

Mathematical modelling as a means and method of problem solving in teaching subjects of branches of mathematics, biology and chemistry

Characterizing the concept of mathematical modelling, we find it appropriate to use as the working definition the one by A. Uyomov¹, who defines model as a system, research of which is a means of getting information about another system. In our situation the system is mathematical notions and their properties, which model environmental phenomena – these are the other systems, presented in Uyomov's definition. Learners actually start using the means as well as the method of mathematical modelling in primary school, but extensive definition with usage of scientific conceptual framework is given in accordance with school program on Algebra only in the 9th form while studying the topic 'Mathematical modelling'. The basis for implementing mathematical modelling as a means of learning is classification of phenomena of environmental character, known to learners due to studying subjects of chemistry and biological branches, which, in their turn, are researched on the basis of mathematical models of a certain kind.

While working on mathematics curriculum it is appropriate for a teacher to plan introducing models (drawings, schemes, tables) to pupils, which can help figuratively convey characteristics of modelled objects, their structure and connections in particular. Thus, for instance, while solving text problems, there appears an opportunity to introduce to pupils various figurative models (drawing, scheme, table with brief expression of conditions of a problem) and mathematical models (equations, equation systems, which are the base of problem solving). The knowledge of different kinds of models becomes for pupils a means of solving applied problems in general and problems with environmental content in particular.

While classifying mathematical notions (algebra, geometry) there is a possibility to involve pupils in creating peculiar models (tables, graphs, schemes) in order to establish existing connections between terms of a topic, unit. The contents of a problem can be helpful for a teacher in pointing motivation, for example, studying the topics 'Volume' in general and 'Volume of a pyramid' in particular. The knowledge, which pupils obtain about various material (real) models and how to apply them while defining notions, observing properties, solving problems, provides a possibility to observe characteristics of notions and to take them as the base for defining, or for formulating theorems, properties or features. Apart from that, the knowledge and skills mentioned above make the base of mathematical modelling as a means of pupils' active cognitive activities during lessons, and after school while doing homework.

Mathematical modelling as a means of cognition is used while forming notions, learning formulae, in particular, to calculate area of plane figures, surface and volume of spatial ones, and while forming skills for solving problems with different levels of difficulty (from standard to heuristic ones). Applied problems in general and those of environmental content in particular are one of the main means of this kind. While studying geometric figures, theorems, their properties and definitions, solving problems of models of geometric figures is a means that can help to create the base for environmental education, namely: to form the correct perception of certain mathematical notions and connections between them, spatial imagination. Drawings as modelling means play the most important part in problem solving. Let us take an illustrative example of solving an applied problem of environmental content, the model of which is the geometric figure pyramid.

¹ Grybyuk Olena. Mathematical modelling as a means of ecological education of pupils in the process of teaching mathematics in forms of specialized study of chemistry and biology. – Manuscript. Dissertation for a Candidate Degree in Pedagogical Science, speciality 13.00.02 – Theory and Methods of Teaching Mathematics. –National Pedagogical Dragomanov University. – Kyiv, 2011 – C.20

The problem. Human life and activities go along with huge amount of organic waste (household waste, sewage runoff, agricultural produce waste, and woodworking). Landfills around big cities occupy immense areas, pollute air and water. For instance, city landfill near New York has reached the volume of the Great Pyramid. Calculate the volume of this landfill, given that in the base of the Great Pyramid there is a square, which side is 227m, and the Pyramid's height is about 147m. (the answer: 63123025 m³). The computer aided solution of the problem on calculating the volume of the Great Pyramid is shown in the picture¹.

Let us consider various ways of teaching purpose of this problem in learning the topic 'Volume of a pyramid' (11th form). While forming the geometric notion, the following will be helpful: 1) problem conditions; 2) models of different kinds of pyramids to form the notion of volume and to deduce a formula for calculating volume of a pyramid; 3) image of a pyramid, which models the situation described above, and only after this pupils are able to solve the problem with the purpose of solving the environmental issue by the method of mathematical modelling.

