fig. 8 we can see how all of the presented singularity exponents behave.

In this case, their dynamics demonstrates behavior similar to $\Delta\alpha$. At the same time, we can observe that the dynamics of α_{min} indicator do not represent synchronous behavior along with other singularity exponents. For pair S&P 500-BTC it becomes higher for last days, indicating a growth of multifractality. This will require additional research, but the signal of this indicator may appear to be false.

Figure 9, where both L and R measures are presented, gives us an idea of how dominance of small and large fluctuations varies.

The growth of L is the most noticeable during crash events such as coronavirus pandemic, whereas R starts to increase for small critical events. It is worth noting that for the pair S&P 500-HSI, the distribution of large and small fluctuations seems relatively uniform. The dynamics of both indicators represent prospects for building effective trading strategies (Fig. 10).

The long tail type ΔS indicator shows us how the difference between left and right tails changes. Here we expect that with the crisis event, the indicator will decrease, which will correspond to the dominance of the left tail, i.e., multifractal properties of large events in the studied signal.

Figure 11 presents the dynamics of asymmetry coefficient A for all of the studied pairs.

Its growth is noticeable during the largest drops in the studied period. At the beginning for the pair S&P 500-BTC some of signals generated by A seem to be spurious as the correlation between them could be negative. For other pairs, most of the time, our indicator behaves in an expectable way (Fig. 12).

The height of $f(\alpha)$ demonstrates dynamics similar to ΔS. In this case we expect higher probability of occurring large fluctuations as Δf decreases, and for small fluctuation we expect opposite behavior. This indicator is also presented to perspective alternative for building reliable trading strategies.

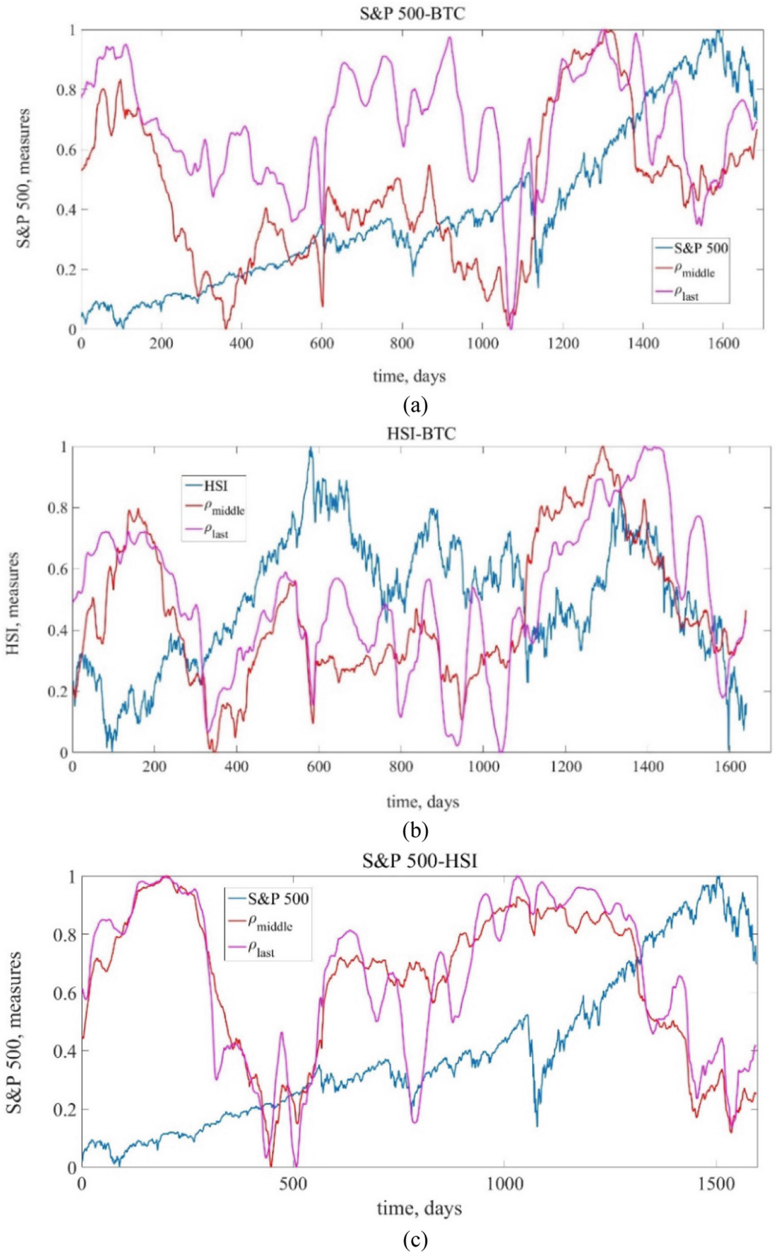
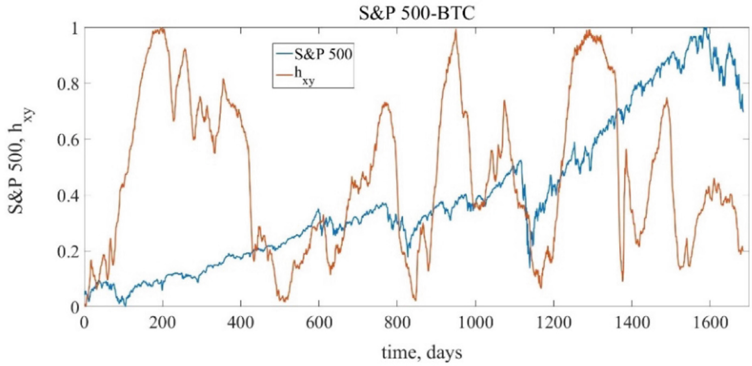
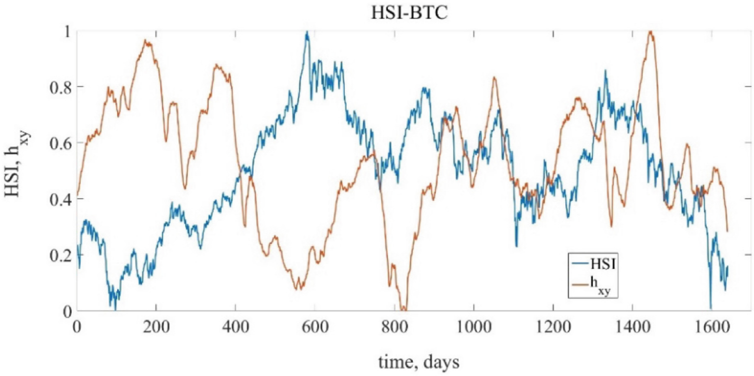


