1.1 Conceptual Bases for the Selection of Textbooks on Mathematics for Lyceums*

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*The article is published in the author's translation

The problem statement

The content selection for textbooks on Mathematics gained special significance as a response to a new social demand concerning the goals and tasks of the school education. Today the educative leitmotif is: the formation of the mathematical competence and the key competences necessary for a person's successful self-realization; the education content reorientation towards a person, the fostering of education subject's active cognition; the education organization basing on the student's environment interaction experience; the education orientation onto young person's abilities, spiritual and creative potential realization as well as onto stable self-education and self-development mechanisms elaboration. The content of teaching Mathematics is affected by its role in economy, technology, management, social processes as well as by the profiling, competence, activity- and personality-oriented approaches wide implementation in schools. The abovementioned factors presuppose development and employment of such conceptual bases for lyceum-oriented Mathematics textbooks content selection that could provide the high-quality mathematical education.

The latest research analysis

The problem of mathematics content at high school and its representation in textbooks was studied by researchers, methodologists and teachers of mathematics (Burda, Tarasenkova, Vasylieva, & Vashulenko[1], 2018; Bevz[2], 1989;

[1] Burda, M., Tarasenkova, N., Vasilieva, D., & Vashulenko, O. (2018). Kontseptsiya matematychnoyi osvity 12-richnoyi shkoly [The Concept of Mathematical Education in a 12-year School]. *Mathematics in Native School*, No. 7-8, 2-8. [In Ukr].

[2] Bevz, G. (1989). *Metodyka vykladannya matematyky [Mathematics Teaching Methods]*. Kyiv: Vyscha shkola. [In Ukr].

Bevz and Vasylieva[3], 2018; Dubinchuk[4], 1992; Malyovany[5], 2015; Slepkan[6], 2000; etc.). However, what still retains relevance is the study of maths textbook selection for lyceum based on the competence approach according to which the result of the mastered subject is the formation of a certain competence as the student's ability to act effectively in academic and existential situations.

The purpose of the article lies in revealing the conceptual bases for the selection of teaching texts, the mathematics textbooks system of tasks and methodology apparatus enhancing the high school student quality mathematical education.

Theoretical framework

The humanistic values of education predetermine the transition from the knowledge oriented educational paradigm to the competence-centered personality-oriented one. The *personality oriented approach* in education is seen as the enhancement of personal interaction in the educational process, of students' personal progress, capability development, self-identification conditions. The chief goal of the personality oriented learning is the formation of the student personality's I-concept as the system of acknowledged and unrealized (unconscious) self-images on which (s)he bases his/her behavior. The personality oriented education's central task is the formation of student personality's positive I-concept as the system of acknowledged and unrealized (unconscious) self-images that serve the basis for his/her behavior. In this regard, the modelling of success situations

[3] Bevz, V. and Vasylieva, D. (2018). The Particularities of Mathematics Curriculum in the New Ukrainian School Concept. *School Curriculum: Problems and Options for Development*. Kishynyov, Moldova. [in Rus.].

[4] Dubynchuk, O. (1992). The Differentiation of Mathematical Profiled College and Professional School Education Content : *Profiled Teaching in Professional Technical Educational Establishments: Research.-method. selection*, 32-37.

[5] Maliovanyi, Yu. (2015). Kontseptual′ni pidkhody do formuvannya zovnishn′oyi struktury profil′noyi osvity [Conceptual Approaches to the Formation of Profile Education External Structure]. *Ukrainian Pedagogical Journal, 1*. 77-84. [In Ukr.]

[6] Slepkan, Z. (2000). *Metodyka navchannya matematyky: pidruchnyk dlya studentiv [Mathematics Teaching Methods: a textbook for stud.*]. Kyiv, *Ukraine*: Zodiak-Eco. [In Ukr.]

– subjective psychic states of student's feeling of pleasure resulting from cognitive activity accomplishments is of primary importance. The success experienced by a student mobilizes his/her hidden abilities, fosters the emotively evaluative attitude to the objects of cognition, the realization of intellectual effort. And the greatest role here belongs to textbooks as the main learning means.

The personality's subjectivity as well as students' individuality reveals itself in the world cognition selectiveness as for the content, kind and form of its presentation, the selectiveness stability, the ways of learning material mastering, the emotionally evaluative treatment of cognition objects. In the personality oriented learning the content, methods, means and organizational forms are to be focused upon every students' experience discovery and use, upon the assistance in working out personally significant cognitive ways by organizing the cognitive activity. In the educative process the student's mastering of the social and historic experience should be accomplished not by ousting his/her individual experience, but through their coordination and through the use of everything accumulated by the student in his/her own life. This principle should serve the basis for working out teaching texts and other constituents of the assimilation organization apparatus.