In the course of studying mathematics we see models of geometric figures as a means of teaching, at geography lessons, respectively, these are cartographic projections, which is a means of depicting the earth surface on a plane. It is very inconvenient to project the earth surface onto a plane in separate parts, and for this reason there were generated tools, which help to project the earth surface or its part onto an auxiliary geometric surface, such as a cylinder, plane, and cone. Pupils 'research' the earth surface having different models – auxiliary projections.

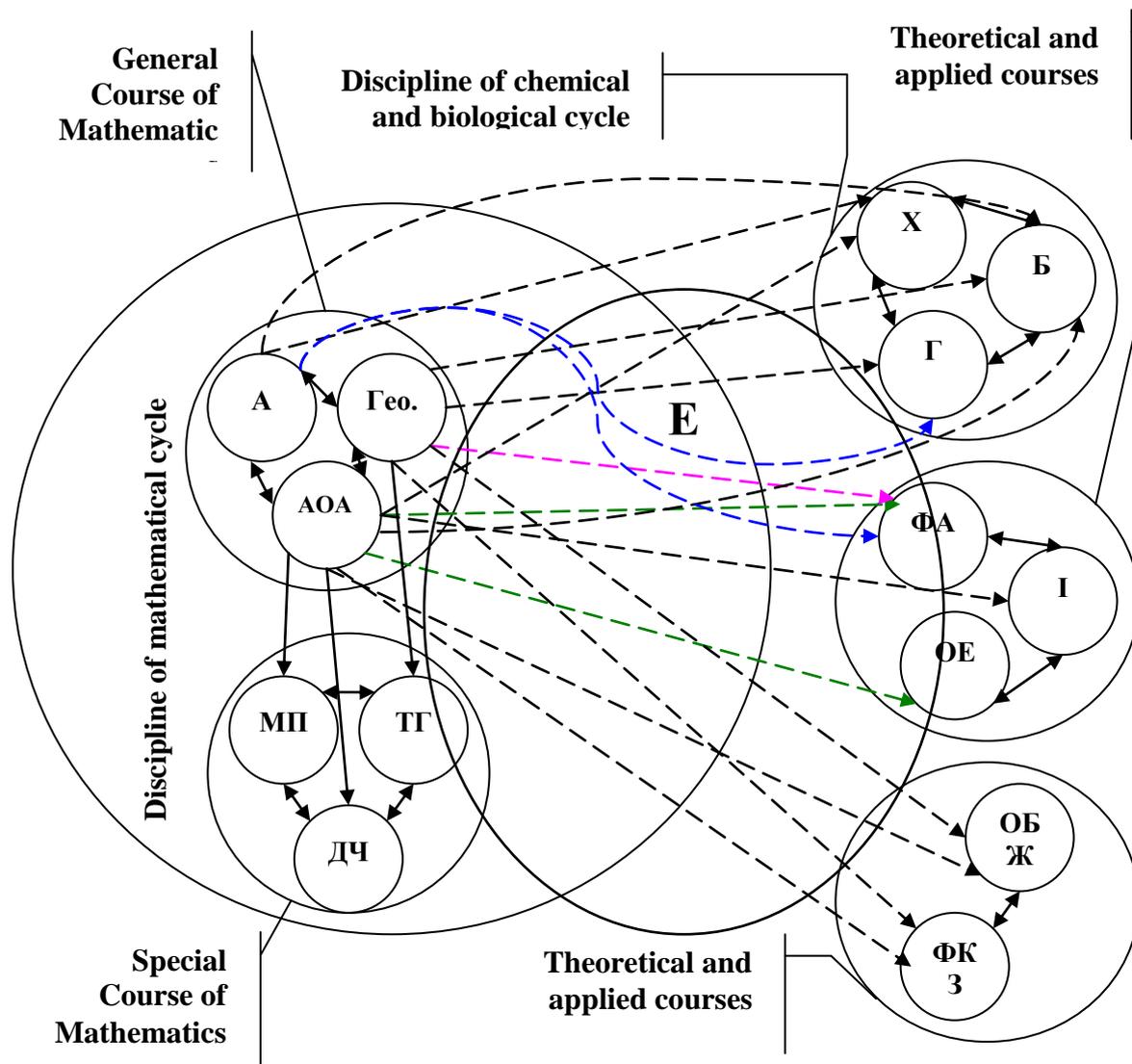
The opportunity to introduce different kinds of models of brief expression of conditions of applied problems to pupils appears in primary school and in 5-6th forms in the course of mathematics studies. For instance, solving an applied problem in the 5th form pupils get acquainted with three different models of a problem conditions. In school textbooks, which are in use now, the number of problems of environmental content is insufficient. We have analyzed the curriculum in subjects of branches of mathematics, biology and chemistry in order to define consistency in conceptual framework, which characterizes certain environmental phenomena, and knowledge about them. As a result, we came to the conclusion: pupils' knowledge about environmental phenomena and types of mathematical models at maths lessons are formed in the course of studying the following subjects: chemistry, botany, zoology, biology, geography, basics of ecology. The connections established between the subjects in the context of the problem being investigated are shown in the scheme interdisciplinary connections disciplines.