Fig. 5. The comparative dynamics of S&P 500, HSI along with mid-term and long-term DCCA cross-correlation coefficients calculated for the pairs: S&P 500-BTC (a), HSI-BTC (b), and S&P 500-HSI (c)



(a)



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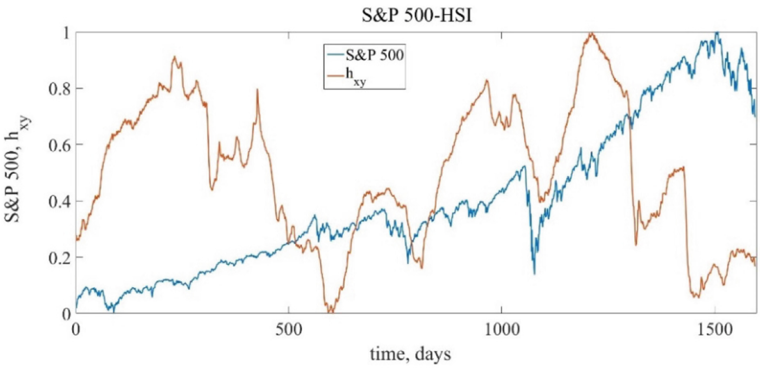
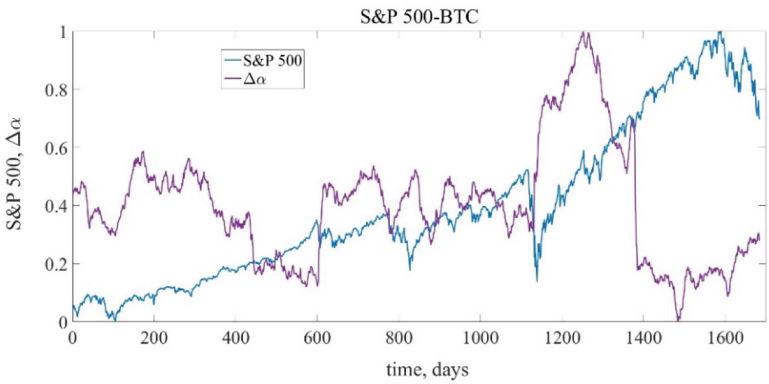
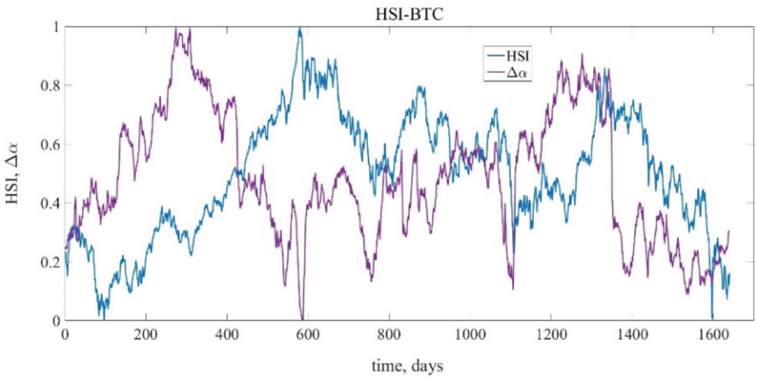