In the process of mathematical knowledge, habits and skills assimilation as well as at the stage of their application the objective prerequisites for the enrichment of not only mathematical but universal cultural students' potential are laid out, ample possibilities are created for the formation and development of students' thinking, memory, ideas and imagination, their scientific outlook, algorithmic, informational and visual culture, their ability to determine causal-consequential relations between separate facts, to substantiate statements and mathematize situations. The employment of didactically weighed Mathematics teaching organization enables a considerable effect on the students' intellectual development, the positive personal features formation, their mental activity, cognitive independence, self-regulation, learning creativity upgrade. So the textbooks

content realizes the personality -oriented learning model and is personality -centered, enhancing the learning model aiming at mathematical education as well as education by means of mathematics, the formation of mental qualities necessary for adapting and functioning at full value in modern society, the assimilation of the mathematical apparatus as a means of modern life problems formulation and solution.

Of primary importance in the textbook's content selection is the consideration of students' specificity in verbal and non-verbal data perception and procession organization, namely: the cognitive stimuli impact is characterized by instability and high degree of activation processes lability; the simultaneous involvement of cerebral cortex different zones at all stages of data perception and procession (sensory analysis, information synthesis, the stimulus categorization) are observed; higher speeds of data procession by the right cerebral hemisphere are revealed; the visual figurative thinking that is approximating the image categorizing operation is predominant, more so when the verbal logical thinking isn't perfect yet, but is in the stage of formation. Thus, the consideration of the logical and the visual beginnings unity in the student's mathematical training, timely detection of the logical and the visual beginnings' hypothetical conflicts, and the didactically balanced selection of the ways to minimize the gap are of primary importance. In this regard, the textbooks combine logical rigor and visual representation, so that deduction and the learning material abstract nature should be based on visuality and students' mathematical intuition.

In the textbook creation the idea of the opportunities *to conduct mathematics training is regarded in two areas* – direct learning and background training.

The latter is represented by propaedeutic and indirect learning. In the course of students' background training the powerful resources of the unconscious are involved – the experience of visual recognition is enriched, the certain intuitive prior knowledge is accumulated, and the experience in the individual subject-practical actions performance is gained. The subsequent expansion of the influences system

becomes possible due to a specially construed visual range of training and a system of exercises aimed at preventive formation of students' abilities to perform certain activities. In general, the semiotic space, which is built in the course of study, should provide conditions for the students' free, psychologically comfortable life in the school Mathematics conditionality dimension, promote their active educational and cognitive activity, as well as encourage them to succeed independently. And this is only possible if the textbooks content meets the specificity of the students' educational and cognitive activity, and textbooks volume corresponds the learning time. Therefore, as we note[7] (Burda & Tarasenkova, 2016), there is currently a decrease in the volume of Mathematics courses that is achieved by avoiding excessive rigor in presentation, by reducing cumbersome calculations and transformations, by excluding the material that is used neither for the course logical deployment, nor problem solution and has no practical application.

The textbook content (textbooks, exercises and methodological apparatus) is aimed at *developing competencies* in students (Tarasenkova & Kirman[8], 2008; Tarasenkova[9], 2016), namely – mathematical, super-mathematical (interdisciplinary and specialist) and key competences. To students' mathematical competencies we refer: *content* or *informative* (has a notion of the ideas and methods of Mathematics, its role in the cognition of reality; possesses formally logical (definition, qualities, features of mathematical objects) and operational (methods, techniques, means of activities) knowledge; understands mathematical formulas and models as such that allow to describe the

[7] Burda, M. & Tarasenkova, N. (2016). Teoretyko-metodychni vymohy do zmistu shkil′nykh pidruchnykiv z matematyky [Theoretical and Methodological Requirements for the School Mathematics Textbook Contents]: *Problems of a Modern Textbook*: *a collection of scientific works*, *16*. 43-51. [In Ukr.].

[8] Tarasenkova, N. & Kirman, V. (2008). Zmist i struktura matematychnoyi kompetentnosti uchniv zahal′noosvitnikh navchal′nykh zakladiv [The Content and Structure of the Secondary School Students' Mathematical Competency]. *Mathematics in School*, *6*. 3-9. [In Ukr.].