The level of pupils' knowledge on structural units of theoretical material helps to learn new mathematical models that are a must for effective use of the method of mathematical modelling at mathematics lessons as well as while studying subjects of branches of biology and chemistry.

Method of mathematical modelling (MMM) is in a way similar to proof by contradiction in geometry, on the base of which in order to form knowledge and skills in using the method of mathematical modelling we can use the conceptual approach, proposed by O. Pogorelov². The essence of the approach is in learning basic information about the method given and forming the skills to apply it in two stages. Thus, at the first **preliminary** stage, pupils are only introduced to the scheme of deductions in the method mentioned above by means of solving problems, at the second stage, there is **detailed** introduction to the method by means of learning conceptual framework, finding out the essence of the method, its **reference rules**.

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² Гриб'юк О. О. Математичне моделювання при навчанні дисциплін математичного та хіміко-біологічного циклів: навчально-методичний посібник для учителів. – Рівне: РДГУ, 2010. – 207 с.



A – Algebra, **АОА** – Algebra and the foundations of analysis, **Б** – Biology,
Г – Geography, **Geo.** – Geometry, **ДЧ** – Fundamentals of differential calculation,
Е – Ecology, **I** – Informatics, **МП** – Foundations of mathematical programming ,
ОБЖ – Basics of life safety, **OE** – Economic fundamentals, **ТГ** – Theory of the graphs,
ФА – Physics, Astronomy, **ФКЗ** – Physical Education, **X** – Chemistry.

Scheme interdisciplinary connections disciplines

In the case studied, the necessary conditions for studying and using the method of mathematical modelling are the following:

- 1) Knowledge about phenomena being investigated, which are the base of an applied problem to be solved;
- 2) Set of mathematical models, which might include the model, suitable for solving the given type of applied problem with environmental content;
- 3) Skills in operating mathematical notions which are used in creating mathematical models.

Using mathematical modelling as a means of forming notions (functions, equations and equation systems, various types of polyhedrons, solids of revolution) encourages the conditions mentioned to be fulfilled. It is appropriate to use this kind of models (material, ideal) on the one hand, with the aim of forming certain mathematical concepts, on the other hand, with the aim of studying the models that will be used at further stages, which is supposed to be the time for solving applied problems with environmental content using the method of mathematical

modelling. Furthermore, both coordinate and vector methods belong to the method of mathematical modelling.

The first preliminary stage is based on introducing the MMM scheme to pupils of up to the 9th form by means of solving applied problems, problems with environmental content in particular. Taking into consideration inter-subject connections between the subjects of branches of mathematics, biology and chemistry, this stage can be implemented while studying topics of the mentioned subjects, which are defined in tables¹.

The systematic stage starts with the first lesson on the topic ‘Mathematical modelling’ (Algebra, 9th form), where it is aimed to present the general concept of the method, learn its content and its reference rule.

Regarding the MMM, we find it appropriate to point out the third stage of **conscious usage of the method of mathematical modelling (MMM)**, which should continue in high school and which implies solving applied problems not only with environmental content. We also see here the usage of the method while carrying out studies of ecology and environment on the scale of a city, town, street, school, house and introducing their solutions by means of graphic calculations.