(c)

Fig. 6. The comparative dynamics of S&P 500, HSI, and h_{xy} calculated for the pairs: S&P 500-BTC (a), HSI-BTC (b), and S&P 500-HSI (c)



(a)



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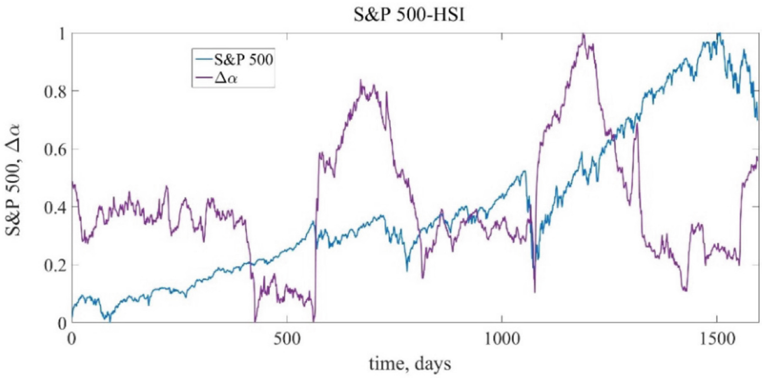
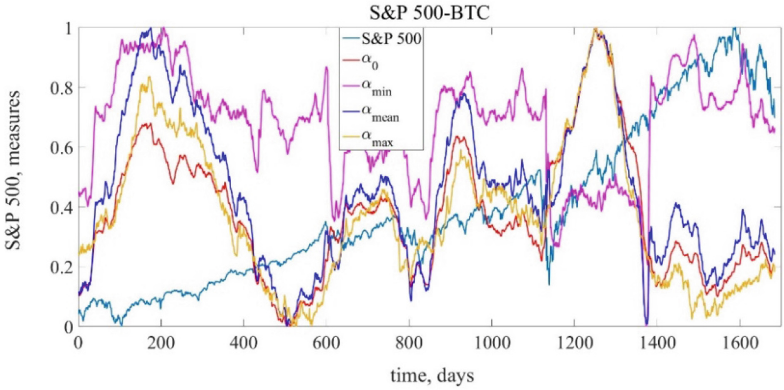


(c)

Fig. 7. The comparative dynamics of S&P 500, HSI, and $\Delta\alpha$ calculated for the pairs: S&P 500-BTC (a), HSI-BTC (b), and S&P 500-HSI (c)