[9] Tarasenkova, N. (2016). Kompetentnisnyy pidkhid u navchanni matematyky: teoretychnyy aspekt [Competency Approach in Teaching Mathematics: Theoretical Aspect]. *Mathematics in the Native School*, *11 (179)*. 26-30. [In Ukr.].

properties of objects, processes and phenomena); *procedural-operational* (depicts mathematical objects, finds out and substantiates their qualities; classifies them by their features; substantiates mathematical statements; uses mathematical objects' definitions, qualities and features to solve problems; measures and calculates the values; applies mathematical methods, techniques and ways of acting in the process of solving purely mathematical or practical problems); *research* or *investigative* (proposes and tests the hypotheses, sets out the program of activities, provides its results, makes decisions in the conditions of incomplete, excess, accurate and probabilistic information, evaluates the correctness and rationality of the tasks solved, interprets the results, taking into account the specific conditions and the objectives of the research); *informational and technological* (uses information and communication technologies in educational activity; finds and develops mathematical information (textbooks, directories, Internet resources), evaluates the obtained information, systematizes and generalizes it, makes the right conclusions). Depending on the goals, objectives and the level of learning mathematics, the degree of these competences mastering is different.

Taking into account Mathematics teaching goal, its role in the study of other subjects, the important task is to develop intersubjective mathematical competencies – interdisciplinary (Geometry, Algebra and Calculus, Mathematics and other subjects) and specializing (as an element of vocational training).

These competences imply that the student: understands the importance of Mathematics for other disciplines successful learning, full-fledged activities in various spheres of public life, in particular in future professional activities; recognizes and formulates the problems that arise in the content of other subjects or in the field of prospective professional activities and can be solved by mathematical methods; applies mathematical models to the study of other school subjects (Physics, Computer Science, Astronomy, Chemistry, Biology, etc.) and to the situations associated with would-be professional activities.

In addition, the textbook content should contribute to the key competencies formation. They are aimed at increasing motivation, interest in learning, developing the ability to apply knowledge and skills in various spheres of real-life activity, as well as gaining experience, creating values and views that can be put into practice. Key competencies are acquired in the process of solving problems of practical kind – the problems arising beyond the boundaries of Mathematics, but solved with the mathematical apparatus employment. There are many tasks of the kind in the published textbooks. However, it is recommended to give preference to the tasks that relate to contemporary socio-economic challenges and values (Vasylieva[10], 2018). They tackle, primarily, energy saving problem (gas, water, electric light and heat are valuable resources requiring economy), the financial literacy (planning and rational spending of personal, family funds, proper cooperation with financial institutions), health and ecology (thrifty expenditure of natural resources, the environment cleanliness, the healthy lifestyle choice, food products quality and correct nutrition, position towards alcohol, nicotine, etc.).

That is, the tasks should be value-orienting, enhance the students' sober behavior concerning energy resources, health, finance, the environment, interpersonal relations, and promote understanding of the mathematical education importance for successful life in modern society.

In the conditions of competence-oriented training, *special requirements are put forward to the textbook's methodological apparatus.* A textbook of a standard or professional level should become not only attractive to students, but also immune from the teacher and his methodological preferences and skills, as a training provider of student's independent learning. First of all, the textbook's structure and all its constituents should be logical and understandable for students, because the structuring method creates the first,

[10] Vasylieva, D. (2018). Matematychni zadachi yak zasib formuvannya klyuchovykh kompetentnostey uchniv [Mathematical Problem as a Means of Forming the Key Competencies in Students]. *Problems of a Modern Textbook: a collection of scientific works*, *21*. 83-92. [In Ukr.].

structural level in the system of the textbook conventions, to which the student is to get used. And this system should be unified for all structural units of the textbook. For example, each section can begin with the section "In the paragraph you will learn", and be ended with control questions and test tasks.

We associate another aspect of the management function of the textbook with the creation of conditions for the development of students' cognitive needs. To this end, each paragraph of the textbook should comprise not only the training material that pupils must learn, but also additional data (for example, in the "Find Out More" section) that contain information on the origin of names and designations, historical, biographical references on outstanding compatriot and foreign mathematicians, etc., as it is done in our Geometry textbook[11] (Burda, Tarasenkova, Bogatyryova, Kolomiyets, & Serdiuk, 2013) and our Mathematics textbook[12] (Burda, Kolesnik, Malyovany, & Tarasenkova, 2018).