The number of possibilities to use the MMM in high school grows due to a increasing set of mathematical models (from expressions, analytical functions, equations, inequalities and their systems to transcendental ones, from geometric figures on plane to spatial ones). Necessity to use the MMM spreads out to phenomena, studied in the course of inorganic chemistry, anatomy, zoology, botany. It is appropriate to put the **algorithmic approach** into the base of studying the MMM. Given that at the first stage of solving applied problems **inductive method** is applied, starting from the second stage, the base of their solving is **deductive method** (from the reference rule of the MMM to its application in a specific situation).

Let us characterize learning activities of pupils at each of these stages (preliminary, systematic, and the stage of conscious usage of the method of mathematic modelling), which ensures assimilation of the MMM.

At the preliminary stage it is reasonable to solve applied problems in general and problems with environmental content in particular, to instruct pupils step by step using the same scheme, which will become the base of content and reference rule of the MMM.

The analysis of the scheme of the method of mathematical modelling shows that pupils are ready to put into practice every position of this scheme due to solutions of applied problems. For instance, at the preliminary stage (4th-9th form) these are the following mental activities:

- At algebra lessons: while solving text problems by equation method, while studying various types of functions, by means of showing functional connections between different types of values (analytically, in tables, graphically);
- At geometry lessons: while solving problems by the test of equality and similarity of triangles, trigonometric functions, methods of coordinates and vector).

To make possible these two mental actions, which are in the base of the method of mathematical modelling, we consider direct and inverse problems.

The modelling process, with the aim of encouraging development of thinking abilities and creativity of pupils, research skills, awakes pupils’ interest in objects by the example of innovative solutions of mathematical problems, using differential and personal approach, encourages self-actualization and growth of creativity of pupils.

It is especially true about the first step, which is about the choice of observing of modelling process and the second one, which implies building informal model. These steps show the difference between the modelling process and the method of mathematical modelling. It is

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convenient to involve pupils in such an activity at an integrated lesson, which combines mathematics with one or more subjects of branches of biology or chemistry and is conducted by two teachers.

The technology of conducting such an integral lesson is creative perception of curriculum material from the point of view of different sciences and formation of generalized picture of phenomena being studied, that is, in our case, when related material of branches of mathematical, biological sciences and chemistry is combined. Conducting such lessons gives pupils an opportunity to get better acquainted with the curriculum material, provides inter-subject connections, develops intellectual, creative skills of pupils, and performs their environmental education. For example, notions of symmetry, proportion, scale are presented at the lessons of mathematical, as well as biological and chemistry branches. Both coordinate and vector methods are also components of the method of mathematical modelling. High school pupils should be ready to use the MMM both at the level of solving applied problems in general and problems with environmental content in particular, and besides, to perform modelling as a process of research of a certain environmental phenomenon, which might not be presented in a form of a specific problem. Regarding the stages of the modelling process, pupils are ready to implement the 1st-3rd and the 5th stages.

Pupils are introduced to the scheme of arguments with the help of the MMM at the preliminary stage in the course of solving applied problems. The topic 'Mathematical modelling' in the 9th form provides more detailed introduction of the method of mathematical modelling to pupils, with the help of mathematical models, for instance, algebraic equations for solving problems on environmental topics that are based on knowledge, obtained in the course of zoology, botany etc.

We can see the appropriateness of the first lesson on the topic 'Mathematical modelling', which is aimed at learning the conceptual framework of this topic and detailed learning of the MMM as an efficient means of solving applied problems, and provided that these problems are on environmental topics, also a means of providing environmental education. The lesson must be of generalizing and systematizing type, the one that broadens knowledge. Knowledge broadening, firstly, is meant to happen by means of introducing the notions of 'model', 'mathematical model', 'mathematical modelling', 'applied problem'¹. At the stage of conscious use of the method of mathematical modelling it is appropriate to suggest high school pupils perform graphic calculations and study natural phenomena, after having analyzed topics from textbooks on chemistry, biology, basics of ecology, and to point out mathematical notions, which were used, describing environmental, chemical, biological phenomena. Thus a teacher involves pupils in inter-subject connections. With such an approach, there is an opportunity to research a number of environmental problems while studying mathematics.

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