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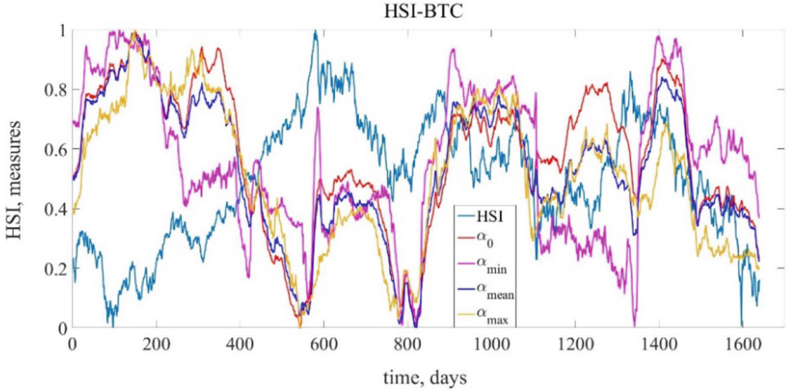

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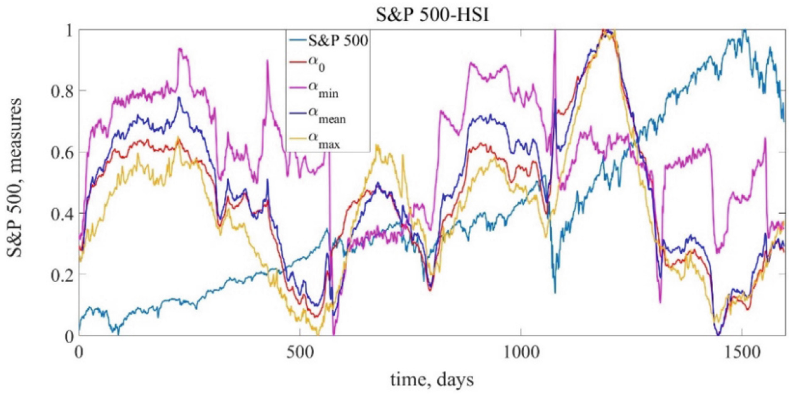
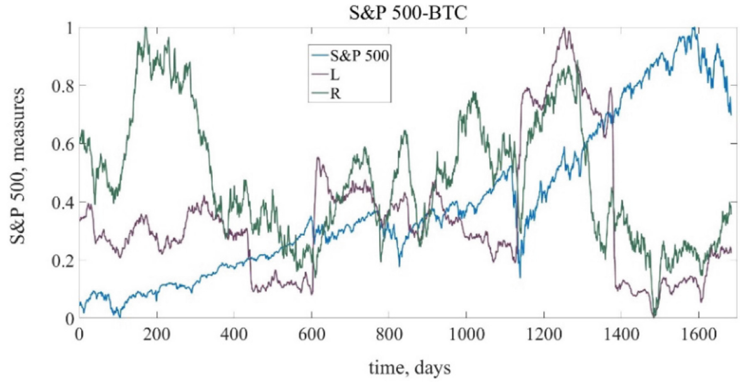
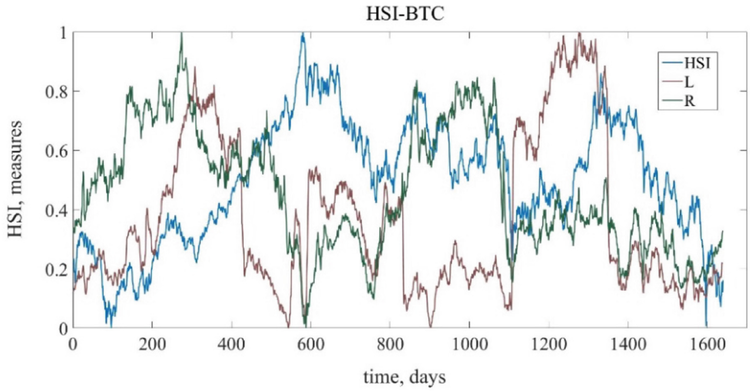


(c)

Fig. 8. The comparative dynamics of S&P 500, HSI, and $\alpha_0, \alpha_{min}, \alpha_{mean}, \alpha_{max}$ measures calculated for the pairs: S&P 500-BTC (a), HSI-BTC (b), and S&P 500-HSI (c)



(a)