The training text unfolds according to a certain plan, which is identical for each paragraph of the textbook. The volume of each semantic text unit should correlate with the students' age, and the content deployment should follow the patterns of the mental process course. To this end, it is advisable to use problem questions, which serve a way of transition to a new idea, contribute to the rethinking of what was learned. The training text should be selected so as to involve students as actively as it is possible into self-dependent speculations, accustom them to posing questions and responding to them. Note that visual accents play an important role in it. Such textbook structure and design enables students' self-dependent work.

[11] Burda, M., Tarasenkova, N., Bogatyryova, I., Kolomiyets, O., & Serdiuk, Z. (2013). *Heometriya. Pidruchnyk dlya 11 klasiv zahal'noosvitnikh navchal'nykh zakladiv (akademichnyy ta profil'nyy rivni.)[Geometry Textbook for the 11th Grade of the Secondary School (academic and profile levels)].* Kyiv, Ukraine: Publishing House "Osvita". [In Ukr.].

[12] Burda, M., Kolesnyk, T., Malovany, Yu., & Tarasenkova, N. (2018). *Matematyka (alhebra i pochatky analizu ta heometriya, riven' standartu): pidruch. dlya 10 klasu zakladiv zahal'noyi seredn'oyi osvity [Mathematics (Algebra and Calculus, and Geometry, standard level): textbook for the 10th form of general secondary education establishments].* Kyiv, Ukraine: UEPC "Orion". [In Ukr.].

In accordance with the scientific foundations of the activity-oriented approach, specially organized substantive activities should serve both the purpose of learning, and its means. *The realization of the activity-oriented approach to mathematics learning in the textbook* implies: students' constant involvement into various types of educational and cognitive activity; assimilation of not only formal and logical but also operational knowledge (how to act in particular situations in order to achieve the set goal); assimilation of methods of reasoning employed in Mathematics; creation of educational situations that stimulate students' independent discovery of mathematical facts. In the textbook (where possible) it is advisable to give advice on how to act in a particular training situation which should be formulated in the form of rules or instructions. That is, the content of the textbook should ensure the presence of the activity-oriented component in any mathematical knowledge the students acquire – they should learn where and how to apply them.

The textbook content's scientificity is provided by the logically sequential placement of the training material, the correct formulation of concepts and theorems definitions, and a sufficient degree of evidence strictness. The logical ordering of the material and the sequence of its presentation must meet the didactic principles and the requirements of Mathematics as a science: modern, substantive, unambiguous terminology; concepts, formulas, qualities are formulated in a correct mathematical language; evidences of statements are at a necessary degree of strictness; the representation in the textbook content of the methods and ways of activities, meeting the mathematical logic of cognition. The mathematical concepts content is clearly distinguished (all essential features are listed) and their volume (the set of objects where the concept is applied) is indicated. The concepts' content is revealed in definitions and their volume is measured with the use of classifications (the division of concepts on a certain basis). The proofs of the theorems in the textbooks should be not only strict, concise, but also feasible, understandable to the students. Before the formulation of the theorem, it is proposed to conduct a small

study, a reduced description of the theorem is given, and its proof is divided into semantic blocks.

The combination of Continuous and Discrete Mathematics is an important feature of its contemporary courses. The development of computerization, information networks puts forward specific requirements for the style of human thinking, and hence for the content of school Mathematics. One of them is connected with the necessity of inclusion in the school course of Discrete Mathematics elements (combinatorics, mathematical logic elements with regards to their application, numerical systems, the theory of graphs elements, etc.).

Accessibility of training texts for students, the possibility to process them independently is one of the features of the textbooks. It is achieved by combining logical and visual means. The training material, as a rule, is based on visuality, students' intuition, their life experience[13] (Burda, 2018); the presentation of mathematical facts, if possible, begins with the empirical material analysis (examples from the environment, models, graphs, drawings, facts from other educational subjects, etc.) or with the description of practical actions; visuality should assume not only the illustrative but also a heuristic role, promote the creation of a pre-emptive idea concerning the new learning material content essence, facilitate the learning material perception and understanding.

The coherence of content and requirements for students' educational material acquisition is implemented in its two functions – the compensatory and the prognostic ones (Tarasenkova[14], 2016). The compensatory function provides

[13] Burda, M. (2018). Metodychni vymohy do pidruchnyka z matematyky rivnya standartu [Methodological Requirements for a Standard Textbook on Mathematics]. *Problems of a Modern Textbook: a collection of scientific works*, *21*. 64 - 72. [In Ukr.].