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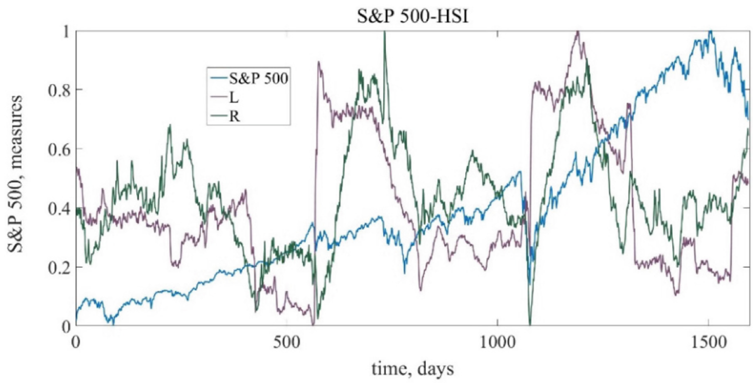
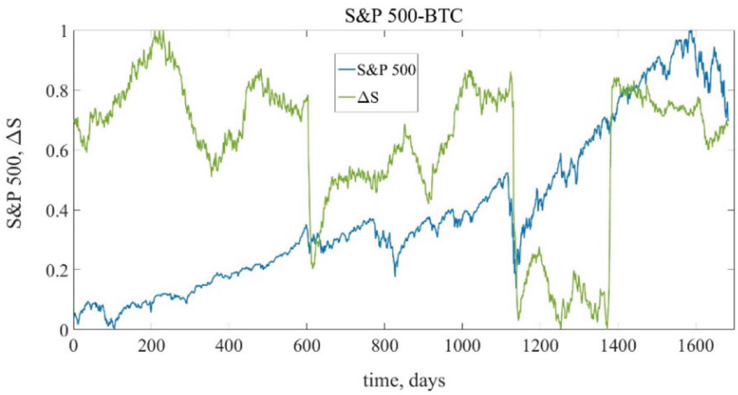
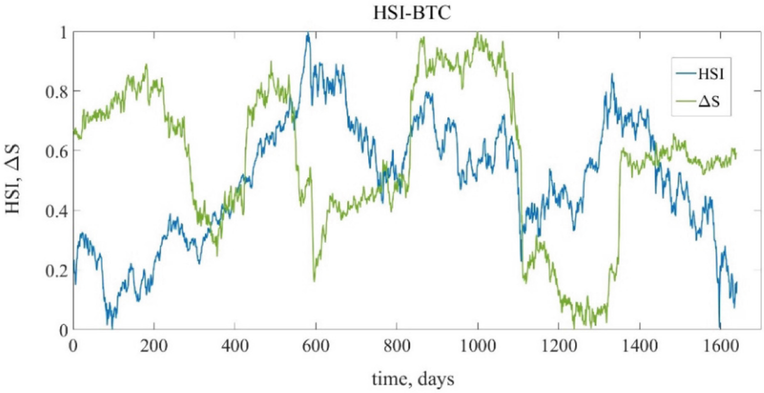


(c)

Fig. 9. The comparative dynamics of S&P 500, HSI, and *L*, *R* measures calculated for the pairs: S&P 500-BTC (a), HSI-BTC (b), and S&P 500-HSI (c)



(a)



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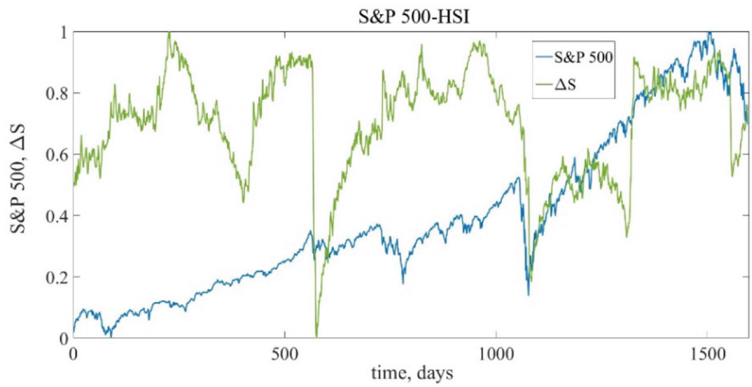
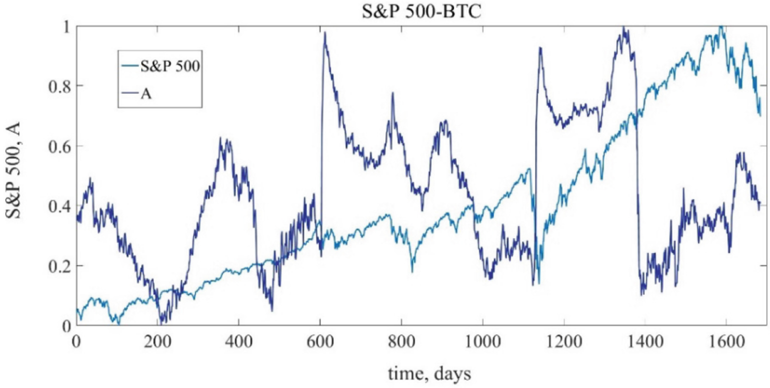
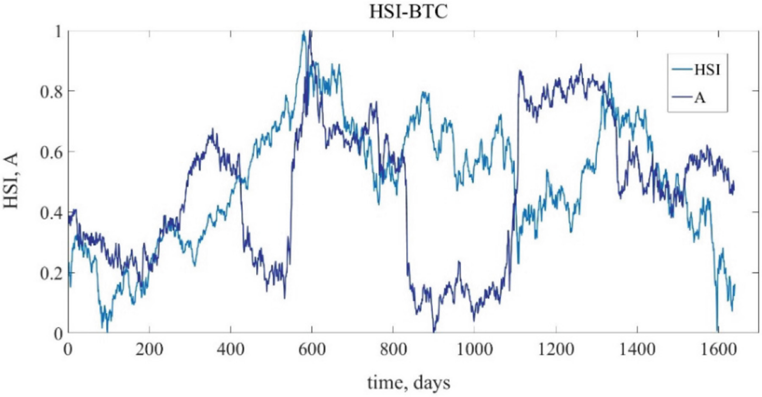


(c)

Fig. 10. The comparative dynamics of S&P 500, HSI, and ΔS measure calculated for the pairs: S&P 500-BTC (a), HSI-BTC (b), and S&P 500-HSI (c)



(a)



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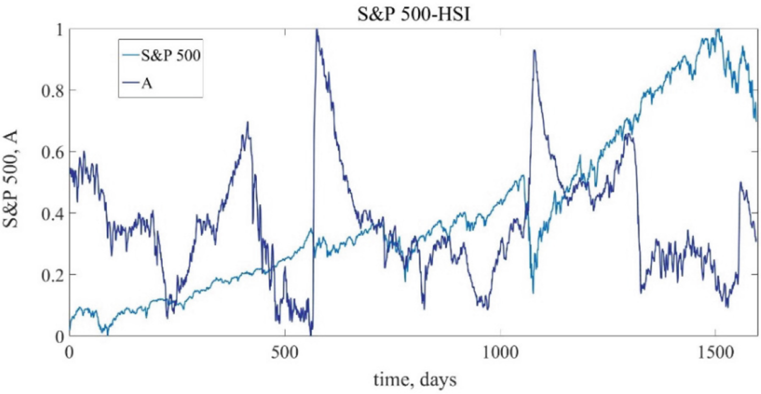

(c)

Fig. 11. The comparative dynamics of S&P 500, HSI, and *A* measure calculated for the pairs: S&P 500-BTC (a), HSI-BTC (b), and S&P 500-HSI (c)