[14] Tarasenkova, N. (2016). Kompetentnisni zasady zabezpechennya nastupnosti navchannya matematyky v riznykh lankakh osvity [The Competency Forming Bases for Enhancing Coherence in Learning Mathematics at Different Education Levels]. Proceedings from the Implementation of continuity in mathematical education: Realities and Prospects: *Vseukrayins'ka naukovo-praktychna konferentsiya (Odesa, 15-16 veresnya 2016 roku) – All-Ukrainian research practical conference*. (pp. 108-110). Kharkiv, Ukraine: «Ranok». [In Ukr.].

a link of current learning process with the previous level of education (the content specification, expansion and deepening, the detection and alignment of disadvantages and gaps in the students' training). The prognostic function ensures the students' training adaptation to studying Mathematics at the next educational level.

The textbook content is aimed at students' creative development. The developmental effect is mostly based on the development of skills to prove assertions and to solve problems, to apply mathematical methods to solving problems of applied content, to the abstract mathematical constructions essence understanding, etc. Still, attention is also paid to students' acquaintance with the significance of Mathematics in present-day human activity, especially, in regard to the historical context. The textbooks, consequently, include materials related to value orientations: Mathematics history glimpses, mathematical theories and methods, some facts concerning the fates of scientists who created the branch, the terms and symbols' origin. The developmental learning function is also realized through the personalized presentation of the material, that is, the presentation, where possible, of mathematical facts in terms of their historical formation and development.

The textbooks are designed for *graded, self-paced learning* of Mathematics. Not only the exercises and tasks of varying complexity tend to be focused on it, but also examples of typical tasks solutions, problem questions and tasks, etc. For those who are interested in the subject and want to deepen their knowledge, the paragraph "Learn more" is assigned. A feature of the textbook's tasks is that high complexity level tasks include elements of medium and high level tasks, and the latter are elements of tasks at the entry level. In the problem settings the described situations vary in terms of personal participation: the problem is formulated by means of this age students' I-vocabulary; the task is formulated by means of the student's closest environment vocabulary; the problem is formulated in terms that are distant from this age students' personal experience.

The competencies formation involves the reinforcement *of the textbook contents pragmatic orientation*. The training material content should be corresponded to the three stages of Mathematics practical application.

The first stage. The study of a mathematical fact, if possible, should begin with the empirical material analysis (examples from the environment, models, graphs, drawings, examples from the field of prospective professional activities, facts borrowed from other educational subjects, etc.) or from the description of practical actions. This enables finding out of the concept's essential features, the mathematical object properties and, on their basis, the corresponding statement independent formulation.

The second stage. The essence of the mathematical fact is clarified and substantiated, then merely mathematical problems are solved. When substantiating mathematical statements, one should neither overindulge in the formal and logical strictness of proofs nor in spending a lot of time on cumbersome transformations and calculations. More attention should be paid to the understanding of concepts, properties, and ideas' meaning.

The third stage. Pragmatic application. Schoolchildren should acknowledge that the process of applying Mathematics to any practical problem solution includes: formalization (the transition from the situation described in the task, to the situation formal mathematical model, and from it, to a clearly formulated mathematical problem); solving the problem within the constructed model framework; interpretation (applying the solution to the original situation). In a separate block of tasks entitled "Apply in Practice" there are practice-oriented tasks, typical practical situations, where it is necessary to employ the material under study. These stages should be inherent in educational activities, since they foster students' creativity, activity and initiative development.

The enhancement of the course's practical application is facilitated by the students' familiarization with the concept of a mathematical model, with the method of mathematical

modeling, the development of ideas about the method's role in the scientific cognition and practice, the simpler mathematical model building skills formation.

Among the most important requirements for the textbook content the systematization of concepts, properties, and methods for solving tasks (tables, diagrams, tasks according to tables, classification) and the content integration (reinforcing the links of Algebra and Calculus with Geometry) should be mentioned. This will enhance the integrity of knowledge and improve its application to solving problems, in particular, of practical content.

Conclusions

The general theoretical and methodological requirements for the selection of educational texts, the system of tasks and the methodological apparatus of Mathematics textbooks for lyceums are offered, in particular: the formation of student personality's positive self-concept, a stable motivation to study the subject; the enrichment of schoolchildren's mathematical as well as general cultural potential; the scientific rigor and accessibility of textbooks; the educational texts scientificity and accessibility; the continuity observed in its two functions – the compensatory and the prognostic ones; the priority of the developmental learning function; practical orientation of educational material; the content meeting the students age and cognitive characteristics; the educational material systematization and integration. The compliance of the above-mentioned requirements to the Mathematics textbooks content, and of the suggested ways of their realization enhances the high-school students' quality mathematical education.