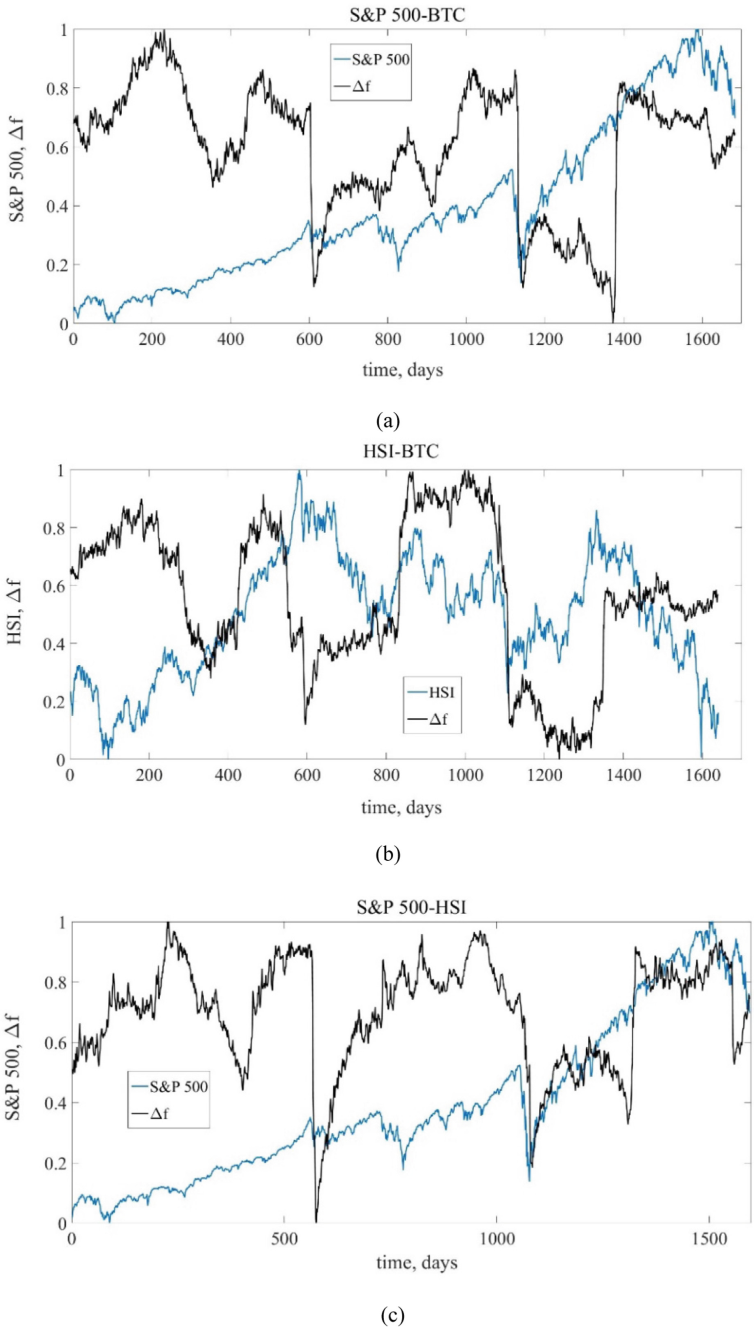

Fig. 12. The comparative dynamics of S&P 500, HSI, and Δf measure calculated for the pairs: S&P 500-BTC (a), HSI-BTC (b), and S&P 500-HSI (c)

4 Discussion and Conclusions

In this study, we have analyzed multifractal cross-correlation characteristics of stock and cryptocurrency markets using multifractal detrended cross-correlation analysis. Using the MF-DCCA and sliding windows approach, we have constructed indicators of cross-correlated behavior in S&P 500, HSI, and BTC. The combination of both approaches gives us the possibility to present such measures as the DCCA correlation coefficient ρ_{DCCA} for short- and long-term behavior, the generalized cross-correlation Hurst exponent (h_{xy}), the width of multifractal spectrum $\Delta\alpha$, the singularity exponents α_0, α_{min}, α_{mean}, α_{max}, the widths of left (L) and right (R) tails of $f(\alpha)$, the long tail type ΔS, the asymmetry coefficient A, and the height of the multifractal spectrum Δf.

In the example of S&P 500, HSI, and BTC we have presented that most of the time the dynamics of stock indices and developing digital market remained anti-persistent during crisis events. Nevertheless, over the last years, their degree of cross-correlations started to demonstrate synchronic behavior. The crashes of both markets are characterized by multifractality, which implies long-term memory for the pair of markets. By analyzing the cross-correlation coefficient ρ_{DCCA} versus time scale s, we have confirmed that in short-term cross-correlations between stock and crypto markets are presented to be weak. Even the mid-term cross-correlations between the Chinese market and the crypto market remain insignificant. Both S&P 500 and HSI indices are highly correlated despite some differences in their structure. Most of our indicators show that after the COVID-19 crisis, and the 2022 Russian invasion into Ukraine that has resulted in a collapse of food supply, we may expect a higher degree of interconnection between the stock market and the cryptocurrencies market.

Our empirical analysis shows further perspectives for constructing effective algorithmic strategies and forecasting models based on complex systems theory. In the future, it would be interesting to consider other methods of classical multifractal analysis or its cross-correlation modifications in combination with other methods of complex systems theory [3, 4, 26, 29, 30, 42–45, 49].

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